Improved moraine age interpretations through explicit matching of geomorphic process models to cosmogenic nuclide measurements from single landforms

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Abstract

The statistical distributions of cosmogenic nuclide measurements from moraine boulders contain previously unused information on moraine ages, and help determine whether moraine degradation or inheritance is more important on individual moraines. Here, we present a method for extracting this information by fitting geomorphic process models to observed exposure ages from single moraines. We also apply this method to 94 \textsuperscript{10}Be apparent exposure ages from 11 moraines reported in four published studies. Our models represent \textsuperscript{10}Be accumulation in boulders that are exhumed over time by slope processes (moraine degradation), and the delivery of boulders with preexisting \textsuperscript{10}Be inventories to moraines (inheritance). For now, we neglect boulder erosion and snow cover, which are likely second-order processes. Given a highly scattered data set, we establish which model yields the better fit to the data, and estimate the age of the moraine from the better model fit. The process represented by the better-fitting model is
probably responsible for most of the scatter among the apparent ages. Our methods should help resolve controversies in exposure dating; we reexamine the conclusions from two published studies based on our model fits.

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**Keywords:** cosmogenic, exposure dating, statistical distribution, moraine, boulder, paleoglaciology, glacial geomorphology, hillslope, process geomorphology

45 **Introduction**

Cosmogenic exposure dating is a powerful method for estimating the ages of glacial landforms. Cosmic rays split atoms in surface rocks at predictable rates, producing measurable concentrations of otherwise-rare isotopes (e.g., $^{10}$Be, $^{26}$Al, and $^{36}$Cl; Gosse and Phillips, 2001; Muzikar et al., 2003). The concentrations of these cosmogenic nuclides in moraine boulders are, under ideal conditions, directly related to the true ages of the host moraines (Lal, 1991).

However, exposure dating assumes that 1) the sampled surface on each boulder was exposed to the full surface flux of cosmic rays since moraine deposition, and 2) the boulders contained no cosmogenic nuclides when deposited on the moraine. Geomorphic processes violate these assumptions, and they affect different boulders on the same moraine to varying degrees (Ivy-Ochs et al., 2007). Thus, the scatter among nuclide measurements from single moraines is often larger than the analytical uncertainty of the method would suggest (Putkonen and Swanson, 2003; Balco and Schaefer, 2006; Balco, 2011).

Many geomorphic processes influence exposure dating of moraine boulders (see Ivy-Ochs et al., 2007; Balco, 2011). Boulders erode, roll downhill, become covered by snow, loess, or vegetation, and are exhumed from the moraine over time. All of these postdepositional
processes reduce nuclide concentrations, causing the apparent ages to underestimate moraine ages. In contrast, some boulders arrive at the moraines with inherited cosmogenic nuclides, yielding apparent ages that are too old. Multiple processes may act on individual boulders, or on different boulders on the same moraine. The relative importance of different processes likely varies among moraines in the same valley and between field sites.

Many interpretive methods

Faced with a collection of widely scattered apparent exposure ages, field workers construct a geomorphic scenario to explain the scatter, and then choose the interpretive method based on this scenario (e.g., Phillips et al., 1990; Zech et al., 2005; Smith et al., 2005; Ivy-Ochs et al., 2007; Balco, 2011). If inheritance is believed to dominate, the youngest apparent age is chosen; if postdepositional processes dominate, then the oldest apparent age is chosen. The mean, weighted mean, or mode of the apparent ages may be preferred if both old- and young-biasing processes are active. Where measurement error produces all the scatter, the mean or weighted mean is the most appropriate true age estimator. Sometimes apparent ages are discarded before these methods are applied. The choice of scenario and interpretive method is usually influenced by field observations and geomorphic expertise.

Conflicting interpretations

Problems arise when two experts construct different geomorphic scenarios to explain the scatter within the same data set. Such disagreements confound efforts to answer important questions with cosmogenic exposure dating. Consider two examples from high-impact journals.

1) Apparent exposure ages from the Manikala moraines in Tibet may indicate a slip rate on the Karakorum Fault of ~11 mm/yr (Chevalier et al., 2005a, b), twice as fast as most previous
estimates, or ~5 mm/yr (Brown et al., 2005). Chevalier et al. (2005a, 2011) presented 27 new
\(^{10}\)Be apparent ages from three moraines offset by the fault \((n = 10, 9, \text{ and } 8)\). Chevalier et al.
(2005a, b) used the modes of their total data set, which appeared to correspond with peaks in the
marine oxygen isotope record, to obtain age estimates of 21 ka and 140 ka for the two moraines
with distinct offsets. Brown et al. (2005) noted that the scatter of apparent ages on individual
moraine crests increased with the moraines' offset along the fault (an independent proxy for
moraine age). Brown et al. (2005) therefore argued that the excess scatter was due to
postdepositional processes, and that the best estimator of moraine age was the oldest apparent
age on each moraine (45 ka and 425 ka). Dividing the measured offset distances (220 m and
1520 m) by these different age estimates yields the fault slip rates mentioned above.

