# Models, Optimization and Control of Collective Phenomena in Power Grids

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Gainsville, Florida, Apr 28, 2011

# Outline

## So What? Impact! Savings!

- $30b\$$ annually is the cost of power (thermal) losses
- 10% efficiency improvement - $3b\$$ savings
- cost of 2003 blackout is $7 - 10b\$$
- $80b\$$ is the total cost of blackouts annually in US
- further challenges (more vulnerable, cost of not doing planning, control, mitigation)

## Grid is being redesigned [stimulus]

- The research is timely: $\sim 2T\$$ in 20 years (at least) in US
- Renewables - Desirable but difficult to handle
- Integration within itself, but also with Other Infrastructures, e.g. Transportation (Electric Vehicles)
- Tons of Interesting (Challenging) Research Problems !

# What is Smart Grid?

**Traditional Power Engineering** (power flows)

+

**New Hardware** (more options, more fluctuations)

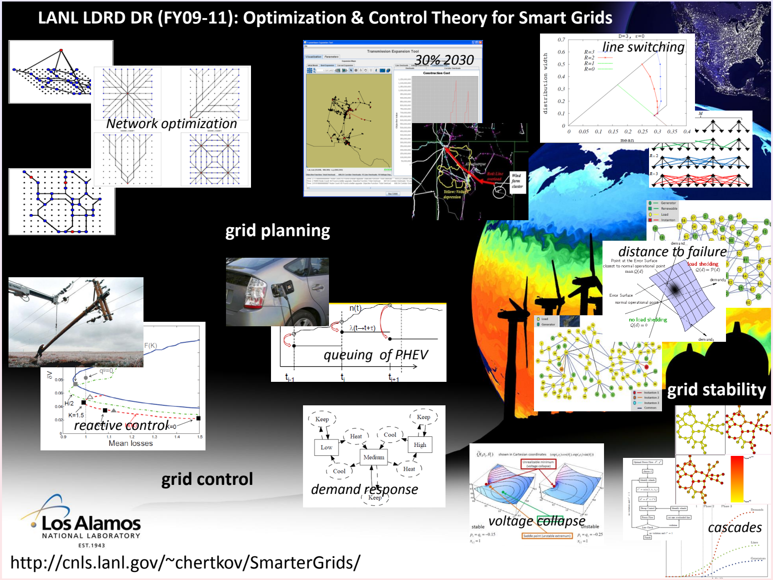
**New Politics & Problems** (blackouts, nuclear, renewables, markets)

**Smart Grid = New Solutions**
[Networks, New Algorithms]
(optimization, control, economics, communications)

**CS/IT/OR** Complexity, Predictability

+

**App. Math & Stat. Physics** new/old phenomena


LANL LDRD DR (FY09-11): Optimization & Control Theory for Smart Grids
Network optimization
30% 2030
line switching
grid planning
distance to failure
queuing of PHEV
reactive control
Mean losses
grid stability
demand response
Keep
Heat
Cool
Low
Medium
High
grid control
voltage collapse
cascades
Los Alamos
NATIONAL LABORATORY
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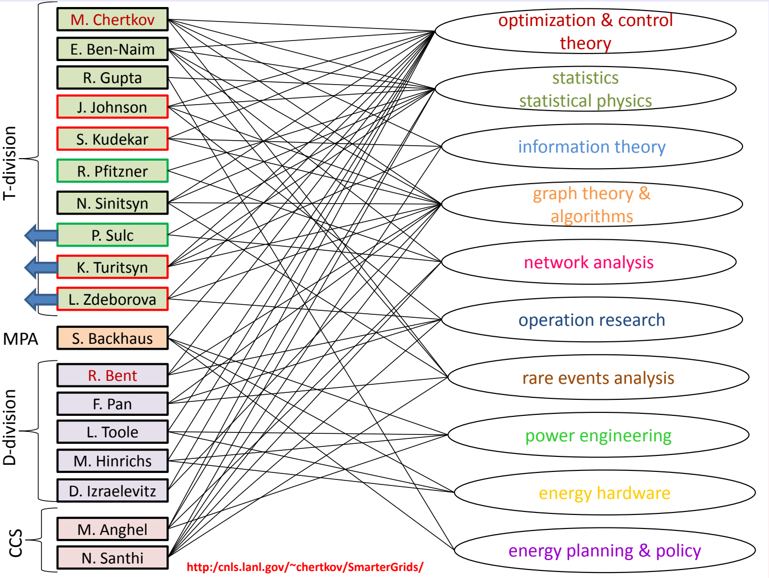
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M. Chertkov
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S. Kudekar
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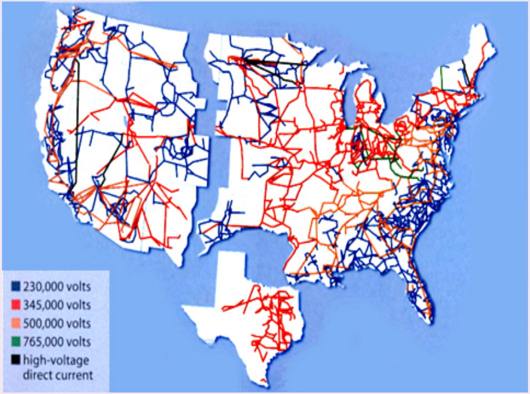
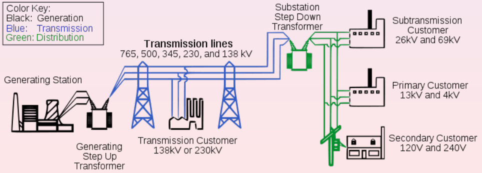



# US power grid

The greatest Engineering Achievement of the 20$^{th}$ century




will require **smart revolution** in the 21$^{st}$ century

# Preliminary Remarks

## The power grid operates according to the laws of electrodynamics

- Transmission Grid (high voltage) vs Distribution Grid (low voltage)
- Alternating Current (AC) ▸ Power Flows ... often considered in linearized (DC) approximation
- No waiting periods $\Rightarrow$ power constraints should be satisfied immediately. Many Scales.
- Loads and Generators are players of two types (distributed renewable will change the paradigm)
- At least some generators are adjustable - to guarantee that at each moment of time the total generation meets the total load
- The grid is a graph ... but constraints are (graph-) global

# Many Scales Involved

## Power & Voltage

- $1KW$ - typical household; $10^3 KW = 1MW$ - consumption of a medium-to-large residential, commercial building; $10^6 KW = 1GW$-large unit of a Nuclear Power plant ($30GW$ is the installed wind capacity of Germany =8% of total, US wind penetration is 5%- [30% by 2030?]); $10^9 KW = 1TW$ - US capacity
- Distribution - $4 - 13$KV. Transmission - $100 - 1000$KV.

