Excitation of Banded Whistler Waves in the Magnetosphere

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Linear kinetic dispersion analysis and a two-dimensional electromagnetic particle-in-cell simulation are performed to demonstrate a possible excitation mechanism of banded whistler waves in the magnetosphere outside of the plasmapause. Whistler waves in the lower and the upper bands can be generated simultaneously by the whistler anisotropy instability driven by two bi-Maxwellian electron components with $T_\perp/T_\parallel > 1$ at different $T_\parallel$, independently, where $\parallel$ and $\perp$ denote directions relative to the background geomagnetic field. Given $\omega_e/\Omega_e$, the ratio of the electron plasma frequency to the electron cyclotron frequency, $T_\parallel$ of each electron component determines the properties of the excited waves. For the typical magnetospheric condition of $1 < \omega_e/\Omega_e < 5$ in regions associated with strong chorus emissions, the present study suggests that upper-band waves can be excited by anisotropic electrons below $\sim 1$ keV, while lower-band waves are excited by anisotropic electrons above $\sim 10$ keV. The resultant lower-band waves are generally field-aligned and substantially electromagnetic. However, the excited upper-band waves generally propagate obliquely to the background geomagnetic field with quasi-electrostatic fluctuating electric fields.
1. Introduction

Whistler waves in the terrestrial magnetosphere outside of the plasmapause are observed in the frequency range of $0.1 - 0.8\Omega_e$ and typically show banded spectra with a gap near $0.5\Omega_e$, where $\Omega_e$ is the equatorial electron cyclotron frequency [Tsurutani and Smith, 1974]. The width of the gap is highly variable, sometimes reaching $0.2\Omega_e$ and sometimes vanishing entirely [Koons and Roeder, 1990]. They are usually composed of discrete narrowband wave elements with rising or falling tones (chorus) often accompanied by banded incoherent whistler waves [e.g. Santolík et al., 2003, Figure 4]. These waves are largely substorm-dependent and are enhanced during geomagnetically disturbed conditions [Meredith et al., 2001] in association with enhanced fluxes of suprathermal electrons [Anderson and Maeda, 1977]. They play an important role in the pitch-angle scattering and the acceleration of radiation-belt electrons [Horne and Thorne, 1998; Shprits et al., 2006; Thorne, 2010], and can be responsible for the generation of plasmaspheric hiss [Bortnik et al., 2009]. These waves are believed to be excited near the geomagnetic equator [LeDocq et al., 1998]. General agreement on their wave normal angles has not been reached, but lower-band waves ($< 0.5\Omega_e$) tend to be field-aligned near the geomagnetic equator and upper-band waves ($> 0.5\Omega_e$) seem to be highly oblique at all latitudes [Haque et al., 2010]. On average, lower-band waves are stronger than upper-band waves [Meredith et al., 2001]. The present study focuses only on the excitation of the banded incoherent whistler waves, while the discrete chorus elements can arise from these incoherent whistler waves through nonlinear wave growth involving the inhomogeneity of the background geomagnetic field [Omura and Summers, 2008].
It is generally accepted that lower-band waves are generated by the whistler anisotropy instability through a cyclotron resonance with anisotropic electrons between a few and tens of keV [Kennel and Petschek, 1966]. However, various mechanisms have been proposed to explain the excitation of upper-band waves. When the banded structure was originally discovered, Tsurutani and Smith [1974] ascribed the gap near $0.5\Omega_e$ to Landau damping as emissions which cut across the gap could clearly be observed. Maeda et al. [1976] found that there is no obvious coherence between upper-band and lower-band waves, and they postulated that whistler waves in the two bands are generated in two separate source regions and the banded structure is due to propagation effects. Curtis [1978] suggested that both bands are generated locally, lower-band being the whistler mode but upper-band being the ordinary mode when $\omega_e < \Omega_e$, where $\omega_e$ is the electron plasma frequency. However, this scenario does not explain many observations when $\omega_e > \Omega_e$. Hayakawa et al. [1984] showed that the observed upper-band waves are quasi-electrostatic and suggested that they are generated through an instability driven by anisotropic electrons of tens eV, following the theory of Hashimoto and Kimura [1981]. Recently, Bell et al. [2009] explained the banded emissions as lower-band and upper-band waves trapped in ducts of enhanced and depleted cold plasma densities, respectively. Omura et al. [2009] proposed that the gap near $0.5\Omega_e$ comes from the nonlinear damping of the longitudinal component of a slightly oblique whistler wave packet propagating along the inhomogeneous geomagnetic field.