2) Apparent exposure ages from the Waiho Loop moraine in the Southern Alps either
definitively rule out Younger Dryas-age cooling in New Zealand (Barrows et al., 2007, 2008), or
are inconclusive (Applegate et al., 2008). Barrows et al. (2007) gave 24 new cosmogenic
apparent ages (sixteen \(^{36}\)Cl, eight \(^{10}\)Be) from 10 boulders on the Waiho Loop. They argued that
their apparent ages were relatively free of geomorphic influences, and took the weighted mean
after discarding outliers (Barrows et al., 2007; Applegate et al., 2008, note 2). This procedure
gave a moraine age estimate about 1 ka after the Younger Dryas. Applegate et al. (2008) argued
that the extreme scatter of the apparent ages (~7.8 ka) and their young-skewed distribution was
more consistent with moraine degradation (a young-biasing process) than either measurement
error alone or inheritance. If so, then the oldest apparent age is likely a better measure of the
moraine's true age than the weighted mean. This oldest apparent age falls close to the end of the
Younger Dryas. Reevaluation of the \(^{10}\)Be production rate in New Zealand (Putnam et al., 2010)
changes this result, as anticipated by Applegate et al. (2008; Discussion).

These examples show that geomorphic insight cannot always provide unique moraine age
interpretations from cosmogenic nuclide measurements. Disagreements arise even among experienced glacial geomorphologists working in the same part of the world; Brown et al. (2002) published a paper on exposure dating of moraines offset by the Karakorum Fault before their comment on the Chevalier et al. (2005a) paper. Careful field geomorphology is crucial for choosing good exposure dating samples and interpreting the apparent ages, but more tools are clearly needed.

A possible solution

Here, we propose a new method for estimating moraine ages from cosmogenic nuclide measurements. This method matches the distributions of apparent exposure ages generated by geomorphic process models to observed exposure ages from single moraines. We apply techniques used successfully in modeling other geomorphic systems, such as hillslopes (e.g., Pelletier et al., 2006). Full model descriptions appear in an earlier paper (Applegate et al., 2010). This work is an advance beyond prior modeling studies of geomorphic influences on exposure dating (e.g., Putkonen and Swanson, 2003) because 1) we make inferences about individual moraines, rather than an aggregate of many moraines with different true ages and geomorphic histories, 2) we use an improved model treatment of inheritance, and 3) we apply well-motivated statistical methods in our data-model comparisons.

Here, apparent age means the exposure time of a single clast, inferred from a nuclide concentration measurement using an estimated local nuclide production rate, and assuming no inheritance or shielding. By true age, we mean the amount of time since a single moraine was deposited, assuming that all the sampled boulders rested on an isochronous surface. We wish to draw a clear distinction between the apparent exposure times of boulders and the true ages of the moraines that the boulders rest on. This usage differs from our earlier papers (Applegate et al.,
2008, 2010; Applegate and Alley, 2011), in which apparent ages were called "exposure dates" and true ages were called "ages." The new terms are more consistent with standard nomenclature (Colman et al., 1987).

We first review earlier modeling efforts to understand geologic scatter among apparent exposure ages before describing our models and statistical methods. To ensure that our methods give sensible answers, we match the models to synthetic data sets where the true age and geomorphic parameters are known. We then examine published data from 11 moraines, including the Waiho Loop and Manikala moraines.

**Prior modeling studies**

Prior modeling work explained unexpected features of observed data sets, suggested sampling strategies, and established the relative importance of different geomorphic processes. For example, the earliest successful exposure dating study (Phillips et al., 1990) contained a stratigraphic reversal: the apparent ages from the Mono Basin moraines at Bloody Canyon, California, were younger than those on a cross-cutting set of moraines. Hallet and Putkonen (1994) explained this reversal through the addition of new boulders to the moraine's surface by the progressive lowering of moraine crests, and the subtraction of old boulders by erosion. In this interpretation, all the original boulders on the Mono Basin moraines were destroyed by erosion; the sampled boulders came to the surface long after the moraine's deposition.

Nuclide concentration measurements are expensive, and many early studies included only a few apparent ages per landform. Putkonen and Swanson (2003) suggested that 2-7 apparent exposure ages are needed to be 95% sure of obtaining at least one apparent age that is within 10% of the moraine's true age (cf. Applegate and Alley, 2011), and that the number of apparent
exposure ages needed increases with moraine height.