## Spatial Scales

- $1mm - 10^3 km$; US grid $= 3 * 10^6 km$ lines (operated by $\sim$ 500 companies)

## Temporal Scales [control is getting faster]

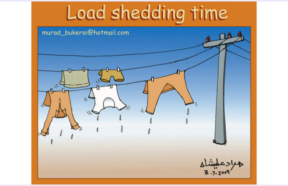
- 17ms -AC (60Hz) period, target for Phasor Measurement Units sampling rate (10-30 measurements per second)
- 1s - electro-mechanical wave [motors induced] propagates $\sim 500km$
- 2-10s - SCADA delivers measurements to control units
- $\sim$ 1 min - loads change (demand response), wind ramps, etc (toughest scale to control)
- 5-15min - state estimations are made (for markets), voltage collapse
- up to hours - maturing of a cascading outage over transmission grids

# Our Publications on Grid Stability

- 22. R. Pfitzner, K. Turitsyn, M. Chertkov, Controlled Tripping of Overheated Lines Mitigates Power Outages, submitted to IEEESmartGridComm 2011, arxiv:1104.4558.
- 21. M. Chertkov, M. Stepanov, F. Pan, and R. Baldick , Exact and Efficient Algorithm to Discover Stochastic Contingencies in Wind Generation over Transmission Power Grids, invited session on Smart Grid Integration of Renewable Energy: Failure analysis, Microgrids, and Estimation at CDC/ECC 2011.
- 16. P. van Hentenryck, C. Coffrin, and R. Bent , Vehicle Routing for the Last Mile of Power System Restoration, submitted to PSCC.
- 15. R. Pfitzner, K. Turitsyn, and M. Chertkov , Statistical Classification of Cascading Failures in Power Grids , arxiv:1012.0815, accepted for IEEE PES 2011.
- 14. S. Kadloor and N. Santhi , Understanding Cascading Failures in Power Grids , arxiv:1011.4098 submitted to IEEE Transactions on Smart Grids.
- 13. N. Santhi and F. Pan , Detecting and mitigating abnormal events in large scale networks: budget constrained placement on smart grids , proceedings of HICSS44, Jan 2011.
- 8. M. Chertkov, F. Pan and M. Stepanov, Predicting Failures in Power Grids, arXiv:1006.0671, IEEE Transactions on Smart Grids 2, 150 (2010).

## MC, F. Pan (LANL) and M. Stepanov (UA Tucson)

- Predicting Failures in Power Grids: The Case of Static Overloads, IEEE Transactions on Smart Grids **2**, 150 (2010).




## MC, FP, MS & R. Baldick (UT Austin)

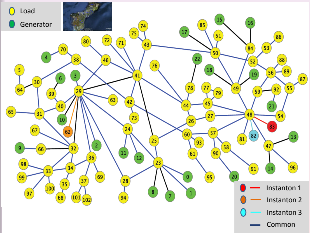
- Exact and Efficient Algorithm to Discover Extreme Stochastic Events in Wind Generation over Transmission Power Grids, invited session on Smart Grid Integration of Renewable Energy at CDC/ECC 2011.

# Failure Probability

- Normally the grid is ok (SATisfied) ... but sometimes failures (UNSATisfied) happens
- How to estimate failure probability (UNSAT)?

## Static overload

- Power Flows. Control=Generation Dispatch. Constraints = Thermal and Generation
- Probabilistic Forecast of Loads (given)
- SAT= Load shedding is avoidable; UNSAT=load shedding is unavoidable
- Find the most probable UNSAT configuration of loads



# Extreme Statistics of Failures

- Statistics of loads/demands is assumed given: $\mathcal{P}(\mathbf{d})$
- $\mathbf{d} \in$ SAT=No Shedding; $\mathbf{d} \in$ UNSAT =Shedding

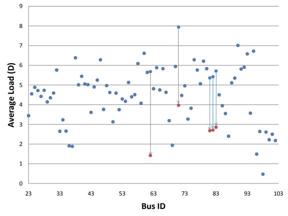
## Most Dangerous Configuration of the demand = the Instanton

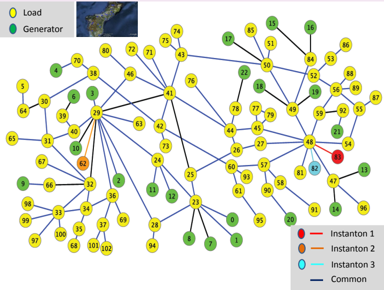
- $\arg\max_{\mathbf{d}} \mathcal{P}(\mathbf{d})|_{\mathbf{d}\notin SAT}$ - most probable instanton
- SAT is a polytope (finding min-shedding solution is an LP); $-\log(\mathcal{P}(\mathbf{d}))$ is (typically) convex

## The task: to find the (rated) list of (local) instantons

- The most probable instanton represents the large deviation asymptotic of the failure probability
- Use an efficient heuristics to find candidate instantons (technique was borrowed from our previous "rare events" studies of a similar problem in error-correction '04-'11)

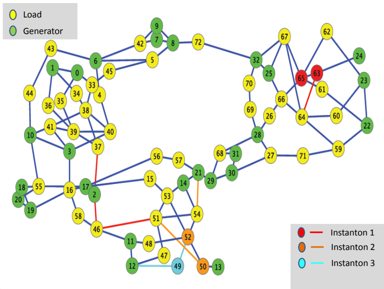
# Example of Guam




- Gaussian Statistics of demands (input) leads to Intermittency (output) = instantons (rare, UNSAT) are distinctly different from normal (typical, SAT)
- The instantons are sparse (difference with "typical" is localized on troubled nodes)
- The troubled nodes are repetitive in multiple-instantons
- Violated constraints (edges) are next to the troubled nodes
- Instanton structure is not sensitive to small changes in statistics of demands

# Example of IEEE RTS96 system




- The instantons are well localized (but still not sparse)
- The troubled nodes and structures are repetitive in multiple-instantons
- Violated constraints (edges) can be far from the troubled nodes: long correlations
- Instanton structure is not sensitive to small changes in statistics of demands

Wind Contingency

# Path Forward (for predicting failures)

## Path Forward

- Many large-scale practical tests, e.g. ERCOT wind integration
- The instanton-amoeba allows upgrade to other (than $LP_{DC}$) network stability testers, e.g. for AC flows and transients
- Instanton-search can be accelerated, utilizing LP-structure of the tester (exact & efficient for example of renewables)
- This is an important first step towards exploration of "next level" problems in power grid, e.g. on interdiction [Bienstock et. al '09], optimal switching [Oren et al '08], cascading outages/extremes [Dobson et al '06], and control of the outages [Ilic et al '05, Bienstock '11]