A more recent case study of banded whistler waves observed by the Cluster spacecraft by Santolík et al. [2010] showed that two anisotropic electron components at different $T_||$.
can drive the whistler anisotropy instability in the lower and the upper bands simultaneously. However, they also found that the linear instability growth rate in the lower band is too small for waves to be observed, although this growth rate may have been underestimated due to the limited time resolution (4 s) and the limited energy range (32 eV to 26.7 keV) of the electron measurements. This event has also been studied by Schriver et al. [2010] with a two-dimensional particle-in-cell (PIC) simulation. They represented the observed anisotropic electrons with essentially a bi-Maxwellian distribution of $T_\perp/T_\parallel = 10$ and $T_\parallel = 300$ eV. This electron component was shown to drive obliquely propagating upper-band waves while the excitation of lower-band waves was explained by nonlinear wave-wave coupling. However, the lower-band waves simulated by Schriver et al. [2010] are at oblique propagation in contrast to the quasiparallel propagation estimated from the observations based on the plane wave approximation [Santolík et al., 2010]. Li et al. [2010] used THEMIS observations to carry out statistical analysis of the equatorial electron distributions responsible for the excitation of whistler waves. Interestingly, there is often a gap of anisotropy for electrons of several keV in the observed electron distributions: electrons above $\sim 10$ keV and below $\sim 1$ keV have $T_\perp/T_\parallel > 1$, but electrons with intermediate energies are isotropic. Li et al. [2010] suggested that the anisotropic electrons below $\sim 1$ keV and above $\sim 10$ keV are responsible for the excitation of upper-band and lower-band waves, respectively, because of the different minimum energies required for electrons to be in cyclotron resonance with waves in the two different bands.

the $\beta_{\parallel e}$ dependence of the whistler anisotropy instability driven by an electron anisotropy of $T_\perp/T_\parallel > 1$, where $\beta_{\parallel e} = n_e T_{\parallel e}/(B_0^2/2\mu_0)$, $n_e$ is the electron density, and $B_0$ is the background magnetic field. For $\omega_e/\Omega_e > 1$, the maximum growth rate of this instability is at parallel propagation at relatively large $\beta_{\parallel e}$ and the resultant fluctuating fields are substantially electromagnetic. At smaller values of $\beta_{\parallel e}$, however, the maximum growth rate shifts to oblique propagation, the excited fluctuating fields become quasi-electrostatic, and the frequency of the excited fluctuations shifts from below to above $0.5\Omega_e$. The critical value of $\beta_{\parallel e}$, at which the transition from the electromagnetic regime to the quasi-electrostatic regime occurs, is around 0.025 as long as $\omega_e/\Omega_e > 1$ [Gary et al., 2011], which corresponds to $\sim 1$ keV for the typical magnetospheric condition of $1 < \omega_e/\Omega_e < 5$ in regions associated with strong chorus emissions [Li et al., 2010]. The electromagnetic regime of this instability at $\beta_{\parallel e} > 0.025$ corresponds to the excitation mechanism of lower-band waves [Kennel and Petschek, 1966], while the quasi-electrostatic regime at $\beta_{\parallel e} < 0.025$ relates to the excitation of upper-band waves [Hashimoto and Kimura, 1981]. Here we carry out linear kinetic analysis and a two-dimensional PIC simulation of the instability to demonstrate that banded whistler waves can be excited when two bi-Maxwellian electron components with $T_\perp/T_\parallel > 1$ at different $T_\parallel$ are present.

2. Linear Kinetic Analysis

Linear kinetic dispersion theory for electromagnetic fluctuations in a homogeneous, magnetized, collisionless plasma is applied to the instability analysis. The Cartesian coordinate system of the analysis admits spatial variations in both the direction parallel to $B_0$ (denoted by $\parallel$) and one direction perpendicular to $B_0$ (denoted by $\perp$), but no spatial
variations in the other perpendicular direction (denoted by \( \perp \perp \)). The wavevector \( k \) is real
and the complex frequency is \( \omega = \omega_r + i\gamma \) where \( \gamma > 0 \) represents temporal growth of a
fluctuating normal mode of the plasma.