Additional quantitative studies showed that the potential impacts of snow cover and grain-by-grain erosion on exposure dating are likely smaller than those of moraine degradation and inheritance. Cover by snow of reasonable density and thickness reduces annually averaged nuclide production by <~10% (Gosse and Phillips, 2001, their Fig. 17), although past snow cover was likely greater (Benson et al., 2004; Schildgen et al., 2005). Likewise, grain-by-grain erosion of moraine boulders reduces $^{10}\text{Be}$ concentrations by <10% on last-glacial and younger moraines (Balco, 2011); boulder spalling may have a larger effect (Zimmerman et al., 1994; Muzikar, 2009). In comparison, moraine degradation produces ranges of apparent exposure ages exceeding 50% of a moraine's true age (Zreda and Phillips, 1994; Hallet and Putkonen, 1994; Putkonen and Swanson, 2003), and inherited nuclide concentrations can be several times larger than the in-situ component (Balco, 2011). Boulder rotation is also likely significant (Balco, 2011), but quantitative treatments of rotation are lacking.

A recent model-data study (Heyman et al., 2011) suggests that moraine degradation is more common than inheritance on alpine glacier moraines. They compared the ranges of apparent exposure ages produced by simple models of moraine degradation and inheritance to the ranges of apparent exposure ages from glacial landforms on the Tibetan Plateau. After tuning, the moraine degradation model shows more skill in explaining the overall data set than the inheritance model. This study is notable because its treatment of both processes is fairly sophisticated, and because of the large data set examined (1361 apparent ages on 342 landforms).

*Inheritance or moraine degradation?*

Despite the contributions of process modeling to the development of exposure dating, none of these studies tells us how to interpret apparent exposure ages on single moraines.
Heyman et al. (2011) suggest that we will usually be correct if we assume that geologic scatter is always due to moraine degradation. However, adjustment of their models’ parameters allows either model to reproduce the range of nearly any observed collection of apparent exposure ages.

The problem is perhaps best illustrated by comparing Putkonen and Swanson (2003) to Benson et al. (2005). Putkonen and Swanson (2003) used a moraine degradation model with a simple treatment of inheritance to argue that the oldest apparent age is always the best estimator of moraine age. Benson et al. (2005) used a more complete treatment of inheritance to argue the opposite, that the youngest apparent age is the best estimator.

Thus, neither geomorphic judgement nor prior modeling studies can reliably determine whether moraine degradation or inheritance is more important on any individual moraine. Correct geomorphic interpretation matters because these processes have opposite implications for estimating moraine ages. If moraine degradation is the only biasing process, the oldest apparent exposure age is the best moraine age estimator; if inheritance is the sole source of geologic scatter, the youngest apparent exposure age is the best moraine age estimator. Given the large scatter in many data sets, we obtain very different moraine ages depending on which estimator we choose. Therefore, the inability to distinguish moraine degradation from inheritance directly impacts the effectiveness of exposure dating.

**Inferring process from the statistical distributions of apparent exposure ages**

However, our earlier work (Applegate et al., 2010; Fig. 1) suggests that moraine degradation and inheritance produce different distributions of apparent exposure ages on single moraines. Specifically, moraine degradation produces distributions of apparent exposure ages that have long tails to the young side (negative skewness), whereas inheritance produces distributions that have long tails to the old side (positive skewness; Applegate et al., 2010).
Data sets affected by moraine degradation or inheritance also have reduced chi-squared scores much greater than 1 (Fig. 1; Bevington and Robinson, 2003; Kaplan and Miller, 2003; Balco, 2011). The reduced chi-squared statistic compares the scatter among a set of apparent exposure ages to the scatter expected from their measurement uncertainties. For a data set with apparent exposure ages \( t_1 \ldots t_n \) and corresponding measurement uncertainties \( \sigma_1 \ldots \sigma_n \), the reduced chi-squared statistic is given by

\[
\chi^2_R = \frac{(n-1) \text{ \sum } (t_i - \text{avg})^2}{\sigma_i^2},
\]

where \( \text{avg} \) is the arithmetic average of the apparent exposure ages (Bevington and Robinson, 2003). Here, \( \sigma \) indicates the "internal uncertainties" from the CRONUS online calculator (Balco et al., 2008); use of the "external uncertainties" will produce a too-small reduced chi-squared score. Measurement error alone yields a normal distribution, with a skewness close to zero and a reduced chi-squared score close to 1 (Fig. 1).