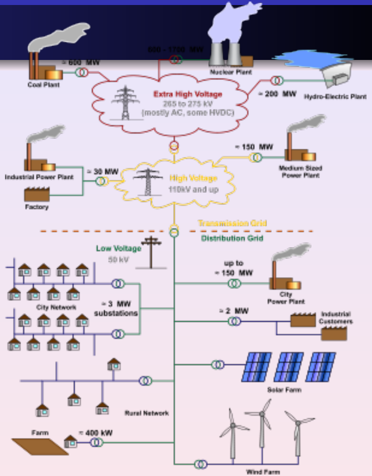
# Our Publications on Grid Control

- 20. K. Turitsyn, S. Backhaus, M. Ananyev and M. Chertkov , Smart Finite State Devices: A Modeling Framework for Demand Response Technologies, invited session on Demand Response at CDC/ECC 2011.
- 19. S. Kundu, N. Sinitsyn, S. Backhaus, and I. Hiskens, Modeling and control of thermostatically controlled loads, submitted to 17th Power Systems Computation Conference 2011, arXiv:1101.2157.
- 16. P. van Hentenryck, C. Coffrin, and R. Bent , Vehicle Routing for the Last Mile of Power System Restoration, submitted to PSCC.
- 12. P. Sulc, K. Turitsyn, S. Backhaus and M. Chertkov , Options for Control of Reactive Power by Distributed Photovoltaic Generators, arXiv:1008.0878, to appear in Proceedings of the IEEE, special issue on Smart Grid (2011).
- 11. F. Pan, R. Bent, A. Berscheid, and D. Izrealevitz , Locating PHEV Exchange Stations in V2G, arXiv:1006.0473, IEEE SmartGridComm 2010
- 10. K. S. Turitsyn, N. Sinitsyn, S. Backhaus, and M. Chertkov, Robust Broadcast-Communication Control of Electric Vehicle Charging, arXiv:1006.0165, IEEE SmartGridComm 2010
- 9. K. S. Turitsyn, P. Sulc, S. Backhaus, and M. Chertkov, Local Control of Reactive Power by Distributed Photovoltaic Generators, arXiv:1006.0160, IEEE SmartGridComm 2010
- 7. K. S. Turitsyn, Statistics of voltage drop in radial distribution circuits: a dynamic programming approach, arXiv:1006.0158, accepted to IEEE SIBIRCON 2010
- 5. K. Turitsyn, P. Sulc, S. Backhaus and M. Chertkov, Distributed control of reactive power flow in a radial distribution circuit with high photovoltaic penetration, arxiv:0912.3281 , selected for super-session at IEEE PES General Meeting 2010.
- 2. L. Zdeborova, S. Backhaus and M. Chertkov, Message Passing for Integrating and Assessing Renewable Generation in a Redundant Power Grid, presented at HICSS-43, Jan. 2010, arXiv:0909.2358
- 1. L. Zdeborova, A. Decelle and M. Chertkov, Message Passing for Optimization and Control of Power Grid: Toy Model of Distribution with Ancillary Lines, arXiv:0904.0477, Phys. Rev. E 80 , 046112 (2009)

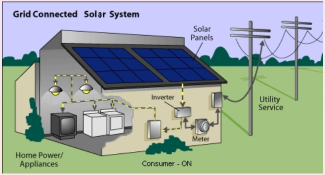
# K. Turitsyn (MIT), P. Sulc (NMC), S. Backhaus and M.C.

- *Optimization of Reactive Power by Distributed Photovoltaic Generators*, to appear in Proceedings of the IEEE, special issue on Smart Grid (2011), `http://arxiv.org/abs/1008.0878`
- *Local Control of Reactive Power by Distributed Photovoltaic Generators*, proceedings of IEEE SmartGridComm 2010, `http://arxiv.org/abs/1006.0160`
- *Distributed control of reactive power flow in a radial distribution circuit with high photovoltaic penetration*, IEEE PES General Meeting 2010 (invited to a super-session), `http://arxiv.org/abs/0912.3281`

# Setting & Question & Idea




- Distribution Grid (old rules, e.g. voltage is controlled only at the point of entrance)
- Significant Penetration of Photovoltaic (new reality)
- How to control swinging/fluctuating voltage (reactive power)?

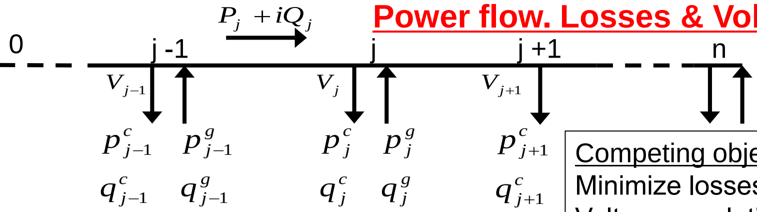



## Idea(s)

- Use Inverters.
- Control Locally.

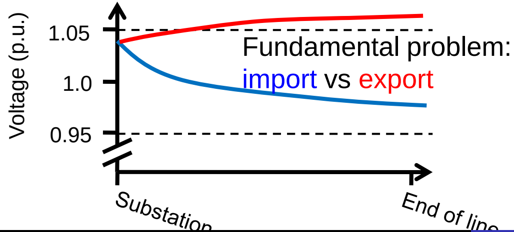
# Optimization & Control Theory for Smart Grids: Control (of reactive power)

## Power flow. Losses & Voltage




$$Loss_j = r_j \frac{P_j^2 + Q_j^2}{V_0^2}$$

$$\Delta V_j = -(r_j P_j + x_j Q_j)$$



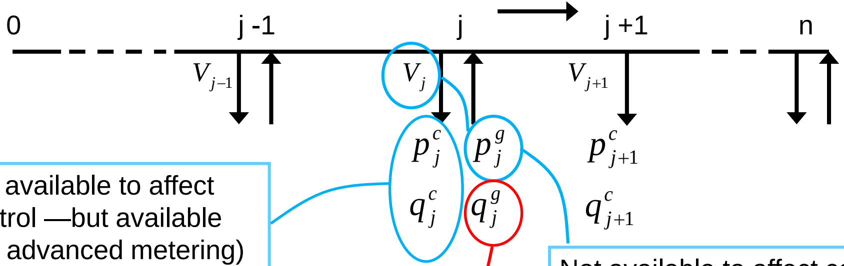

Competing objectives

Minimize losses → $Q_j=0$

Voltage regulation → $Q_j=-(r_j/x_j)P_j$

- **Rapid reversal of real power flow can cause undesirably large voltage changes**
- **Rapid PV variability cannot be handled by current electro-mechanical systems**
- **Use PV inverters to generate or absorb reactive power to restore voltage regulation**
- **In addition… optimize power flows for minimum dissipation**

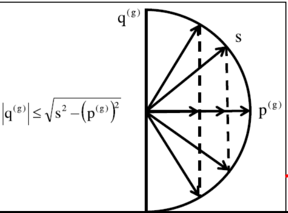
# Parameters available & limits for control




Not available to affect control —but available (via advanced metering) for control input

Not available to affect control — but available (via inverter PCC) for control input

