Transmission Network Expansion Planning:
Bridging the Gap between AC Heuristics and DC Approximations

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***Abstract*—It was recently observed that a significant gap exists between the costs of Transmission Network Expansion Planning (TNEP) solutions produced by the DC power flow approximation and AC power flow heuristics. This paper confirms the existence of that gap and shows that DC-based TNEP solutions exhibit significant constraint violations when converted into AC power flows. The paper then demonstrates how to bridge this gap with the LPAC power flow model, a linear-programming approximation of the power flow that captures reactive power and voltage magnitudes. Indeed, LPAC-based TNEP solutions have minimal violations in AC power flows and provide high-quality solutions. The strength of the LPAC formulation is further demonstrated on the joint optimization of line expansion and VAr compensation, as well as a competitive market study. These studies suggest that the underling TNEP formulation has significant impacts on the proposed expansion plans.**

Keywords*—*Power System Planning, AC Power Flow, LPAC Power Flow, DC Power Flow, Nonlinear Programming, Optimization methods

NOMENCLATURE

$\widetilde{V} = v + i\theta$	AC voltage
$\widetilde{S} = p + iq$	AC power
$\widetilde{Y} = g + ib$	Line admittance
$\widetilde{V} = \lvert\widetilde{V}\rvert\angle\theta^{\circ}$	Polar form
$\widetilde{T} = \lvert\widetilde{T}\rvert\angle\theta^{s}$	Voltage Transformer
$\mathcal{PN}$	Power network
N	Set of buses in a power network
L	Set of lines $\langle n, m\rangle$ in a power network where n is the from node
L^r	Set of lines $\langle n, m\rangle$ in a power network where n is the to node
L^+	Set of new lines $\langle n, m\rangle$ that can be added to the power network
$L(n)$	Set of buses connected to bus n by a line
s	Slack bus
$\overline{x}$	Upper bound of x
$\underline{x}$	Lower bound of x
$\Re()$	Real part of a complex number
$\Im()$	Imaginary part of a complex number

I. INTRODUCTION

Transmission Network Expansion Planning (TNEP) is a well-studied optimization problem which consists of finding the least expensive way of increasing the capacity of a transmission network to meet some projected future energy delivery requirements. Due to its computational complexity, TNEP problems have been traditionally studied with active-power-only approximations of the transmission system [1]–[5]. Such approximations are appealing as they yield Mixed Integer Linear Programs (MIPs), which exploit decades of research in network design optimization and existing commercial tools [6].

Many TNEP studies have adopted the widely-used DC power flow model. However, its applicability for power flow applications is a point of significant discussion: Some papers take an optimistic outlook, (e.g., [7], [8]) while others are more pessimistic (e.g., [9], [10]). For TNEP applications, some major shortcomings of DC model were identified in [11]. Recognizing these potential limitations, recent work has started to consider the TNEP problem with the complete AC power flow equations [11]–[16].

This paper explores the use of the LPAC model [17] for transmission planning. The LPAC model was proposed recently for approximating the AC power flow equations. Contrary to the DC model, the LPAC approximation captures both reactive power and voltage magnitudes; Yet it is linear program which is highly desirable computationally. The LPAC model thus provides an appealing tradeoff between computational efficiency and solution accuracy: It was instrumental in identifying new best solutions on existing test cases and bridging the significant gap between DC-based models and AC heuristics on transmission planning. The key findings in this paper are summarized as follows:

1) DC-based transmission planning significantly underestimates the cost of expansion; The resulting plans, when converted into an AC plan, exhibits large violations of line capacities and voltage bounds.
2) LPAC-based transmission planning provides AC plans with little or no violations and more reasonable expansion costs; Yet the use of the LPAC model may significantly improve the quality of AC-based heuristics.
3) The results in (1) and (2) still hold when VAr compensation is considered, including in the case where each bus has unlimited reactive power injection.
4) The LPAC model can be used for the joint optimization of line and VAr compensation costs, steps which are typically separated in prior work.

Model 1 The Core AC-TNEP Problem.

Inputs:

$\langle N, L, s\rangle$	- the power network
$\underline{\lvert\widetilde{V}_n\rvert}, \overline{\lvert\widetilde{V}_n\rvert}$	- voltage limits for bus n
$\overline{p}_n^g$	- active limits for bus n
$\underline{q}_n^g, \overline{q}_n^g$	- reactive limits for bus n
p_n^l, q_n^l	- demands at bus n
g_{nm}, b_{nm}	- admittance of line $\langle n, m\rangle$
$\overline{\lvert\widetilde{S}_{nm}\rvert}$	- thermal limit of line $\langle n, m\rangle$
c_{nm}	- cost of expanding corridor $\langle n, m\rangle$
$\overline{z}$	- maximum expansions in a corridor

Variables:

$\widetilde{V}_n$	- voltage at bus n
$p_n^g \in (0, \overline{p}_n^g)$	- active generation on bus n
$q_n^g \in (\underline{q}_n^g, \overline{q}_n^g)$	- reactive generation on bus n
$p_{nm} \in (-\overline{\lvert\widetilde{S}_{nm}\rvert}, \overline{\lvert\widetilde{S}_{nm}\rvert})$	- active flow on line $\langle n, m\rangle$
$q_{nm} \in (-\overline{\lvert\widetilde{S}_{nm}\rvert}, \overline{\lvert\widetilde{S}_{nm}\rvert})$	- reactive flow on line $\langle n, m\rangle$
$z_{nm} \in \{0, ..., \overline{z}\}$	- expansion variable for line $\langle n, m\rangle$