Consistent with the typical magnetospheric condition of \( 1 < \omega_e/\Omega_e < 5 \) in regions associ-
ciated with strong chorus emissions, the present study uses \( \omega_e/\Omega_e = 4 \). The protons have a
Maxwellian velocity distribution and \( \beta_p = 0.0001 \) (which corresponds to a temperature of
1.6 eV). Two bi-Maxwellian electron components are considered: the first component has
\( \tilde{\beta}_j = 0.01 \) with \( T_{\perp}/T_{\parallel} = 5.0 \) while the second component has \( \tilde{\beta}_j = 1.0 \) with \( T_{\perp}/T_{\parallel} = 2.0 \),
where \( \tilde{\beta}_j = n_0 T_j/(B_0^2/2\mu_0) = (2T_j/m_e c^2)(\omega_e/\Omega_e)^2 \) (\( n_0 \) is the total plasma density and
subscript \( j \) represents the various electron components). The two electron components
have \( T_{\parallel} \) of 0.16 keV and 16 keV and hereafter are referred to as warm (subscript \( w \)) and
hot (subscript \( h \)) electrons, respectively.

First, we follow Gary et al. [2011] to analyze plasmas with a single bi-Maxwellian
electron component. When the single electron component is the hot electrons specified
above, the maximum growth of the instability has \( \omega_{\text{max}}/\Omega_e = 0.36 \), \( \gamma_{\text{max}}/\Omega_e = 0.10 \) at
\( k\lambda_e = 0.65 \) and \( \theta = 0^\circ \), where \( \lambda_e = \sqrt{m_e/n_0\mu_0 c^2} \) is the electron inertial length, and
\( \theta \) is the wave normal angle with regard to \( B_0 \). So the maximum growth is at parallel
propagation and the excited fluctuations are in the lower band. Linear analysis also shows
that the resultant fluctuating fields are substantially electromagnetic and the amplitude
relations among electric and magnetic field components of the excited waves are
\( |\delta E_{\parallel}| \ll |\delta E_{\perp \perp}| \leq |\delta E_{\perp}| \) and \( |\delta B_{\parallel}| \ll |\delta B_{\perp}| \leq |\delta B_{\perp \perp}| \). On the other hand, when the single electron
component is the warm electrons described above, \( \omega_{\text{max}}/\Omega_e = 0.69 \), \( \gamma_{\text{max}}/\Omega_e = 0.031 \) at
$k\lambda_e = 3.8$ and $\theta = 48^\circ$. The maximum growth shifts to oblique propagation and the excited fluctuations are in the upper band. Linear theory also shows that the resultant fluctuating fields are quasi-electrostatic, $|\delta E_\perp| \ll |\delta E_\parallel| < |\delta E|$, and $|\delta B_\parallel| \sim |\delta B_\perp| < |\delta B_\perp\perp|$.

When the plasma has both the warm and hot electron components with concentrations of $n_w/n_0 = 90\%$ and $n_h/n_0 = 10\%$, respectively, the unstable modes driven by the two components do not interfere with each other much and they grow simultaneously at rates reduced almost linearly proportional to $n_w/n_0$ and $n_h/n_0$. Figure 1 displays the calculated instability growth rate as a function of $k\lambda_e$ and $\theta$ for this case. There are two unstable zones as marked by the black contour lines, which represent the contour of $\gamma/\Omega_e = 0.01$. The weaker unstable zone near the lower-left corner is driven by the hot electron component: $\omega_{\text{max}}/\Omega_e = 0.29$, $\gamma_{\text{max}}/\Omega_e = 0.016$ at $k\lambda_e = 0.63$ and $\theta = 0^\circ$.

The other unstable zone is driven by the warm electron component: $\omega_{\text{max}}/\Omega_e = 0.69$, $\gamma_{\text{max}}/\Omega_e = 0.030$ at $k\lambda_e = 3.8$ and $\theta = 48^\circ$.

Our linear analysis suggests that, when more than one electron components are considered, the major factor which determines the properties of the excited waves is $\tilde{\beta}_{ij}$. In other words, when $\omega_e/\Omega_e$ is given, the properties of the excited waves are controlled mainly by $T_{\parallel j}$ of the electron component which drives the instability. For the typical magnetospheric condition of $1 < \omega_e/\Omega_e < 5$ in regions associated with strong chorus emissions, this suggests that upper-band waves can be excited by anisotropic electrons below $\sim 1$ keV, while lower-band waves are excited by anisotropic electrons above $\sim 10$ keV.