Available—minimal impact on customer, extra inverter duty



# Schemes of Control

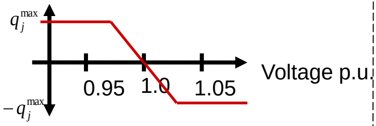
- Base line (do nothing)

$$q_j^g = 0$$

- Unity power factor

$$q_j^g = q_j^c \qquad F^{(L)}$$

- Proportional Control (EPRI white paper)




- voltage control heuristics

$$q_j^g = q_j^c + \frac{r_j}{x_j}(p_j^c - p_j^g)$$

- composite control

$$q_j^g = Kq_j^c + (1-K)[q_j^c + \frac{r_j}{x_j}(p_j^c - p_j^g)]$$

$$= KF_j^{(L)} + (1-K)F_j^{(V)}$$

- Hybrid (composite at V=1 built in proportional)

$$q_j^g = F_j(K) + (q_j^{\max} - F_j(K))\left(1 - \frac{2}{1+\exp(-4(V_j-1)/\delta}\right)$$

$$F_j(K) = Constr_j\left(KF_j^{(L)} + (1-K)F_j^{(V)}\right)$$

$$\mathrm{Constr}_j[q] = \begin{cases} q, & |q| \le q_j^{max} \\ (q/|q|)q_j^{max}, & \text{otherwise} \end{cases}$$

## *Prototypical distribution circuit: case study*

**Import—Heavy cloud cover**

- $p^c$ = uniformly distributed 0-2.5 kW
- $q^c$ = uniformly distributed $0.2p^c$-$0.3p^c$
- $p^g$ = 0 kW
- Average import per node = 1.25 kW

**Export—Full sun**

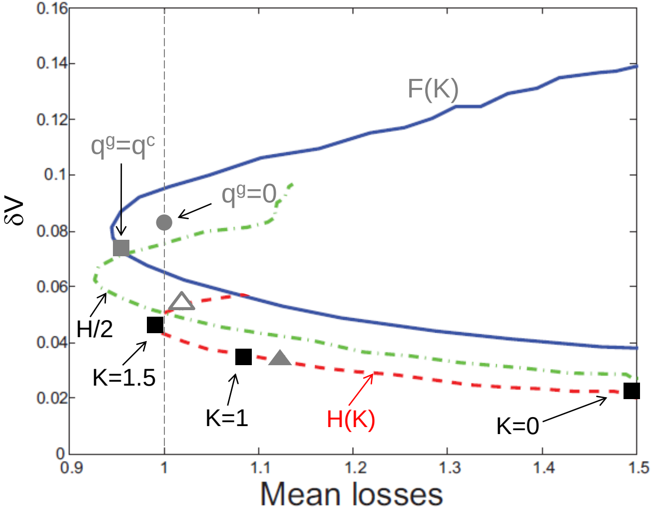
- $p^c$ = uniformly distributed 0-1.0 kW
- $q^c$ = uniformly distributed $0.2p^c$-$0.3p^c$
- $p^g$ = 2.0 kW
- Average export per node = 0.5 kW

- $V_0$=7.2 kV line-to-neutral
- n=250 nodes
- Distance between nodes = 200 meters
- Line impedance = 0.33 + i 0.38 Ω/km
- 50% of nodes are PV-enabled with 2 kW maximum generation
- Inverter capacity s=2.2 kVA – 10% excess capacity

**Measures of control performance**

- $\delta V$—maximum voltage deviation in transition from export to import
- Average of import and export circuit dissipation relative to "Do Nothing-Base Case"

## Performance of different control schemes




### Hybrid scheme

- **Leverage nodes that already have $V_j$~1.0 p.u. for loss minimization**
- **Provides voltage regulation and loss reduction**
- **K allows for trade between loss and voltage regulation**
- **Scaling factor provides related trades**

## *Conclusions:*

- **In high PV penetration distribution circuits where difficult transient conditions will occur, adequate voltage regulation and reduction in circuit dissipation can be achieved by:**
  - Local control of PV-inverter reactive generation (as opposed to centralized control)
  - Moderately oversized PV-inverter capacity ($s \sim 1.1\, p^{g,max}$)

- **Using voltage as the only input variable to the control may lead to increased average circuit dissipation**
  - Other inputs should be considered such as $p^c$, $q^c$, and $p^g$.
  - Blending of schemes that focus on voltage regulation or loss reduction into a hybrid control shows improved performance and allows for simple tuning of the control to different conditions.

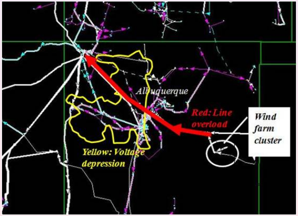
- **Equitable division of reactive generation duty and adequate voltage regulation will be difficult to ensure simultaneously.**
  - Cap reactive generation capability by enforcing artificial limit given by $s \sim 1.1\, p^{g,max}$

# Our Publications on Grid Planning

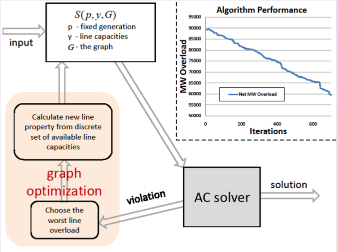
- 18. R. Bent, A. Berscheid, and L. Toole , Generation and Transmission Expansion Planning for Renewable Energy Integration, submitted to Power Systems Computation Conference (PSCC).
- 17. R. Bent and W.B. Daniel , Randomized Discrepancy Bounded Local Search for Transmission Expansion Planning, accepted for IEEE PES 2011.
- 11. F. Pan, R. Bent, A. Berscheid, and D. Izrealevitz , Locating PHEV Exchange Stations in V2G, arXiv:1006.0473, IEEE SmartGridComm 2010
- 6. J. Johnson and M. Chertkov, A Majorization-Minimization Approach to Design of Power Transmission Networks, arXiv:1004.2285, 49th IEEE Conference on Decision and Control (2010).
- 4. R. Bent, A. Berscheid, and G. Loren Toole, Transmission Network Expansion Planning with Simulation Optimization, Proceedings of the Twenty-Fourth AAAI Conference on Artificial Intelligence (AAAI 2010), July 2010, Atlanta, Georgia.
- 3. L. Toole, M. Fair, A. Berscheid, and R. Bent, Electric Power Transmission Network Design for Wind Generation in the Western United States: Algorithms, Methodology, and Analysis , Proceedings of the 2010 IEEE Power Engineering Society Transmission and Distribution Conference and Exposition (IEEE TD 2010), April 2010, New Orleans, Louisiana.