Minimize:

$$\sum_{\langle n,m\rangle \in L} c_{nm}(z_{nm} - 1) + \sum_{\langle n,m\rangle \in L^+} c_{nm} z_{nm} \tag{M1.1}$$

Subject To:

$$\Im(\widetilde{V}_s) = 0 \tag{M1.2}$$

$$\underline{\lvert\widetilde{V}_n\rvert} \le \lvert\widetilde{V}_n\rvert \le \overline{\lvert\widetilde{V}_n\rvert};\ \forall n \in N \tag{M1.3}$$

$$p_n^g - p_n^l = \sum_{m \in L(n)} z_{nm} p_{nm} \quad \forall n \in N \tag{M1.4}$$

$$q_n^g - q_n^l = \sum_{m \in L(n)} z_{nm} q_{nm} \quad \forall n \in N \tag{M1.5}$$

$$1 \le z_{nm} \quad \forall \langle n, m\rangle \in L \tag{M1.6}$$

$\forall \langle n, m\rangle \in L \cup L^r \cup L^+ \cup L^{+r}$

$$p_{nm} = g_{nm}\lvert\widetilde{V}_n\rvert^2 - g_{nm}\Re(\widetilde{V}_n\widetilde{V}_m^*) - b_{nm}\Im(\widetilde{V}_n\widetilde{V}_m^*) \tag{M1.7}$$

$$q_{nm} = -b_{nm}\lvert\widetilde{V}_n\rvert^2 + b_{nm}\Re(\widetilde{V}_n\widetilde{V}_m^*) - g_{nm}\Im(\widetilde{V}_n\widetilde{V}_m^*) \tag{M1.8}$$

$$p_{nm}^2 + q_{nm}^2 \le \overline{\lvert\widetilde{S}_{nm}\rvert}^2 \tag{M1.9}$$

Additionally, the paper also suggests that, given the LPAC models accuracy, it may be informative to revisit previous TNEP studies. This point is illustrated on a simple case study about the implications of competitive markets for the TNEP.

The rest of the paper is organized as follows. Section II introduces the TNEP problem and the notations. Section III discusses potential solution methods, including prior art, and uses a 3-bus example to illustrate the tradeoffs of the approaches. The next three sections conduct the case studies: Section V establishes the baseline by considering the classic TNEP formulation; Section VI considers the implications of VAr compensation; and Section VII studies the implications of a competitive market. Section VIII concludes the paper.

II. The TNEP Problem

From a mathematical perspective, the TNEP task is a network design problem that aims at modifying the network topology to increase the total capacity and supply the necessary power. Flow conservation, ensured by Kirchhoff's Current Law (KCL), is the same as in classic network design problems. However, Ohm's Law, which governs power flows, is a departure from classic network design problems. A TNEP formulation based on the nonlinear AC power flow equations is presented in Model 1. The input data and decision variables are described in the model and hence only the constraints are discussed in detail. The objective (M1.1) minimizes the costs of building all of the transmission lines, discounted for the lines that already exists. Constraint (M1.2) fixes the slack-bus phase angle to 0, which simplifies the comparison between different models. Constraints (M1.3) ensure reasonable voltage magnitude limits are enforced. Constraints (M1.4–M1.5) ensure flow conservation and constraints (M1.6) ensure existing lines stay in the network. Constraints (M1.7–M1.8) capture Ohm's Law and constraints (M1.9) enforce the thermal line loading limits. Implicitly, the TNEP problem assumes that the total power demand of the network $\left(\text{i.e., } \sum_{n \in N} p_n^l\right)$ cannot be satisfied by the current network topology. Hence, new lines must be introduced.

A. Extensions

Model 1 is a natural extension of the DC-TNEP model to the AC power flow equations. However, classic power flow test cases [18] feature additional parameters such as (1) line charging b_{nm}^{sh}; (2) bus-shunts $\widetilde{Y}^s = g^s + ib^s$; and (3) Voltage transformers $\widetilde{T}$. As suggested by [14], for improving the AC power flow accuracy, Model 1 can be modified to incorporate these values as follows (only reactive power equations are shown, active power is similar).

Bus Shunts:

$$q_n^g - q_n^l + b_{nm}^s\lvert\widetilde{V}_n\rvert^2 = \sum_{m \in L(n)} z_{nm} q_{nm}$$

Line Charging & Voltage Transformers:

$$q_{nm} = -\left(b_{nm} + \frac{b_{nm}^{sh}}{2}\right)\frac{\lvert\widetilde{V}_n\rvert^2}{\lvert\widetilde{T}_n\rvert^2} + b_{nm}\Re\left(\frac{\widetilde{V}_n\widetilde{V}_m^*}{\widetilde{T}_n^*}\right) - g_{nm}\Im\left(\frac{\widetilde{V}_n\widetilde{V}_m^*}{\widetilde{T}_n^*}\right)$$

$$q_{mn} = -\left(b_{nm} + \frac{b_{nm}^{sh}}{2}\right)\lvert\widetilde{V}_m\rvert^2 + b_{nm}\Re\left(\frac{\widetilde{V}_m\widetilde{V}_n^*}{\widetilde{T}_n}\right) - g_{nm}\Im\left(\frac{\widetilde{V}_m\widetilde{V}_n^*}{\widetilde{T}_n}\right)$$

These additions may not fully capture the detailed components of modern AC power networks (such as changing transformer settings and FACTS devices), but they represent the most detailed model with readily available network data.