These results suggest that we can determine which process gives rise to the scatter among apparent exposure ages on individual moraines (Fig. 1). The reduced chi-squared statistic tells us whether geomorphic processes are important for a given data set; values close to 1 indicate that measurement error is the dominant source of scatter, and in such cases we should take the mean of the apparent ages. For highly scattered data sets, the skewness provides a crude guide to whether moraine degradation or inheritance is more important. However, the use of skewness to diagnose process often fails (Applegate et al., 2010). Thus, more explicit statistical methods are needed to estimate the true ages of moraines from highly scattered data sets.

Modeling the effects of geomorphic processes on exposure dating, and matching models to data

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Before describing our methods for comparing modeled and observed distributions of apparent exposure ages, we provide brief explanations of how the models work. Our earlier paper (Applegate et al., 2010) includes full descriptions of our models, including the assumptions involved in constructing the models and the results of sensitivity tests.

The moraine degradation model is based on Putkonen and Swanson (2003). It couples an analytical solution for the evolution of moraine slopes over time to a parameterization of nuclide production at depth (Granger and Muzikar, 2001). The model's slope evolution equation describes the height of the moraine crest over time. With this curve, we can determine the burial depth as a function of time for any boulder, given its initial depth. The boulder’s final nuclide concentration is then the integral of production within the boulder over time, correcting for nuclear decay.

Our inheritance model tracks nuclide concentrations in boulders that had significant cosmic ray exposure before being deposited on the moraine, where they are eventually sampled. The model formulation is identical for boulders that were derived from subglacial material and boulders that fell onto the glacier from the adjacent valley walls. The final concentration in each boulder is the sum of production during the predepositional and postdepositional exposure periods, corrected for nuclear decay. The model calculates the concentration acquired during the preexposure period for each boulder, given the length of that boulder’s preexposure time and the depth to which the boulder was buried during that time. This calculation is performed using the Granger and Muzikar (2001) parameterization of production as a function of depth, with a correction for surface slope during the predepositional exposure time (Dunne et al., 1999).

Modeling statistical distributions of apparent exposure ages

For each model run, we generate a large number of synthetic apparent exposure ages,
each corresponding to a modeled boulder. In the moraine degradation model, the nuclide concentration in each boulder depends only on the initial depth of the boulder within the moraine. In the inheritance model, each boulder's nuclide concentration depends on the predepositional exposure time and the predepositional burial depth of each boulder. These values are selected from uniform distributions using Latin hypercube sampling (Saltelli et al., 2008; Urban and Fricker, 2010), which provides more even sampling of parameter space than does Monte Carlo (Bevington and Robinson, 2003). ~5,000 Latin hypercube samples per model run produced acceptably consistent statistical distributions for the applications discussed here.

*Model input parameters*

Each model has three important parameters (Applegate et al., 2010). Moraine age is a parameter in both models. The output of the moraine degradation model also depends on the moraine’s initial height, initial slope, and topographic diffusivity. However, moraine slope and topographic diffusivity have a much larger influence on the modeled distributions than the initial height of the moraine (Applegate et al., 2010, their Fig. 6; cf. Putkonen and Swanson, 2003). The inheritance model requires the maximum exposure time and maximum burial depth for boulders during the predepositional exposure time, as well as the slope of the surface from which the boulders are derived. Sensitivity testing shows that the slope of the surface from which boulders with inherited nuclides are derived has little effect for reasonable slopes (Applegate et al., 2010, their Fig. 7). For boulders derived from subglacial material, the maximum burial depth corresponds to the depth of glacial erosion; for supraglacially transported boulders, this parameter is related to the depth of landsliding onto the glacier surface. Because the depth parameter refers to the point on each boulder that is eventually sampled, this parameter is a minimum estimate for the maximum depth of erosion in both the subglacial and supraglacial
Matching models to data

Given a collection of apparent exposure ages from a single moraine and a process model, we run the model repeatedly, searching for the set of parameter values that produces the best match between the modeled and observed distributions. Because histograms of apparent exposure ages usually contain gaps, it is challenging to compare histograms objectively. Instead, we minimize the Kolmogorov-Smirnov test statistic, or the maximum distance between the modeled and observed cumulative density functions, measured parallel to the y-axis (Press et al., 1992; Croarkin and Tobias, 2006; Clauset et al., 2009). By definition, the KS statistic ranges between 0 and 1.

To search for the model parameter values that minimize the KS statistic, we use the Differential Evolution genetic algorithm (Price et al., 2005). Differential Evolution is a fast, widely applied numerical solution method that reliably identifies global (rather than local) minima. For each search, we used 25-30 generations of 10 members each, with a step size of 0.85 and a crossover probability of 1.0. This inverse method ignores the measurement uncertainties of the apparent ages, which we hope to treat in the future through resampling methods.