# Grid Design: Motivational Example

- Cost dispatch only (transportation,economics)
- Power flows highly approximate
- Unstable solutions
- Intermittency in Renewables not accounted




An unstable grid example




Hybrid Optimization - is current "engineering" solution developed at LANL: Toole,Fair,Berscheid,Bent 09 extending and built on NREL "20% by 2030 report for DOE

**Network Optimization ⇒**

- Design of the Grid as a tractable global optimization

## Network Optimization (for fixed production/consumption $\mathbf{p}$)

$$\underbrace{\min_{\hat{g}} \mathbf{p}^{+}\left(\hat{G}(\hat{g})\right)^{-1}\mathbf{p},}_{\substack{\text{minimize losses}\\ \text{convex over}\hat{g}}} \qquad \underbrace{G_{ab}=\begin{cases} 0, & a\neq b,\quad a\nsim b\\ -g_{ab}, & a\neq b,\quad a\sim b\\ \sum_{c\neq a}^{c\sim a} g_{ac}, & a=b.\end{cases}}_{\text{Discrete Graph Laplacian of conductance}}$$

## Network Optimization (averaged over $\mathbf{p}$)

$$\min_{\hat{g}}\langle \mathbf{p}^{+}\left(\hat{G}(\hat{g})\right)^{-1}\mathbf{p}\rangle = \min_{\hat{g}} \mathrm{tr}\left(\left(\hat{G}(\hat{g})\right)^{-1}\langle \mathbf{p}\mathbf{p}^{+}\rangle\right) =$$

$$\underbrace{\min_{\hat{g}} \mathrm{tr}\left(\left(\hat{G}(\hat{g})\right)^{-1}\hat{P}\right),}_{\text{still convex}} \quad \hat{P} - \text{covariance matrix of load/generation}$$

Boyd,Ghosh,Saberi '06 in the context of resistive networks
also Boyd, Vandenberghe, El Gamal and S. Yun '01 for Integrated Circuits

# Network Optimization: Losses+Costs [J. Johnson, MC '10]

## Costs need to account for

- "sizing lines" - grows with $g_{ab}$, linearly or faster (convex in $\hat{g}$)
- "breaking ground" - $l_0$-norm (non convex in $\hat{g}$) but also imposes desired sparsity

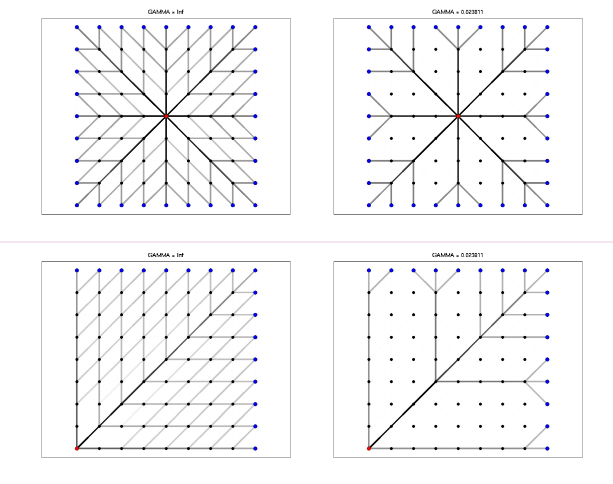
## Resulting Optimization is non-convex

$$\min_{\hat{g}>0}\left(\mathrm{tr}\left(\left(\hat{G}(\hat{g})\right)^{-1}\hat{P}\right)+\sum_{\{a,b\}}\left(\alpha_{ab}g_{ab}+\beta_{ab}\phi_\gamma(g_{ab})\right)\right),\quad \phi_\gamma(x)=\frac{x}{x+\gamma}$$

## Tricks (for efficient solution of the non-convex problem)

- "annealing": start from large (convex) $\gamma$ and track to $\gamma\to 0$ (combinatorial)
- Majorization-minimization (from Candes, Boyd '05) for current $\gamma$: $\hat{g}^{t+1}=\mathrm{argmin}_{\hat{g}>0}\left(\mathrm{tr}(\mathcal{L})+\hat{\alpha}.*\hat{g}+\hat{\beta}.*\phi'_\gamma(g^t_{ab}).*g_{ab}\right)$

# Single-Generator Example




▸ Other Examples

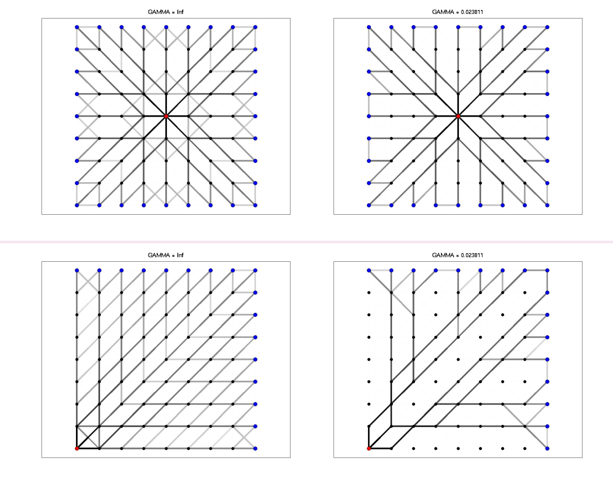
# Adding Robustness

To impose the requirement that the network design should be robust to failures of lines or generators, we use the worst-case power dissipation:

$$\mathcal{L}^{\backslash k}(\hat{g}) = \max_{\forall\{a,b\}: z_{ab}\in\{0,1\} | \sum_{\{a,b\}} z_{ab}=N-k} \mathcal{L}(\hat{z}.*\hat{g}))$$

- It is tractable to compute only for small values of $k$.
- Note, the point-wise maximum over a collection of convex function is convex.
- So the linearized problem is again a convex optimization problem at every step continuation/MM procedure.

# Single-Generator Examples [+Robustness]




Other Examples

# Conclusion (for the Network Optimization part)

A promising heuristic approach to design of power transmission networks. However, cannot guarantee global optimum.

- CDC10: `http://arxiv.org/abs/1004.2285`

## Future Work:

- Applications to real grids, e.g. for 30/2030
- Bounding optimality gap?
- Use non-convex continuation approach to place generators
- possibly useful for graph partitioning problems
- adding further constraints (e.g. don't overload lines)
- extension to (exact) AC power flow?

# Bottom Line

- A lot of interesting collective phenomena in the power grid settings for Applied Math, Physics, CS/IT analysis
- The research is timely (blackouts, renewables, stimulus)

### Other Problems the team plans working on

- Efficient PHEV charging via queuing/scheduling with and without communications and delays
- Power Grid Spectroscopy (power grid as a medium, electro-mechanical waves and their control, voltage collapse, dynamical state estimations)
- Effects of Renewables (intermittency of winds, clouds) on the grid & control
- Load Control, scheduling with time horizon (dynamic programming +)
- Price Dynamics & Control for the Distribution Power Grid
- Post-emergency Control (restoration and de-islanding)

### For more info - check:

http://cnls.lanl.gov/~chertkov/SmarterGrids/
https://sites.google.com/site/mchertkov/projects/smart-grid

Thank You!