3. Simulation Results
A two-dimensional electromagnetic PIC simulation is performed to examine the simultaneous excitation of lower-band and upper-band whistler waves suggested by Figure 1. The simulation domain lies in the $x - y$ plane and $B_0$ is in the $x$ direction ($x - \parallel$, $y - \perp$, $z - \perp\perp$). Periodic boundary conditions are used in both dimensions. Following the linear analysis results, we use $L_x = L_y = 51.4\lambda_e$, $N_x = N_y = 256$, and $\Delta t\Omega_e = 0.018$. In each cell, the present simulation has 9600 simulation particles to represent the warm (90%) and hot (10%) electrons, respectively. Since our concern here is whistler fluctuations at $\omega_r \gg \Omega_p$ (the proton cyclotron frequency), protons in the simulation are treated as an immobile, charge neutralizing background to conserve computation time. The other parameters are the same as in the linear analysis of section 2.

Figure 2 illustrates the time evolution of the simulation. The increase of the electric field energy around $t\Omega_e = 200$ in Figure 2c corresponds to the development of the instability driven by the warm electrons, as confirmed by the simultaneous decrease of $T_{\perp w}/T_{\parallel w}$ shown in Figure 2a. According to the linear analysis results, this should lead to enhanced quasi-electrostatic fluctuations in the upper band. The magnetic components associated with these fluctuations are indeed small as shown in Figure 2d. The electric field energy of these fluctuations starts to decrease slightly after it reaches a peak around $t\Omega_e = 300$. This is because these fluctuations have significant $\delta E_\parallel$ and therefore are subject to strong Landau damping, as discussed in Gary et al. [2011]. The increase of the electric field energy after $t\Omega_e = 600$ is accompanied by the decrease of $T_{\perp h}/T_{\parallel h}$ shown in Figure 2b. This corresponds to the development of the instability driven by the hot electrons, which grows slightly slower as shown in Figure 1. Linear theory predicts that the enhanced
fluctuations are in the lower band and predominantly electromagnetic. The simultaneous
growth of the magnetic fluctuations in Figure 2d confirms the electromagnetic feature of
these enhanced waves.

The simultaneous excitation of lower-band and upper-band waves is explicitly demon-
strated in Figure 3, which displays the energy spectral density of $\delta E_\perp$ at $x = y = 25.7\lambda_e$
from $t\Omega_e = 900$ to $1800$. The two spectral peaks in the lower and the upper bands agree
well with the prediction of the linear kinetic analysis and demonstrate the growth of the
unstable modes driven by the warm and hot electron components, respectively. The four
discrete spikes around the lower-band spectral peak are due to the limited size of the sim-
ulation domain, which only allows waves of $k = 2n\pi/L$ ($n=0, 1, 2, ...$). It can be readily
shown from the linear analysis results of section 2 (and even the cold plasma dispersion
relation) that the frequencies at these spikes closely correspond to $k_\parallel = 4, 5, 6,$ and
$7 \times 2\pi/L_x$ ($k_\perp = 0$) in the simulation. The spectral peak would be continuous if a much
larger simulation domain can be used, which is limited by the available computational
resources.

The $\delta E_\perp (x, y)$ at $t\Omega_e = 1800$ and the corresponding spatial power spectrum are shown
in Figure 4a and Figure 4b, respectively. Figure 4a suggests the superposition of short-
wavelength obliquely-propagating waves on top of long-wavelength field-aligned waves in
the system. This is clearly confirmed by Figure 4b where the strongest peak is at parallel
propagation associated with the lower-band waves while the second peak is at oblique
propagation corresponding to the upper-band waves. The locations of the spectral peaks
in Figure 4b are in good agreement with the linear theory prediction. Interestingly, the
weak enhancement of waves around $k_\parallel \lambda_e = 0.5$ and $k_\perp \lambda_e = 2.5$ suggests that the nonlinear wave-wave coupling mechanism in Schriver et al. [2010] is also operating. This has been clearly confirmed by a simulation with 100% warm electrons (all other parameters remain unchanged).

4. Summary and Discussion

A possible excitation mechanism of banded whistler waves is investigated using both linear kinetic analysis and a PIC simulation. Whistler waves in the lower and the upper bands can be generated simultaneously by the whistler anisotropy instability driven by two bi-Maxwellian electron components with $T_\perp/T_\parallel > 1$ at different $T_\parallel$, independently. For the typical magnetospheric condition of $1 < \omega_e/\Omega_e < 5$ in regions associated with strong chorus emissions, the present study suggests that upper-band waves can be excited by anisotropic electrons below $\sim 1$ keV, while lower-band waves are excited by anisotropic electrons above $\sim 10$ keV. The resultant lower-band waves are generally field-aligned and substantially electromagnetic. However, the excited upper-band waves propagate obliquely to $B_0$ and have quasi-electrostatic fluctuating electric fields.