Separation of moraine degradation and inheritance; additional processes not treated

Figure 1 suggests that moraine degradation and inheritance are end members along a continuum. If this hypothesis is true, we expect to find some moraines that are dominated by one process, even though most moraines will be a mixture of the two. We can distinguish the old-skewed distributions produced by inheritance from the young-skewed distributions produced by...
moraine degradation. Moreover, past work shows that boulder erosion and snow cover are second-order processes (~10% or less) on most moraines (Gosse and Phillips, 2001; Balco, 2011).

For these reasons, we match the moraine degradation and inheritance models to the data separately, neglecting boulder erosion and snow cover. There are technical reasons for these decisions, also; a model that incorporated all these processes would have too many parameters to estimate from present data sets. In effect, we set up two working hypotheses (Chamberlin, 1897; Hilborn and Mangel, 1997) and try to determine which hypothesis is more consistent with the data. The hypotheses are:

1) Inheritance is more important than moraine degradation on this moraine,

OR

2) Moraine degradation is more important than inheritance on this moraine.

Testing the inverse method

Can we recover moraine ages and geomorphic parameters from synthetic data sets?

A necessary step in any inverse problem is to ensure that the inverse method can recover parameter values that were used to generate synthetic data generated by the forward model (Hilborn and Mangel, 1997). To generate synthetic data, we took the $n$ quantiles ($0.5n^{-1}$), ($1.5n^{-1}$), ($2.5n^{-1}$)... ($[n-0.5]n^{-1}$) of modeled distributions from both models, where $n$ is the number of apparent exposure ages (5, 10,... 25). These synthetic data sets are representative of the underlying distribution, which may not be true for real data (Murphy, 1964). We then fitted the corresponding model to each synthetic data set.
The inverse method recovered moraine ages within 5% of the correct values for all the test data sets (Figure 2; Table S2). We also obtained consistent moraine age estimates even among repeated evaluations of the 5-apparent age cases, where we expect the greatest variability among fitting attempts. However, the inverse method often fails to reconstruct the geomorphic parameter values, especially moraine slope and topographic diffusivity in the degradation model (Table S2). We can still determine whether a particular data set is biased by moraine degradation or inheritance; see “What if we choose the wrong model?”, below.

Tradeoffs among parameters

To examine why reconstructed geomorphic parameter estimates often lie far from the correct values, we evaluated the shape of the KS statistic response surfaces in the vicinity of the true solutions. We successively held one model parameter constant at its correct value and varied the others over a regular grid extending to +/- 10% of their correct values, evaluating the fit between each parameter combination and the corresponding 25-apparent age synthetic data set.

For both models, the moraine age is tightly constrained, but there are tradeoffs among the geomorphic parameters (Fig. 3). In the right-hand panel of the inheritance figure, the dark area is inclined relative to the axes, indicating that the data are almost equally consistent with a large preexposure time and a large predepositional burial depth, or small values of both parameters. The right-hand panel of the degradation figure is almost the same shade everywhere, indicating that the synthetic data are consistent with many combinations of the geomorphic parameters.

What if we choose the wrong model?

For our synthetic data sets, we know which process causes the scatter, but this relationship is unknown for real data. Therefore, we examined what happens if we choose the
In our tests, the KS statistic is always worse (larger) for these “wrong model” fits than for fits of the correct model to the same data set (Fig. 2, Table S2). Figure 2 emphasizes the difference in convexity between the cumulative distribution functions produced by the two models; the inheritance model produces convex-up curves, whereas the degradation model produces concave-up curves.

These results suggest that we can identify which process is more important on a particular moraine by fitting both models to each data set. The dominant process is the one whose model yields the smaller best-fit KS statistic.

Applying the models to real data

These tests of the inverse method increase our confidence that we can reconstruct the true ages of moraines from cosmogenic nuclide measurements. Our methods recover the correct moraine ages for synthetic data sets where this value is known, even for small data sets ($n \sim 5$). They also help us determine which process (moraine degradation or inheritance) is more important on individual moraines.

Having shown that our inverse methods pass this crucial test, we now match our models to real data. We chose four studies that report $^{10}$Be apparent exposure ages on 11 moraines and moraine groups (Chevalier et al., 2005a; Munroe et al., 2006; Barrows et al., 2007; Kelly et al., 2008). We have direct experience with the field sites for two of these papers (Munroe et al., 2006; Kelly et al., 2008). The other two papers (Chevalier et al., 2005a; Barrows et al., 2007) describe efforts to date the Waiho Loop and Manikala moraines, as mentioned in the Introduction. Our chosen studies are well distributed geographically and climatologically.
Valleys on the southern slope of Utah’s Uinta Mountains contain prominent moraines of the Pinedale-age Smiths Fork glaciation (Munroe et al., 2006; Laabs et al., 2009). In particular, the Lake Fork drainage contains two prominent Smiths Fork-age moraines, the maximal moraine and a smaller one inset into it. The maximal moraine may be several thousand years younger than the time of the global Last Glacial Maximum (Munroe et al., 2006).