# Basic AC Power Flow Equations (Static)

## The Kirchhoff Laws (linear)

$$\forall a \in \mathcal{G}_0: \quad \sum_{b\sim a} J_{ab} = J_a \text{ for currents}$$
$$\forall (a,b) \in \mathcal{G}_1: \quad J_{ab} z_{ab} = V_a - V_b \text{ for potentials}$$
$$\Rightarrow \forall (a,b) \in \mathcal{G}_1: \quad J_a = \sum_{b\in\mathcal{G}_0} Y_{ab} V_b$$

$$\hat{Y} = (Y_{ab} | a, b \in \mathcal{G}_0), \quad \forall \{a,b\}: \; Y_{ab} = \begin{cases} 0, & a \neq b, \quad a \nsim b \\ -y_{ab}, & a \neq b, \quad a \sim b \\ \sum_{c\neq a}^{c\sim a} y_{ac}, & a = b. \end{cases}$$

$$\forall \{a,b\}: \quad y_{ab} = g_{ab} + i\beta_{ab} = (z_{ab})^{-1}, \quad z_{ab} = r_{ab} + x_{ab}$$

## Complex Power Flows [balance of power, nonlinear]

$$\forall a \in \mathcal{G}_0: \quad P_a = p_a + iq_a = V_a J_a^* = V_a \sum_{b\sim a} J_{ab}^* = V_a \sum_{b\sim a} \frac{V_a^* - V_b^*}{z_{ab}^*}$$
$$= \sum_{b\sim a} \frac{\exp(2\rho_a) - \exp(\rho_a + \rho_b + i\theta_a - i\theta_b)}{z_{ab}^*}$$

- Nonlinear in terms of Real and Reactive powers
- Reactive Power needs to be injected to maintain reasonably stable voltage
- Quasi-static (transients may be relevant on the scale of seconds and less)
- Different (injection/consumption/control) conditions on generators $(p, V)$ and loads $(p, q)$
- $(\theta, \rho)$ are conjugated (Lagrangian multipliers) to $(p, q)$, energy landscape

Preliminary Remarks

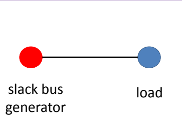
# Energy Functional Landscape (Static)

## Transmission Networks
## (resistance is much smaller than inductance, $r_{ab} \ll x_{ab}$)

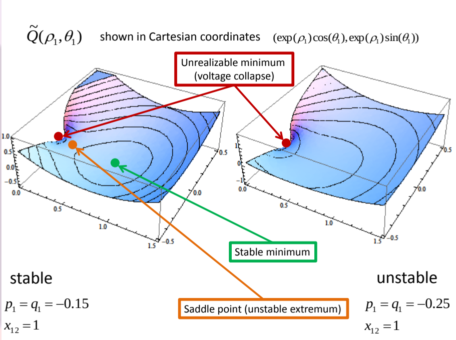
$$Q(\boldsymbol{\rho}, \boldsymbol{\theta}) = \underbrace{\sum_{\{a,b\}\in\mathcal{G}_1} \frac{\exp(2\rho_a) + \exp(2\rho_b) - 2\exp(\rho_a+\rho_b)\cos(\theta_a-\theta_b)}{2x_{ab}}}_{\text{reactive power ``lost'' in lines}} - \sum_{a\in\mathcal{G}_0} \theta_a p_a - \sum_{a\in\mathcal{G}_{\text{loads}}} \rho_a q_a$$

## Single Load $(p_1, q_1)$ and Slack Bus $(\rho_0 = \theta_0 = 0)$

$$Q = \frac{1+\exp(2\rho_1)-2\exp(\rho_1)\cos(\theta_1)}{2x} - \theta_1 p_1 - \rho_1 q_1$$




voltage collapse = (nonlinear) PF equations do not have a solution



# DC [linearized] approximation (for AC power flows)

(0) The amplitude of the complex potentials are all fixed to the same number (unity, after trivial re-scaling): $\forall a: \quad \rho_a = 0$.

(1) $\forall\{a,b\}: \quad |\theta_a - \theta_b| \ll 1$ - phase variation between any two neighbors on the graph is small

(2) $\forall\{a,b\}: \quad r_{ab} \ll x_{ab}$ - resistive (real) part of the impedance is much smaller than its reactive (imaginary) part. Typical values for the $r/x$ is in the $1/27 \div 1/2$ range.

## It leads to

- Linearized relation between powers and phases (at the nodes):
$$\forall a \in \mathcal{G}_0: \quad p_a = \sum_{b\sim a} \frac{\theta_a - \theta_b}{x_{ab}}$$
- Losses of real power are zero in the network (in the leading order) $\sum_a p_a = 0$
- Reactive power needs to be injected (lines are inductances - only "consume" reactive power=accumulate magnetic energy per cycle)

Preliminary Remarks

# Model of Load Shedding

## Minimize Load Shedding = Linear Programming for DC

$$LP_{DC}(\mathbf{d}|\mathcal{G};\mathbf{x};\mathbf{u};\mathbf{P}) = \min_{\mathbf{f},\varphi,\mathbf{p},\mathbf{s}} \left(\sum_{a\in\mathcal{G}_d} s_a\right)_{COND(\mathbf{f},\varphi,\mathbf{p},\mathbf{d},\mathbf{s}|\mathcal{G};\mathbf{x};\mathbf{u};\mathbf{P})}$$

$$COND = COND_{flow} \cup COND_{DC} \cup COND_{edge} \cup COND_{power} \cup COND_{over}$$

$$COND_{flow} = \left(\forall a: \quad \sum_{b\sim a} f_{ab} = \begin{cases} p_a, & a\in\mathcal{G}_p \\ -d_a + s_a, & a\in\mathcal{G}_d \\ 0, & a\in\mathcal{G}_0\setminus(\mathcal{G}_p\cup\mathcal{G}_d) \end{cases}\right)$$

$$COND_{DC} = \left(\forall\{a,b\}: \varphi_a - \varphi_b + x_{ab} f_{ab} = 0\right), \quad COND_{edge} = \left(\forall\{a,b\}: -u_{ab} \le f_{ab} \le u_{ab}\right)$$

$$COND_{power} = \left(\forall a: \quad 0 \le p_a \le P_a\right), \quad COND_{over} = \left(\forall a: \quad 0 \le s_a \le d_a\right)$$

$\varphi$ -phases; $f$ -power flows through edges; $x$ - inductances of edges

Instantons

# Instantons for Wind Generation

## Setting

- Renewables is the source of fluctuations
- Loads are fixed (5 min scale)
- Standard generation is adjusted according to a droop control (low-parametric, linear)

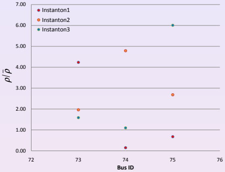
## Results

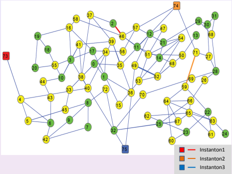
- The instanton algorithm discovers most probable UNSAT events
- The algorithm is EXACT and EFFICIENT (polynomial)
- Illustrate utility and performance on IEEE RTS-96 example extended with additions of $10\%, 20\%$ and $30\%$ of renewable generation.