III. Prior Work and Solution Methods

The AC-TNEP problem presented in Model 1 (and its extensions) are challenging Mixed Integer Non-Convex Non-Linear Programs (MINLP) and are outside the scope of current global optimization solvers. As a result, one must resort to solving alternative, computationally tractable versions of the problem. There are three main approaches to making the AC-TNEP more tractable: (1) using heuristics [11], [14], [15]; (2) approximating the power flow equations [1], [17]; (3) relaxing the power flow equations to a convex set [12], [13], [16]. Heuristics are often very fast to compute, but they provide no quality guarantees. Approximations can alter the computational complexity (e.g., moving from a MINLP to a MIP) and provide quality guarantees within the confines of the approximation. In practice, approximations may be sufficiently accurate to provide feasible solutions to the original problem, but they provide no such guarantees. In the context of the AC-TNEP problem, it was recently demonstrated that feasibility can be quite challenging when using the DC power flow approximation [11]. Relaxations provide provable dual bounds to the original problem. In the context of the AC-TNEP, relaxations compute an optimistic value for the number of lines required to meet the future demands. The rest of this section introduces various heuristics, relaxations, and approximations of the AC-TNEP problem, and evaluates them on a simple 3-bus example to illustrate their properties. All the models presented below rely on a continuous relaxation of the discrete variable z_{nm} and use the *binarization* of z_{nm} from [13].

A. Heuristics

Several heuristics have been proposed [11], [14], [15] for solving the AC-TNEP. In contrast to these constructive heuristics, we consider a destructive heuristic HAC-TNEP which maintains feasibility in each iteration. HAC-TNEP outperforms the constructive heuristic proposed in [11], produces high-quality primal solutions, and thus provides a good basis for comparison. HAC-TNEP starts with all of the lines in the network (there are sufficiently many lines to guarantee feasibility). On each iteration, HAC-TNEP attempts to remove one line from the network and tests if the network is still feasible in an AC Optimal Power Flow model (AC-OPF). Lines are selected in increasing order of relative loads. HAC-TNEP completes when no line can be removed.

B. DC Power Flow

The DC model is a popular approximation of the AC power flow model motivated by design and operational considerations. It uses the polar voltage formulation of the AC power flow equations, i.e., $\widetilde{V} = |\widetilde{V}|\angle\theta^\circ$, and makes the following simplifications: (1) the susceptance is large relative to the conductance $|g| \ll |b|$; (2) the phase angle difference is small enough to ensure $\sin(\theta^\circ_n - \theta^\circ_m) \approx \theta^\circ_n - \theta^\circ_m$ and $\cos(\theta^\circ_n - \theta^\circ_m) \approx 1.0$; and (3) the voltage magnitudes $|\widetilde{V}|$ are close to 1.0 and do not vary significantly. Under these assumptions, equations (M1.7–M1.8) reduce to

$$p_{nm} = -b_{nm}(\theta^\circ_n - \theta^\circ_m) \tag{1}$$

and the resulting model is called the DC-TNEP formulation. Due to the voltage approximation, this formulation cannot capture any of the extensions presented in Section II-A. From a computational standpoint, the DC model is much more appealing than the AC power flow: It forms a system of linear equations and can be naturally embedded in Mixed-Integer Linear Programs (MIPs). For this reason, many TNEP works have focused on the DC-TNEP variant [1]–[4]. The DC-TNEP model however has no notion of line losses, reactive power flow, or bus voltage magnitudes. As noticed in [11], these inaccuracies may make it difficult to convert DC-TNEP solutions to AC-TNEP solutions.

C. LPAC Power Flow

The LPAC power flow model [17] bridges the gap between the DC power flow and the AC power flow model without an increase in computational complexity (i.e., it can still be embedded in MIPs). The LPAC model approximates the AC power flow equations (also in the polar voltage formulation) with following modifications: (1) $\sin(\theta^\circ_n - \theta^\circ_m) \approx \theta^\circ_n - \theta^\circ_m$ (2) the voltage magnitude at each bus is based on the deviation form a nominal voltage $|\widetilde{V}| = 1.0 + \phi$; (3) the non-convex cosine function is replaced with a polyhedral relaxation ($\widehat{\cos}_{nm}$); (4) the remaining non-linear terms are factored and approximated with a first order taylor expansion. These modifications yield the following power flow equations:

$$p_{nm} = g_{nm} - g_{nm}\widehat{\cos}_{nm} - b_{nm}(\theta^\circ_n - \theta^\circ_m)$$
$$q_{nm} = -b_{nm} + b_{nm}\widehat{\cos}_{nm} - g_{nm}(\theta^\circ_n - \theta^\circ_m) - b_{nm}(\phi_n - \phi_m)$$

and the resulting model is called the LPAC-TNEP formulation. The LPAC model captures line losses, reactive power flows, and an approximation of bus voltage magnitudes and is significantly more accurate than the DC power flow model. The LPAC-TNEP can also incorporate the extensions discussed in Section II-A. The line loading constraint (M1.9) can be embedded in the LPAC-TNEP through a polyhedral outer approximation.