In Gurreholm Dal, eastern Greenland, four groups of moraines lie between the present-day ice margin and the seaward end of the valley (Kelly et al., 2008). From youngest to oldest, these groups are the G-I (Little Ice Age-equivalent), G-II, G-III, and G-IV moraines. Each group contains moraines corresponding to 2-4 distinct positions of the glacier margin. For groups G-II through G-IV, the maximum distance between the innermost and outermost moraines in each moraine group (~1 km) is small compared to the length of the valley (~20 km). The G-I moraines are spread out over a larger area because they represent the splitting of the formerly continuous ice body into the smaller valleys in the upper catchment. All four groups must be younger than the Last Glacial Maximum (Håkansson et al., 2007; Kelly et al., 2008).

The Manikala valley cuts the western wall of Tibet’s Gar valley (Chevalier et al., 2005a). The prominent Manikala M1, M2W, and M2E moraines (youngest to oldest) occupy the piedmont area on the Gar valley floor. The right-lateral Karakorum fault has moved the M2E moraine ~1.52 km and the M1 moraine ~0.22 km from the valley mouth. The offset for the M2W moraine likely falls between these two values, but the moraine segment nearest the valley wall is not preserved.

The western slope of the Southern Alps receives many meters of precipitation yearly, sustaining large glaciers such as the Franz Josef (Barrows et al., 2007). The Waiho Loop is a prominent, forested ridge, probably a moraine, rooted in postglacial outwash on the western coastal plain on the South Island, New Zealand (Denton and Hendy, 1994). The deposit might
have originated as a landslide onto the glacier surface (Tovar et al., 2008), but the debris from such a landslide would have spread out into a diffuse carpet, instead of a ridge (Vacco et al., 2010a, 2010b; cf. Shulmeister et al., 2010). Radiocarbon dating of organic material upvalley from the Waiho Loop suggests that it was deposited sometime after ~13 ka (Denton and Hendy, 1994; Turney et al., 2007).

The $^{10}$Be apparent exposure ages from the moraines discussed above are presented in Figure 4. These apparent ages were calculated using recent versions of the CRONUS online calculator and the Lal/Stone time-independent scaling method (Balco et al., 2008). The eastern Greenland apparent ages include a postglacial uplift correction. The Waiho Loop apparent ages incorporate an improved production rate estimate from New Zealand (Putnam et al., 2010). We neglect the sixteen $^{36}$Cl apparent ages from Barrows et al. (2007), because $^{36}$Cl production is not represented in our models.

**Interpretive framework**

Figure 5 suggests an interpretive framework for estimating moraine ages from cosmogenic nuclide measurements. If the reduced chi-squared score (Bevington and Robinson, 2003; Balco, 2011) is close to 1, then measurement uncertainty likely accounts for the observed scatter, and the average of the apparent ages provides a good estimate of the moraine's true age. If not, we fit both models to the data. Because we treat moraine degradation and inheritance separately, the true age might lie anywhere between the two models’ age estimates. However, the model with the smaller best-fit KS statistic value likely provides a better estimate of the moraine’s true age.

**Model fitting and moraine age estimation**
Only the proximal Lake Fork moraine has a degree of scatter consistent with measurement error alone. This data set requires no model fitting. Following our interpretive scheme (Fig. 5), we average the apparent exposure ages from the inner Lake Fork moraine to arrive at a age estimate.

Figure 6 shows model fits to the other data sets, together with the KS statistic values and inferred true ages associated with each fit. Table S1 shows the parameter values for each fit. For the Manikala M1, distal Lake Fork, Yellowstone, and Waiho Loop moraines, the degradation model provides a better fit than the inheritance model (Fig. 6). On the other hand, the inheritance model provides a better fit for the Manikala M2E, M2W, and Gurreholm G-II, G-III, and G-IV moraines. Selection of the best model is sometimes difficult; we have little confidence in our ability to distinguish between moraine degradation and inheritance on the G-I moraines.

Comparison of model fit age estimates to other methods

We compared the age estimates from the model fits to other methods of estimating moraine age from apparent exposure ages (Table 1). Where possible, we reported the age estimates from the original studies (Kelly et al., 2008), or from the original authors’ later reinterpretations (Laabs et al., 2009; Chevalier et al., 2011). For the Waiho Loop, we calculated the weighted average of the seven oldest $^{10}$Be apparent ages, approximately consistent with the authors’ original interpretive method (Barrows et al., 2007; Applegate et al., 2008, note 2). Our "preferred" age estimate for each data set is given by the model fit with the lower KS statistic value (Fig. 5). We normalized each age estimate by the median, a measure of central tendency that is insensitive to the shape of the underlying distribution.