Load Contingency

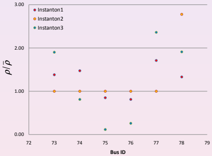
# Simulations: IEEE RTS-96 + renewables

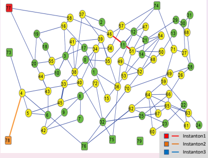
10% of penetration - localization, long correlations



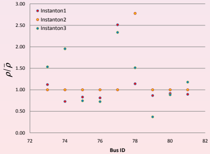



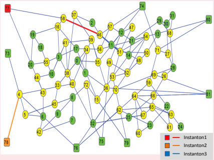

20% of penetration - worst damage, leading instanton is delocalized



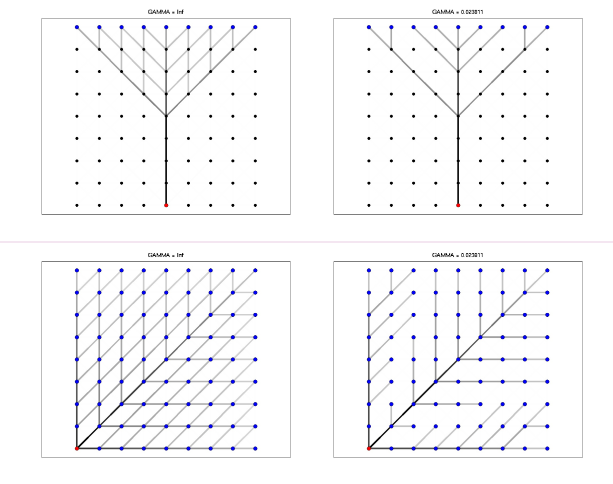

30% of penetration - spreading and diversifying decreases the damage, instantons are localized



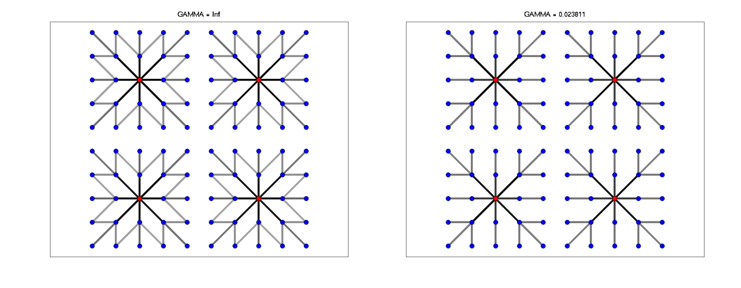

Load Contingency

# Single-Generator Examples (II)



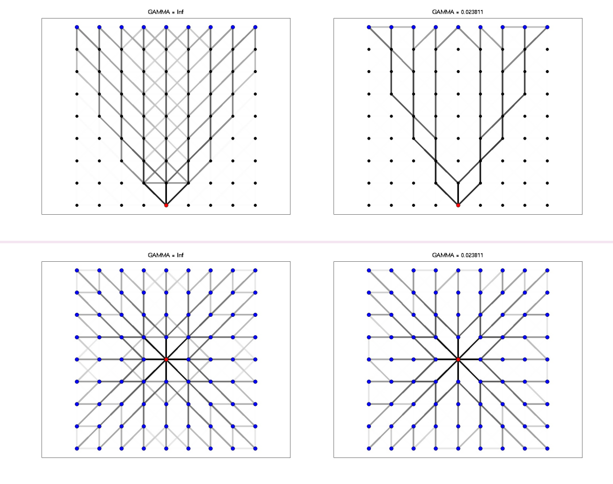

Single Generator Example

# Multi-Generator Example



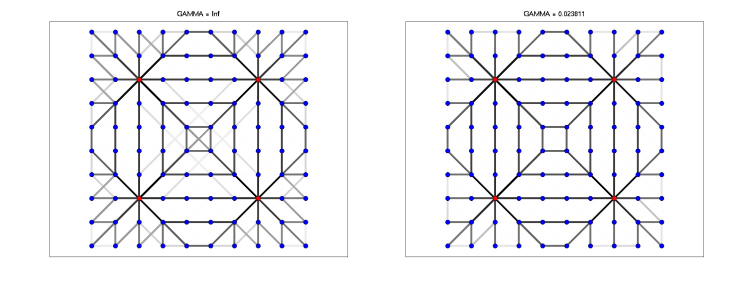

◀ Single Generator Example

# Single-Generator Examples [+Robustness] (II)




Single Generator Example [+Robustness]

# Multi-Generator Example [+Robustness]




◂ Single Generator Example [+Robustness]

# Outline

## Rene Pfitzner (NMC), Konstantin Turitsyn (MIT) & MC

- Statistical Classification of Cascading Failures in Power Grids, accepted to IEEE PES 2011, http://arxiv.org/abs/1012.0815

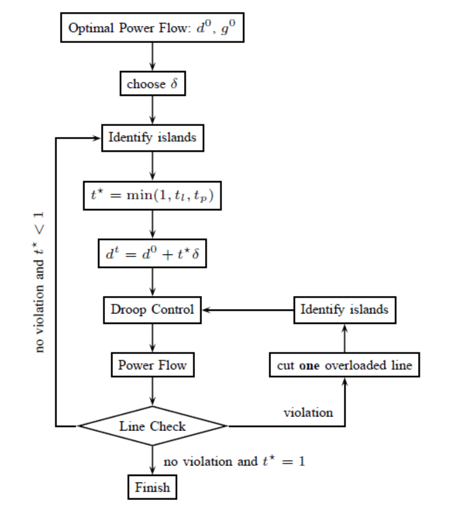
## Objectives:

- Have a realistic microscopic model of a cascade [not (!!) a "disease-spread" like phenomenological model]
- Resolve discrete events dynamics (lines tripping, overloads, islanding) explicitly
- Address (first) the current reality of the transmission grid operation, e.g. automatic control on the sub-minute scale
- Consider (first) fluctuations in demand as a source of cascade in the overloaded (modern) grid
- Analyze the results, e.g. in terms of phases observed, on available power grid models [IEEE test beds]

## Building on

- I. Dobson, B. Carreras, V. Lynch, and D. Newman, *An initial model for complex dynamics in electric power system blackouts*, HICSS-34, 2001