D. SOCP Power Flow

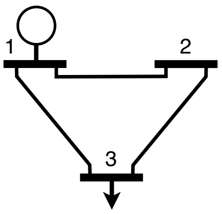
A Second Order Cone Problem (SOCP) relaxation of the rectangular power flow equations was proposed in [19]. This was used in [13], [16] to build a SOCP-TNEP formulation. This formulation is appealing for two reasons: (1) High-quality industrial solvers exist for SOCPs; and (2) it is a relaxation and can be used for bounding the AC-TNEP problem. The primary disadvantage of the SOCP formulation is that it assumes the network has a sufficient number of *virtual* phase-shifting transformers (at least one for every cycle in the network). This assumption means that the SOCP formulation degenerates into a transportation model at the limit. In addition to extending [19] to TNEP, the formulation in [13] also adds an $\epsilon = \pi/720$ term for limiting the effects of the virtual phase-shifting transformers. It is not clear, however, whether the resulting formulation is still a relaxation. In this paper, we refer to this modified SOCP model as SOCP*-TNEP. The SOCP*-TNEP formulation in [13] included the line charging extension, but bus shunts and transformers were not mentioned.

E. Comparing Expansion Plans

An expansion plan produced by any approximation or relaxation of the AC-TNEP problem can be infeasible for the original AC-TNEP problem (i.e., Model 1). To compare expansion plans, their constraint violations can be measured by computing an AC power flow in the expanded network for the generator dispatches and the voltage magnitudes obtained in the solution (the voltage magnitudes are fixed to 1.0 in the DC-TNEP). Two types of constraint violations are used to characterize the quality of an expansion plan: Line capacity violations and voltage magnitude violations. Line capacity violations measure how overloaded a line is. The relative capacity violation of a line $\langle n, m\rangle$ is the maximum relative violation at either side of the line:

$$\max\left(\frac{\sqrt{p^2_{nm} + q^2_{nm}}}{|\widetilde{S}_{nm}|}, \frac{\sqrt{p^2_{mn} + q^2_{mn}}}{|\widetilde{S}_{mn}|}\right)$$

where p_{nm} and q_{nm} denote the real and reactive power on the line in the AC power flow. Obviously the line is overloaded



Bus Parameters

bus	p^l	q^l	$\lvert\underline{\widetilde{V}}\rvert$	$\lvert\overline{\widetilde{V}}\rvert$
1	0	0	0.9	1.1
2	0	0	0.9	1.1
3	500	0	0.9	1.1

Line Parameters

From–To bus	R	X	b	Cost
1–2	0.00	0.05	0.01	1
2–3	0.00	0.05	0.01	1
1–3	0.05	0.10	0.01	1

Fig. 1. Three-Bus System Diagram and Network Data (100 MVA Base).

TABLE I. THREE-BUS SYSTEM AC POWER FLOW SOLUTION.

Bus Values					Line Values		
Bus	p^g	q^g	$\lvert\widetilde{V}\rvert$	$\theta°$, deg.	Line	From MVA	To MVA
1	528	126	1.100	0.000	1–2	294	281
2	0	0	1.048	-6.597	2–3	281	271
3	0	0	1.011	-13.78	1–3	263	242

when this value is greater than 100%. Voltage magnitude violations are defined as

$$\max\left(0, |\underline{\widetilde{V}_n}| - |\widetilde{V}_n|, |\widetilde{V}_n| - |\overline{\widetilde{V}_n}|\right)$$

which captures violations above or below the desired limits.

IV. A Case Study on TNEP Solution Methods

This section studies the behavior of each TNEP solution method on a simple 3-bus example. The network and its parameters are presented in Figure 1: It is designed so that the power flows on the paths 1–3 and 1–2–3 are roughly the same. This is illustrated on the AC load flow study presented in Table I. To understand the properties of the methods, two variants of this simple 3-bus system are considered: One with a low capacity line (5MVA thermal limit on 2–3) and one with tight voltage magnitude bounds (± 0.01 V p.u.). The results of each TNEP solution method on these two cases are compared to our best-known AC feasible solution (AC*-TNEP) in Tables II and III. The quality of the solutions produced by the approximation and relaxation methods are evaluated using the constraint violations metrics discussed previously. We now review the results of each method in detail.

In the low capacity case (Table II), AC*-TNEP features an expansion plan with 27 additional lines. Because the approximate methods effectively capture line congestion caused by cycles in the network, both LPAC-TNEP and DC-TNEP correctly identify expansions with 27 lines. In contrast, the SOCP model does not capture cycle-based line congestion and SOCP-TNEP greatly underestimates the expansions needed (only expanding 1 line) and leads to huge violations on line 2–3. SOCP*-TNEP does better by adding 19 lines but still exhibits significant violations. Since similar trends were observed on the SOCP*-TNEP solutions to the other benchmarks studied in this paper, this solution method is not discussed further.