Where the inheritance model is the better-fitting model, its age estimate is slightly younger than the youngest apparent age, except for the Manikala M2W moraine (Fig. 6, Table
1). Otherwise, its age estimate lies between the youngest apparent age and the median. Where the degradation model fits the real data sets better, its age estimate is always between the oldest and second-oldest apparent ages (Fig. 6, Table 1); in other cases, its age estimate lies between the median and the oldest apparent age.

Discussion

This work suggests that widely scattered and apparently unintelligible collections of apparent exposure ages (Fig. 4) actually have a coherent structure (Fig. 6). Our methods yield moraine age estimates that are consistent with geomorphic knowledge, represented by simplified models that are independent of individual field sites. The model fitting procedure uses all the data available from each moraine, rather than a limited subset. Moreover, the methods help answer a longstanding question within exposure dating, “Is moraine degradation or inheritance more important on this moraine?”

Two observations increase our confidence in the results. The better-fitting model in each panel of Figure 6 reproduces both the range and the curvature of the data better than the worse-fitting model. Second, where we have multiple moraines deposited by the same glacier (G1-G4; LFp, LFd; M1, M2E, M2W), the age estimates fall in stratigraphic order, except for G2 and G3 (Fig. 6; Table S1).

Caveats

Our age estimates are subject to several limitations. First, the models include many simplifying assumptions (Applegate et al., 2010). The model fits will yield misleading results for field sites where these assumptions are violated. Our models treat moraine degradation and
inheritance separately, and we neglect various second-order processes, such as boulder erosion and snow cover (Gosse and Phillips, 2001; Balco, 2011). Problems in estimating the $^{10}\text{Be}$ production rate will lead to errors in moraine age estimates, regardless of any improvement in geomorphic understanding (Balco, 2011, section 7). Additional observations might change our answers. Finally, we cannot be certain that we have identified the best-fitting parameter combination in all cases.

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Reexamining controversial age interpretations

In this section, we reevaluate the two controversial studies we described in the Introduction (Chevalier et al., 2005a; Barrows et al., 2007). Because our preferred age estimates are also uncertain, we simply evaluate how the conclusions of the original studies would change if our interpretations were correct. Note that our inferred moraine ages will change as in situ $^{10}\text{Be}$ production rates are reevaluated (Balco, 2011).

Chevalier et al. (2005a) used apparent exposure ages on the fault-displaced Manikala M1 and M2E moraines to argue that the Karakorum fault slips at a rate of ~10 mm/yr, whereas Brown et al. (2005) argued for a lower slip rate (~5 mm/yr). Chevalier et al. (2005a) report offset distances for these moraines of ~220 m and ~1520 m. Dividing these distances by our preferred age estimates of 39.8 ka and 110 ka, respectively, suggests that the time-averaged slip rate on the fault has diminished considerably, from ~18 mm/yr between deposition of the M2E and M1 moraines, to ~6 mm/yr after deposition of the M1 moraine. This result is consistent with the original interpretation of Chevalier et al. (2005a) that the fault has slowed down.

In contrast to Brown et al. (2005), we find that the nature of geomorphic biases on the Manikala moraines changes with stratigraphic age (Fig. 6). Moraine degradation appears to be the dominant geomorphic process on the youngest moraine, M1, but inheritance takes over on
the older moraines M2 and M3. Thus, an increase in scatter among apparent exposure ages with stratigraphic age does not necessarily indicate that all the moraines are affected primarily by postdepositional processes. Larger, more extensive glaciers sample rarely-visited parts of their catchment that contain clasts with large inherited nuclide concentrations. Thus, we expect moraines farther from a glacier’s accumulation area to show more inheritance than moraines closer to the cirques (see also Gosse et al., 2003).

Barrows et al. (2007a, 2008) used their apparent exposure ages to argue for a post-Younger Dryas true age for the Waiho Loop moraine. After recalculating their $^{10}$Be apparent ages with an updated, local production rate (Putnam et al., 2010), our model fits suggest a true age for the Waiho Loop moraine of 13.0 ka. This age estimate falls near the end of the Antarctic Cold Reversal, which primarily affected the Southern Hemisphere and preceded the Younger Dryas in the Northern Hemisphere. This age estimate is also consistent with radiocarbon apparent ages from Canavan’s Knob, which require the moraine to be younger than 13.0-13.1 ka (Denton and Hendy, 1994; Turney et al., 2007). Thus, the Waiho Loop was likely deposited before the Younger Dryas, rather than after it.