The excitation mechanism discussed requires two anisotropic electron components with $\tilde{\beta}_\parallel$ above and below $\sim 0.025$, respectively, when $\omega_e/\Omega_e > 1$. The mechanism operates over a range of $\omega_e/\Omega_e$ as long as $T_\parallel$, which corresponds to the critical value of $\tilde{\beta}_\parallel \sim 0.025$ and changes with $\omega_e/\Omega_e$, is still between $T_\parallel$ and $T_\parallel$. The properties of the excited waves change gradually when $\tilde{\beta}_\parallel$ of the electron component which drives the instability varies [Gary et al., 2011]. Changes in $\omega_e/\Omega_e$, $T_\parallel$ and $T_\parallel$ would consequently modify the width of the spectral gap near $0.5\Omega_e$ and may even affect the location of the gap if the parameter...
changes are sufficiently large. For example, the frequency of the upper-band waves excited in the present study is relatively high compared with observations that upper-band waves tend to have a peak intensity near $0.53\Omega_e$ [Burtis and Helliwell, 1976]. A lower frequency of the upper-band waves can be achieved if $T_{\parallel w}$ or $\omega_e/\Omega_e$ in the present study is higher. Subsequently, the excited upper-band waves would become more field-aligned and more electromagnetic. A relatively cool warm electron component has been used in the present study to highlight the oblique-propagation and quasi-electrostatic features of the excited upper-band waves.

Due to the quasi-electrostatic feature of the excited upper-band waves, the proposed excitation mechanism suggests that upper-band waves may be more easily identified in electric field observations than in magnetic field observations. Furthermore, upper-band waves are liable to Landau damping due to their significant $\delta E_{\parallel}$ component. The simulation results suggest that the saturation level of upper-band waves is lower than lower-band waves. This is consistent with observations that lower-band waves are stronger than upper-band waves on average [Meredith et al., 2001]. In addition, the oblique propagation, the lower saturation level, and the more severe Landau damping together would make the propagation of upper-band waves to high latitude regions more difficult compared with lower-band waves. This is in agreement with observations that upper-band waves are more tightly confined to the geomagnetic equator ($|\lambda_m| < \sim 10^\circ$) than lower-band waves [e.g. Meredith et al., 2001, Figures 5 and 6].

The cold background electrons $\sim 1$ eV often present in the magnetosphere are not included in this study to facilitate the PIC simulation. We have carried out linear ki-
netic analysis including the background electrons but the results show little change in the properties of the unstable modes except that the instability growth rate is reduced proportional to the reduction of the warm and hot electron concentrations. In addition, the present study uses uniform background and represents only the vicinity of the geomagnetic equator. So the propagation effects as well as the nonlinear wave growth involving the inhomogeneity of the geomagnetic field \cite{Omura and Summers, 2008; Omura et al., 2009} are not included.

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**References**


Figure 1.  Linear dispersion theory results: Instability growth rate as a function of $k\lambda_e$ and $\theta$ for a plasma with 90% of warm electrons and 10% of hot electrons. The two asterisks represent the locations of the local maximum growth rates. The black contour lines are the contour of $\gamma/\Omega_e = 0.01$. Note that the minimum value of the color scale corresponds to $\gamma/\Omega_e = -0.03$ so larger damping rates saturate in the plot.
Figure 2. Simulation results: The time evolution of $T_\perp/T_\parallel$ of (a) the warm electrons, and (b) the hot electrons, as well as the wave energies in different (c) electric and (d) magnetic field components. In panels c and d, the red, green, blue, and black lines denote $x-\parallel$ component, $y-\perp$ component, $z-\perp\perp$ component, and the total of the corresponding field, respectively.
Figure 3. Simulation results: the energy spectral density of $\delta E_{\perp}/cB_0$ at $x = y = 25.7\lambda_e$ from $t\Omega_e = 900$ to $1800$. The two vertical dashed lines mark the locations of the most unstable modes predicted by the linear kinetic analysis in section 2.
Figure 4. Simulation results: The wave component $\delta E_{\perp}/cB_0$ at $t\Omega_e = 1800$. (a) The contour plot of $\delta E_{\perp}/cB_0$. (b) The spatial power spectrum of $\delta E_{\perp}/cB_0$ in logarithmic scale. The black contour lines are the same ones in Figure 1 but are now plotted in the $k_{\perp}-k_{\parallel}$ coordinates. They represent the contour of $\gamma/\Omega_e = 0.01$ given by linear kinetic dispersion theory.