The results on the tight voltage magnitude bound case in Table III are entirely different. AC*-TNEP is an expansion plan requiring 5 expansions. DC-TNEP has no notion of the voltage magnitudes; Hence it adds no lines and exhibits significant voltage violations. The SOCP model proposes only 2 expansions, producing a network with less severe voltage violations. The remaining methods (HAC-TNEP, SOCP*-TNEP, and LPAC-TNEP) all suggest adding 5 lines and have no violations.

TABLE II. TNEP RESULTS ON THE THREE-BUS SYSTEM WITH A 5MVA THERMAL LIMIT ON LINE 2–3.

	Cost	Investment			Capacity Violation (MVA)
		1–2	2–3	1–3	
AC*-TNEP	27	0	15	12	0
HAC-TNEP	72	0	72	0	0
SOCP*-TNEP	19	0	8	11	31.3
SOCP-TNEP	1	0	0	1	184
LPAC-TNEP	27	0	15	12	0
DC-TNEP	27	0	14	13	0

TABLE III. TNEP RESULTS ON THE THREE-BUS SYSTEM WITH TIGHT VOLTAGE BOUNDS (± 0.01 V P.U.).

	Cost	Investment			Voltage Violation (V p.u.)
		1–2	2–3	1–3	
AC*-TNEP	5	2	3	0	0
HAC-TNEP	5	3	2	0	0
SOCP*-TNEP	5	2	3	0	0
SOCP-TNEP	2	1	1	0	0.0256
LPAC-TNEP	5	2	3	0	0
DC-TNEP	0	0	0	0	0.1567

In summary, on this 3-bus network, LPAC-TNEP provides an appealing tradeoff of accuracy and computational complexity: It finds the best-known AC solution in both cases. Similar observations also hold for the other benchmarks discussed in this paper. Hence, although there are many options for solving the AC-TNEP problem, we selected LPAC-TNEP as a reasonable approximation of the challenging AC-TNEP MINLP for the remainder of the paper. LPAC-TNEP will be compared with HAC-TNEP and DC-TNEP.

V. Evaluation on Classic Test Systems

The three most popular TNEP test cases are networks with 6, 24, and 46 buses (from [1], [3], [5] respectively). Since there are some variations on these test cases in the literature, we review our versions in detail. Our 6-bus benchmark corresponds to the one from [14] with bus 6 as the slack bus and $0.95 \leq |\widetilde{V}_n| \leq 1.05$. Our 24-bus case is from MATPOWER's distribution case files with cost data from [14], $b^{sh}_{6,10}/100$, and the load increasing strategy discussed next. Our 46-bus case is from [5] with bus 16 as the slack bus. The reactive injection capacity of generators is set to half of the active capacity, and line resistances are assigned one fifth of the reactance.

To understand the TNEP problem on a wide collection of networks, we design the following procedure for generating TNEP instances from any MATPOWER OPF test case. The loads and the capacity of generating units are scaled by a factor of 3 (except for synchronous condensers). The reactive injection capacity of each generator is set to half of the active capacity (i.e., $\underline{q}^g_n = -0.5 \cdot \overline{p}^g_n, \overline{q}^g_n = 0.5 \cdot \overline{p}^g_n$). For lines with a capacity set to 9900 MVA, we use the value of the line loaded at a phase angle difference of 15 degrees instead. Finally, the cost of adding each line is set to 1 (i.e., the goal is to minimize the number of lines added). This procedure is used to build the 9, 14, 30, 39, 57, and 118 bus TNEP problems from the MATPOWER cases.[1] In all of our benchmarks, expansions can select up to 6 lines per corridor ($\overline{z} = 6$) and their quality

[1]The voltage constraints on case 57 dominate this network and we widen the bounds to $0.9 \leq |\widetilde{V}_n| \leq 1.1$ to make the optimization task interesting.

TABLE IV. SOLUTIONS PRODUCED BY HAC-TNEP, DC-TNEP, AND LPAC-TNEP

	HAC-TNEP	DC-TNEP			
			Capacity vio.		Vol. vio.
Case	Cost	Cost	Max (%)	Avg. (%)	Max
6	160 (6)	110 (4)	137.32	(5) 121.90	0.13188
24	2310 (43)	152 (5)	AC-PF did not converge		
46	569810 (47)	89889 (9)	AC-PF did not converge		
9	3 (3)	2 (2)	136	(5) 124.07	0.18994
14	15 (15)	1 (1)	120.44	(2) 116.17	0.11955
30	13 (13)	5 (5)	192.86	(16) 128.21	0.06847
39	47 (47)	26 (26)	113.46	(8) 108.21	0.08713
57	49 (49)	1 (1)	AC-PF did not converge		
118	37 (37)	5 (5)	172.03	(17) 113.36	0.08198

	LPAC-TNEP			
		Capacity vio.		Vol. vio.
Case	Cost	Max (%)	Avg. (%)	Max
6	130 (5)	0	(0) 0.00	0.00347
24	689 (17)	101.02	(1) 101.02	0.00435
46	310688 (29)	102.96	(1) 102.96	0
9	2 (2)	104.45	(1) 104.45	0.01587
14	4 (4)	0	(0) 0.00	0
30	10 (10)	110.84	(5) 103.55	0.00303
39	28 (28)	103.01	(10) 100.80	0.01368
57	39 (39)	111.15	(2) 106.98	0.01785
118	7 (7)	116.97	(13) 105.96	0.00259

TABLE V. SOLUTIONS PRODUCED BY HAC-TNEP, DC-TNEP, AND LPAC-TNEP WITH LINE CAPACITIES REDUCED BY 10%.