*Further work*

In the future, we hope to combine our models of moraine degradation and inheritance and to estimate the uncertainties of our moraine age estimates. Estimating uncertainties using resampling techniques requires many hundreds of model evaluations, so the revised model will need to be written in a faster programming language such as Fortran. We expect that the geomorphic uncertainties of exposure dating will prove to be larger than measurement error or production rate uncertainty at many sites.

The topographic profiles of moraines contain information on the product of moraine age
and topographic diffusivity (Hanks, 2000; Phillips et al., 2003; Putkonen and O’Neal, 2006). We have already measured topographic profiles on the Lake Fork and Yellowstone moraines; we plan to incorporate this information into our model inversions.

Despite the work that still needs to be done, we believe that our methods represent a promising avenue for future investigation. Any final answer to the problem of how to interpret cosmogenic nuclide measurements from moraine boulders must explicitly account for the statistical distributions of the nuclide measurements. Our work represents a step toward this eventual solution.

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Figures

Figure 1. Comparison of distributions produced by models of moraine degradation and inheritance (Applegate et al., 2010) to a measurement error-only case. The distributions are shown as histograms and cumulative density functions (black lines; Chambers et al., 1983). Each distribution includes $10^5$ synthetic observations. The model histograms use the default parameter values given in Applegate et al. (2010); the measurement error-only case assumes 3% uncertainty. The true age of the moraine in each panel is 20 ka (dashed lines). Both moraine degradation and inheritance give very large reduced chi-squared scores ($\chi^2_R$) relative to the measurement error-only case. We can distinguish moraine degradation from inheritance using the skewness and the curvature of the cumulative density functions (convex-up for degradation, convex-down for inheritance). Compare to Figure 4.

Figure 2. Fits of the models to synthetic data (black dots with error bars). The inverse method fits the data successfully, if we have guessed the underlying process correctly (black lines). If we use the wrong model (dashed, gray curves), the visual match between the best-fit curve and the data is poor, and the KS statistic value is larger than if the correct model is used. Thick, vertical lines on the x-axis indicate the age estimate from each model fit. Compare to Figure 6.

Figure 3. Tradeoffs among parameters when fitting the models to synthetic data. In each panel, one model parameter is held constant at its true value, and the other two parameters are varied over a grid (see text). The grayscale patches indicate the quality of the resulting fit, scaled linearly between the best and worst observed KS statistic values. The KS statistic is sensitive to variation about the true age for both models. However, there is a strong tradeoff between the geomorphic parameters for the inheritance model, and almost any combination of the geomorphic parameters will produce a good fit in the degradation model (right-hand panels).

Figure 4. Published beryllium-10 apparent exposure ages, arranged by moraine or moraine group (Greenland, Kelly et al., 2008; Uinta Mountains, Munroe et al., 2006; Tibet, Chevalier et al., 2005a; New Zealand, Barrows et al., 2007). See Table 1 for moraine name abbreviations. Each histogram contains $n+1$ evenly-spaced bins, with the first and last bins centered on the youngest and oldest apparent exposure ages, respectively. Dots with error bars represent the individual apparent ages, arranged as cumulative distribution functions and scaled to match the vertical extent of the histograms. Compare to Figure 1.

Figure 5. Suggested procedure for interpreting collections of apparent exposure ages from single moraines. See text for discussion.

Figure 6, part 1. Fits of the moraine degradation and inheritance models to published beryllium-10 apparent exposure ages from eastern Greenland (Kelly et al., 2008) and the southern Uinta Mountains (Munroe et al., 2006). The proximal Lake Fork moraine (LFp in Fig. 4) is omitted; the scatter in the data are consistent with the measurement uncertainty of the apparent ages, so no fitting is needed (see text). Thick, vertical lines on the x-axis indicate the age estimate from each model fit. The true age of the moraine might lie anywhere between the age estimates from the two model fits, but we prefer the age estimate from the better-fitting model (solid, black curve). Compare with Figure 2.
Figure 6, part 2. As part 1, but shows fits of the moraine degradation and inheritance models to published beryllium-10 apparent exposure ages from the Manikala moraines, Tibet (Chevalier et al., 2005a) and the Waiho Loop moraine, New Zealand (Barrows et al., 2007).

**Table**

Table 1. Comparison of age estimates from the model fits (Fig. 6) to other methods of inferring moraine age.

**Appendix Tables**

Table S1. Beryllium-10 apparent exposure ages from the 11 moraines treated in this study, plus best-fit model parameter estimates.

Table S2. Parameter estimates from fitting models to synthetic data sets.


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