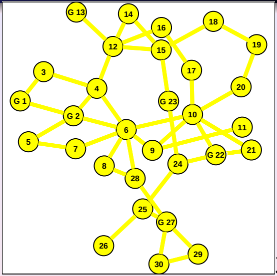
# Algorithm of the Cascade




- *Optimum Power Flow* finds (cost) optimal distribution of generation (decided once for $\sim$ 15 min - in between state estimations)
- *DC power flow* is our (simplest) choice
- *Droop Control* = equivalent (pre set for 15 min) response of all the generators to change in loads
- *Identify islands* with a proper connected component algorithm(s)
- *Discrete time Evolution of Loads* = (a) generate configuration of demand from given distribution (our enabling example = Gaussian, White); (b) assume that the configuration "grow" from the typical one (center of the distribution) in continuous time, $t \in [0; 1]$; (c) project next discrete event (failure of a line or saturation of a generator) and jump there

# Tests on IEEE systems (30, 39, 118 buses)

- The base configuration of demand, $\mathbf{d}^0$ is a part of the system description. Contingency (in demand) is generated according to
- $\mathcal{P}(\delta_i) =$
$$\begin{cases} \frac{\exp(-(\delta_i)^2/(2d_i^0\Delta))}{\sqrt{\pi d_i^0 \Delta/2}}, & d_i^0+\delta_i > d_i^0 \\ 1/2, & d_i^0+\delta_i = d_i^0 \\ 0, & d_i^0+\delta_i < d_i^0 \end{cases}$$
- $\Delta$ is the governing parameter, measuring level of fluctuations
- Collect statistics averaging over multiple (200) samples for each $D$

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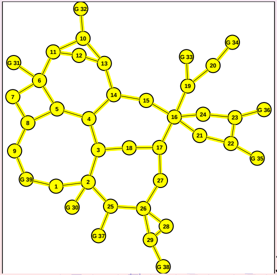


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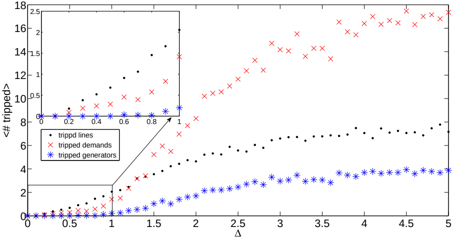
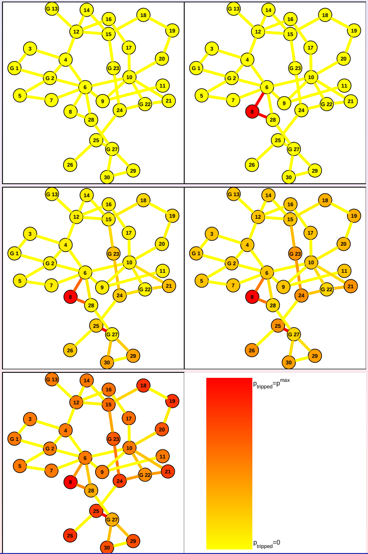
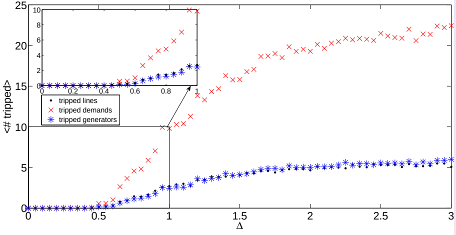
# Tests on IEEE 30 system

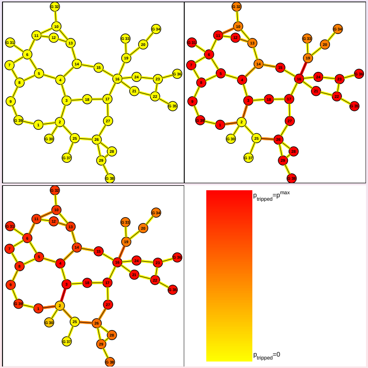



- Average # vs level of fluctuations.
- Stress Diagram. Average # of failures per edge/node. $\Delta = 0.1, 0.2, 0.9, 1.2, 2.0 \Rightarrow$



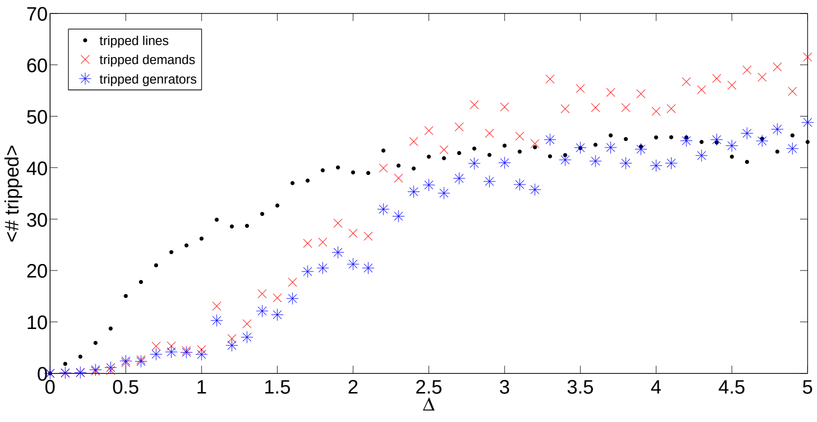
# Tests on IEEE 39 buses



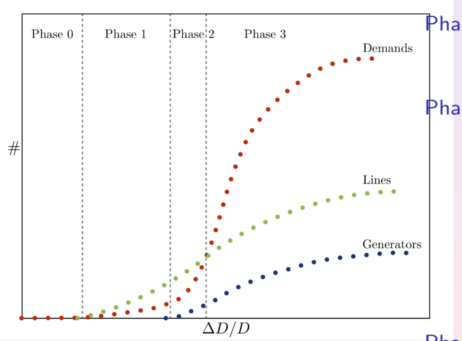

- Average # vs level of fluctuations.
- Stress Diagram. Average # of failures per edge/node. $\Delta = 0.3, 0.4, 0.6 \Rightarrow$

# Tests on IEEE 118 system




- 25 samples
- observed (run into) interesting sensitivity to distribution of line capacities

# General Conclusions (3 phases)




- **Phase #0** The grid is resilient against fluctuations in demand.
- **Phase #1** shows tripping of demands due to tripping of overloaded lines. This has a overall "de-stressing" effect on the grid.
- **Phase #2** Generator nodes start to become tripped, mainly due to islanding of individual generators. With the early tripping of generators the system becomes stressed and cascade evolves much faster (with increase in the level of demand fluctuations) when compared with a relatively modest increase observed in Phase #1.
- **Phase #3** Significant outages are observed. They are associated with removal from the grid of complex islands, containing both generators and demands.

# Path Forward (Cascades)

- From DC solver to AC solver
- Mixed models - combining fluctuations in demands and incidental line tripping
- More detailed study of effect of capacity inhomogeneity (e.g. on islanding)
- Towards validated (derived from micro-) phenomenological model and theory of cascades [power tails, scaling, dynamic mechanisms]