	HAC-TNEP	DC-TNEP			
			Capacity vio.		Vol. vio.
Case	Cost	Cost	Max (%)	Avg. (%)	Max
6	160 (6)	130 (5)	105.91	(3) 105.91	0.05778
24	2378 (44)	266 (8)	AC-PF did not converge		
46	569810 (47)	130110 (13)	AC-PF did not converge		
9	3 (3)	2 (2)	126.24	(3) 121.07	0.17817
14	15 (15)	1 (1)	0	(0) 0.00	0.11628
30	21 (21)	8 (8)	158.68	(13) 119.80	0.07081
39	48 (48)	32 (32)	108.32	(1) 108.32	0.07016
57	50 (50)	4 (4)	AC-PF did not converge		
118	39 (39)	7 (7)	158.42	(6) 120.16	0.07591

	LPAC-TNEP			
		Capacity vio.		Vol. vio.
Case	Cost	Max (%)	Avg. (%)	Max
6	130 (5)	0	(0) 0.00	0.00498
24	681 (15)	0	(0) 0.00	0.00484
46	316551 (30)	0	(0) 0.00	0.00097
9	3 (3)	0	(0) 0.00	0.01219
14	4 (4)	0	(0) 0.00	0
30	12 (12)	106.62	(2) 105.03	0.0057
39	34 (34)	0	(0) 0.00	0.00696
57	42 (42)	0	(0) 0.00	0.01268
118	11 (11)	112.99	(2) 107.70	0.00294

is evaluated using an AC power flow as discussed in Section III-E. Each model was executed on a 2 x 2.00GHz Intel Quad Core Xeon E5405 with 2x6MB Cache and 16 GB RAM using GUROBI 5.5 on 4 cores.

The HAC-TNEP heuristic was run until completion without a time limit, while the DC-TNEP and LPAC-TNEP algorithms were terminated after 2 hours. Our analysis considers the following metrics to characterize the quality of a TNEP solution (see Table IV): (1) the expansion cost (i.e., the objective value) with the number of lines added in parenthesis; (2) for the DC-TNEP and the LPAC-TNEP, the maximum and average line violations with the number of lines with violations in parenthesis, and (3) the maximum voltage bound violations. Note that the AC power flow algorithm (AC-PF) used to convert the DC-TNEP and the LPAC-TNEP plans into AC-feasible solutions is not guaranteed to converge to a solution. This is noted when it occurs.

Table IV shows that there is a significant gap in the cost between the TNEP solution found by the HAC-TNEP heuristic and the DC-TNEP models, confirming the results of [11]. The table also shows that DC-TNEP solutions have significant violations to the original AC-TNEP constraints. In three cases, the DC-TNEP solutions cannot be converted into a AC-feasible plan. In contrast to the DC-TNEP, the LPAC-TNEP solutions have significantly higher costs but also significantly less violations in both maximum and average values. The LPAC-TNEP model also produces significant improvements over the HAC-TNEP heuristic.

When using an approximate TNEP solution method, it is common to use a second corrective stage to eliminate violations [16]. Here we propose an alternative approach called *constraint tightening*, where the model constraints are modified before the solution process. For example, reducing the line capacity by 10% may mitigate small line loading violations and lead to AC-feasible solutions. Table V evaluates the quality of the DC-TNEP and LPAC-TNEP solutions when the line capacities are reduced by 10%. In the DC-TNEP model, the constraint-tightening procedure makes marginal improvements in the violations and the solutions still have major issues with AC-PF feasibility. In contrast, the constraint-tightening procedure has a great impact on the LPAC-TNEP solutions. It reduces the line violations in all cases and eliminates the violations in most cases. The violations occurring on the 30 and 118 cases can be reduced further by a more aggressive tightening than 10%.

Tables IV and V highlight three key points: (1) the DC-TNEP may significantly underestimate the expansion cost; (2) the LPAC-TNEP provides a nice compromise between accuracy and computational complexity; (3) constraint tightening is effective for eliminating line-loading violations in LPAC-TNEP. It is thus reasonable to conclude that LPAC-TNEP is an excellent vehicle for studies in transmission planning.

VI. TNEP WITH VAR COMPENSATION

The classic TNEP formulation presented in Section II assumes that transmission lines are the only components to be added to the network. Section IV indicated that tight voltage magnitude bounds may necessitate the addition of a significant number of lines. Although the AC-TNEP with voltage bounds is a common AC-TNEP formulation [11], [12], [16], tight voltage magnitude constraints may be unrealistic. As noted in [20], VAr compensation equipment is much cheaper than transmission lines and may be installed throughout a network to satisfy voltage magnitude bounds. Inspired by [20], this section investigates the transmission models assuming *unlimited* VAr compensation at every bus, which can be modeled by transforming every bus into a synchronous condenser with unlimited reactive injection capacity and a voltage set-point of 1.0. We call this model the perfect voltage profile AC power flow (AC-PVP) and our goal is to study how the DC-TNEP and LPAC-TNEP behave, and how much cheaper a TNEP solution might be in this context. The DC-PVP-TNEP has the same first step as the DC-TNEP: It is only when converting the resulting expansion plan that the VAr compensation plays a role. In contrast, the LPAC-PVP-TNEP exploits VAr compensation in

TABLE VI. SOLUTIONS PRODUCED BY HAC-PVP-TNEP, DC-PVP-TNEP, AND LPAC-PVP-TNEP

	HAC-PVP-TNEP	DC-PVP-TNEP			
			Capacity vio.		Vol. vio.
Case	Cost	Cost	Max (%)	Avg. (%)	Max
6	130 (5)	110 (4)	104.65	(4) 104.52	0
24	573 (10)	152 (5)	111.97	(9) 104.79	0
46	277592 (22)	89889 (9)	129.86	(10) 108.26	0
9	2 (2)	2 (2)	105.86	(4) 102.36	0
14	2 (2)	1 (1)	113.41	(2) 108.38	0
30	8 (8)	5 (5)	119.61	(9) 109.88	0
39	24 (24)	20 (20)	100.41	(7) 100.31	0
57	2 (2)	0 (0)	122.95	(3) 112.11	0
118	2 (2)	1 (1)	160.31	(17) 113.99	0

	LPAC-PVP-TNEP			
		Capacity vio.		Vol. vio.
Case	Cost	Max (%)	Avg. (%)	Max
6	130 (5)	0	(0) 0.00	0
24	218 (6)	0	(0) 0.00	0
46	128948 (13)	100.41	(1) 100.41	0
9	2 (2)	0	(0) 0.00	0
14	1 (1)	0	(0) 0.00	0
30	7 (7)	100.49	(2) 100.49	0
39	22 (22)	0	(0) 0.00	0
57	2 (2)	0	(0) 0.00	0
118	2 (2)	0	(0) 0.00	0

TABLE VII. POWER INJECTION FOR SOLUTIONS TO THE PERFECT VOLTAGE PROFILE BENCHMARKS.

	HAC-PVP-TNEP		DC-PVP-TNEP		LPAC-PVP-TNEP	
Case	P (MW)	Q (MVAr)	P	Q	P	Q
6	772.79	279.9	781.9	370.98	772.46	276.58
24	8819.93	4140.34	8802.53	3979.02	8758.64	3824.76
46	7313.82	3618.02	7806.4	5740.78	7549.26	4443.6
9	977.22	659.86	973.11	635.66	965.52	626.66
14	826.62	630.82	821.3	510.91	785.68	414.99
30	586.74	671.41	595.57	695.46	586.61	623.85
39	18932.56	8398.51	18937.74	8249.37	18937.61	8518.99
57	3890.33	1985.7	3926.14	2113.43	3900.46	1986.25
118	13170.39	7615.2	13249.42	8023.94	13047.29	7067.46

the first stage as well, since it models reactive power and voltage magnitudes.

Table VI revisits the evaluation of Section V with the AC-PVP model. Since the AC-PVP model integrates the nominal voltage assumption of the DC power flow, it is no surprise that the gap between the HAC-PVP-TNEP solution is much smaller than in Section V. However, despite the gap reduction, there are still significant violations in the DC-PVP-TNEP solutions. In contrast, the LPAC-PVP-TNEP does amazingly well and has almost no violations without constraint tightening. Although the cost of line expansion is greatly reduced in these PVP solutions, the amount of required reactive injection capability is significant: Table VII indicates that the total reactive injection capacity is roughly half the total active injection. Depending on the cost of VAr compensation devices, it may be advantageous to jointly optimize of expansion lines and VAr compensation as in [13], [15]. The main message however is that LPAC model is an excellent vehicle for these studies, given its accuracy and computational advantages.

The classic TNEP formulation and the PVP-TNEP formulation are special cases of a multi-objective optimization problem. The former assumes lines are cheap and VAr compensation is very expensive, while the later assumes the reverse. It is possible to enumerate some of the solutions along the Pareto Frontier to understand the tradeoff between line expansions and VAr compensation. Table VIII uses a scaling parameter

TABLE VIII. SOLUTIONS PRODUCED BY LPAC-PVP-TNEP ALONG THE PARETO FRONTIER ON THE 24 BUS CASE.

	LPAC-PVP-TNEP				
		Q injection	Capacity vio. (%)		Vol. vio.
λ	Cost	(MVar)	Max	Avg.	Max
0	10834 (204)	231.82	0	(0) 0.00	0
0.1	5609 (112)	1201.39	0	(0) 0.00	0
0.2	2310 (51)	1856.36	0	(0) 0.00	0
0.3	1642 (39)	2126.24	0	(0) 0.00	0
0.4	1376 (34)	2270.73	0	(0) 0.00	0
0.5	1214 (29)	2344.62	0	(0) 0.00	0
0.6	910 (21)	2511.46	0	(0) 0.00	0
0.7	745 (17)	2620.82	0	(0) 0.00	0
0.8	645 (15)	2672.72	0	(0) 0.00	0
0.9	573 (14)	2722.19	0	(0) 0.00	0
1	509 (13)	2810.23	0	(0) 0.00	0

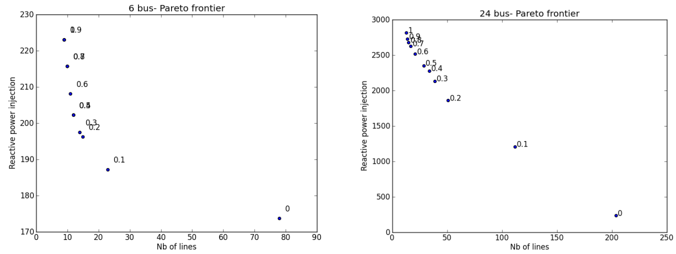


Fig. 2. Pareto frontier for 6 bus case (left) and 24 bus case (right).

λ to balance the tradeoff between expansion costs an VAr compensation. The classic TNEP formulation is captured by $\lambda = 1$ while an extreme VAr minimization model is captured by $\lambda = 0$. The Table illustrates that there is a huge range of network design possibilities spanning from as few as 13 new lines to an amazing 200 new lines. However, in all of these design possibilities, the LPAC-TNEP is producing high-quality solutions with no violations.

To illustrate the tradeoff of joint line and VAr expansion further, Figure 2 shows part of the Pareto frontier for the 6 and 24 bus cases as found using the LPAC-TNEP model. The non-linear shape of both plots suggests that the tradeoff of line expansions and VAr compensation is non-trivial and the resulting network expansion plan may be heavily influenced by the cost models of line expansion and VAr compensation. Hence, it is best to optimize these two quantities jointly (e.g., [13], [15]), possibly under several cost models.

VII. POWER MARKET CONSIDERATIONS

Since the LPAC-TNEP provides an appealing tradeoff between solution quality and computational efficiency, it can used for studies of more complex models of transmission expansion, producing results that can be trusted in contrast to the DC-TNEP. This section provides such an illustration. As observed in [1], the classic TNEP formulation presented in Section II is meaningful for a power system based on a regulated monopoly. However, its solutions may not be suitable for emerging competitive power markets, as physical transmission limitations may prevent competitive power generators from entering the market. In [1], an economic model was used to build four different competitive generation scenarios (called g1, g2, g3, and g4). Each scenario indicates the relative contribution of each generator to supplying the required demand (called contribution factors). These contribution factors amount

TABLE IX. LPAC-TNEP SOLUTIONS WITH CAPACITIES REDUCED BY 10%.

Case	LPAC-TNEP			
		Capacity vio.		Vol. vio.
	Cost	Max (%)	Avg. (%)	Max
24 g1	1065 (25)	0	(0) 0.00	0.00296
24 g2	1171 (29)	0	(0) 0.00	0.00133
24 g3	1007 (21)	0	(0) 0.00	0.00458
24 g4	1084 (23)	0	(0) 0.00	0.00038
24 gf	705 (15)	0	(0) 0.00	0.00562

to placing an additional constraint in Model 1 (a lower bound on the active generation p_n^g), and hence they can only increase the cost of the expansion plan. Consider, for instance, the application of these contribution factors to our 24 bus case and the effect on solution cost depicted in Table IX (the last line indicates the cost without contribution factors). The table illustrates that the cost of expansion can increase as much as 50% when a competitive market is introduced. This suggest that, as power systems move from monopolies to competitive markets, the TNEP must also move from the static TNEP (i.e., Model 1) to a more dynamic expansion model based on generation scenarios [1], incorporating an economic dispatch model or probabilistic formulations [21], [22] grounded in robust optimization.

VIII. CONCLUSION

This paper revisited the gap between the Transmission Network Expansion Planning (TNEP) solutions produced by using the DC power flow approximation and AC power flow heuristics. It was shown that the TNEP solutions produced by the DC power flow approximation significantly underestimate the expansion costs and have significant violations in the AC power flow model. The recent LPAC power flow model was proposed to bridge the gap between infeasible DC-TNEP solutions and AC heuristics. It was demonstrated that the LPAC-TNEP solutions have minimal constraint violations and, with a constraint tightening procedure, these violations can often be eliminated entirely.

The strength of the LPAC formulation was further demonstrated on additional studies on the joint optimization of line expansion and VAr compensation, as well as a competitive market study. The VAr planning study showed that the gap between the DC approximation and AC heuristics may be reduced through two-stage VAr planning, but significant AC violations still remain. In contrast, the LPAC-TNEP solutions have no violations in this two-stage approach. A study on the co-optimization of line expansions and VAr planning enabled by the LPAC-TNEP formulation illustrated the delicate balance of line and VAr cost models on the resulting expansion plans. Finally, a competitive market study indicated that the cost of the TNEP solutions may increase significantly to accommodate emerging energy markets.

Overall, this study indicates that the LPAC power flow model provides a good tradeoff between computational benefits and model accuracy for solving TNEP problems. Furthermore, it indicated that AC feasible expansion plans can be heavily influenced by the underlying TNEP formulation (e.g., adding VAr planning or market considerations). Great care should be taken in selecting an appropriate TNEP formulation for the study at hand. Future works utilizing the LPAC TNEP model should consider extending the models proposed here by incorporating network faults with recourse to ensure N–1 reliability as well as incorporating multiple generation and loading scenarios to produce more flexible and robust expansion plans.

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