Quantum chromodynamics, resonances, and the Riemann-Hilbert problem

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20 April 2010
Motivation

- Data Analysis Center/Center for Nuclear Studies
  - **SAID**: suite of programs to analyze 2 \( \rightarrow \) 2 & 3 body scattering and reaction data
  - Routines: database, fit, and analysis
  - Reactions: \( \pi N \rightarrow \pi N, \pi \pi N; \quad KN \rightarrow KN; \quad NN \rightarrow NN'; \quad \pi d \rightarrow \pi d; \quad \pi d \rightarrow pp; \quad \gamma N \rightarrow \pi N, \eta N, \eta' N, \eta N, \gamma N; \quad eN \rightarrow e\pi N \)

- Current studies
  - Meson-nucleon reactions
  - Electromagnetic meson production: photo- & electro-production

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**Objective: Learn about QCD**

- Strongly interacting
- Infinitely many degrees-of-freedom
- Non-linear

\( \Rightarrow \) QCD is a challenging theory to solve

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In our terminology, we sometimes use 'reaction' to include elastic scattering.

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\( ^a \text{Web: } \text{http://gwdac.phys.gwu.edu/} \)

\( ^b \text{ssh: } \text{ssh -C -X said@said.phys.gwu.edu [passwordless]} \)
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Quantum physics

Quantum mechanics

- Dynamical variables \( \{x(t), p(t)\} \rightarrow [\hat{x}, \hat{p}] = i\hbar \) – ‘uncertainty’ principle (HUP)
- Quantum “weirdness” – position and velocity not definite simultaneously
- Wave-particle duality
  - wave ↔ continuum properties in propagation
  - particle ↔ energy exchanged discretely
- Fixed number of particles

Quantum field theory

- Dynamical variables → fields: \( \phi_\alpha(t, r) \)
- Predicts antiparticles: same mass, spin; opposite charge(s)
- Arises inevitably if:
  - Local: ‘action at a distance’ isn’t allowed
  - Poincaré (⊂ Lorentz xform) invariant: relativistic
  - Cluster decomposition: distant experiments are not correlated
  - CPT invariance
- Variable number of particles necessitated: HUP & Lorentz invariance
Outline

1. Quantum chromodynamics
   - Quantum field theory
   - Gauge theory

2. Resonance
   - Phenomena of resonance
   - Description of resonance
   - Resonance structure

3. Reaction theory
   - Experiments
   - Formalism

4. Amplitude parameterization
   - Complex energy plane
   - Analytic structure
   - SAID Parameterization

5. Modeling
   - Particles and fields
   - Dynamics
   - Results

6. Conclusion
The “strong force”
Empirical considerations

- Strong & short ranged compared to electromagnetic, weak, and gravity

- Quarks (and gluons) aren’t directly observed

- Hadrons interact weakly at small momenta
Strong & short ranged compared to electromagnetic, weak, and gravity → Mass gap

Quarks (and gluons) aren’t directly observed

Hadrons interact weakly at small momenta

\[ E(p) = \sqrt{|p|^2 + m^2} \]
The “strong force”

Empirical considerations

- Strong & short ranged compared to electromagnetic, weak, and gravity → Mass gap
- Quarks (and gluons) aren’t directly observed → Color confinement
- Hadrons interact weakly at small momenta

Illustration: Typoform

Indication of confinement from perturbation theory. The running strong coupling constant $\alpha_s$ as a function of the energy, $E$ at which it is measured.
The “strong force”
Empirical considerations

- Strong & short ranged compared to electromagnetic, weak, and gravity
  → Mass gap
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  → Color confinement
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Non-perturbative confinement via monopoles. Possible monopole configurations.
The “strong force”
Empirical considerations

- Strong & short ranged compared to electromagnetic, weak, and gravity → Mass gap
- Quarks (and gluons) aren’t directly observed → Color confinement
- Hadrons interact weakly at small momenta → Chiral symmetry breaking
Gauge theory & QCD

Gauge principle [Weyl, Lee, Yang]

- All four fundamental forces are governed by the gauge principle
  - Electromagnetic: phase invariance
  - Weak: broken non-Abelian symmetry
  - Strong: color invariance
  - Gravity: diffeomorphism invariance

- Invariance under some local symmetry transformations, *eg.*
  1-dim Abelian symmetry electrodynamics:

  \[ \psi(x) \rightarrow e^{i\varphi(x)} \psi(x) \Rightarrow D_\mu \psi(x) = [\partial_\mu - ieA_\mu(x)]\psi(x) \]

  \( A_\mu(x) \), the four-vector electromagnetic potential, ‘compensates’ for possible changes in the phase and has its own dynamics

- QCD
  - Internal quantum number “color”: \( R, G, B \)
  - Invariance under local changes of color
  - ‘Compensating’ field are *gluons*, \( G^A_\mu(x) \) – come in 8 types

- Gauge fields are massless, vector bosons\(^1\)

\(^1\)Unless the ground state of the theory breaks the gauge symmetry as in, the Higgs mechanism in the electroweak sector.
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Classical atomic resonance

Dispersion characteristics of (low-density) dielectrics: Classical EOM for an electron ($e > 0$) bound harmonically within a non-conducting material

\[-\frac{e}{m} \mathbf{E}(t, \mathbf{r}) = \ddot{\mathbf{r}}(t) + \gamma \dot{\mathbf{r}}(t) + \omega_0^2 \mathbf{r}(t)\]

\[\mathbf{E}(t) \sim e^{-i\omega t}\]

\[\epsilon(\omega) = 1 + \frac{4\pi Ne^2}{m} \frac{1}{\omega_0^2 - \omega^2 - i\omega \gamma}\]

Response function

- $\text{Re } \epsilon(\omega)$: related to phase velocity ($v = \frac{c}{\text{Re} \sqrt{\mu \epsilon}}$)
- $\text{Im } \epsilon(\omega) \neq 0$: energy dissipation EM wave $\rightarrow$ medium

\[\text{The dielectric constant as a function of frequency.}\]
Quantum atomic resonance

Resonance fluorescence

\[ i\hbar \frac{\partial \psi(t)}{\partial t} = [H_{\text{free}} + H_{\text{int}}] \psi(t) \]

\[ \psi(t) = \sum_k c_k(t) u_k(r) e^{-iE_k t} \]

\[ \dot{c}_m(t) = -i \sum_k \langle m|H_{\text{int}}|k\rangle e^{i(E_m-E_k)t} c_k(t) \]

\[ \dot{c}_I(t) = -i \langle I|H_{\text{int}}|0\rangle c_0 e^{i(E_I-E_0)t} - \frac{\Gamma_I}{2} c_I(t) \]

\[ |c_I|^2 = \frac{\langle I|H_{\text{int}}|0\rangle}{(E_I-E_0-\omega)^2 + \Gamma_I^2/4} \]

Breit–Wigner response fn.
Hadronic^2 resonance

Formation

Associated production

2Necessarily quantum.

M. Paris

QCD, N^*, & R–H
Background/non-resonant vs. resonant

Folklore

Setup:
- target at rest in the lab
- projectile impinges upon the target with energy $E_L$
- interact over (very) short range [neglect, eg. Coulomb]
- scattering elastically or inelastically, receding to infinity

Qualitatively:

Non-resonant
The target–projectile system interact via an attractive force, remaining in proximity for a time, $\tau$ all the while retaining their individual identities, then move off to infinity.

Resonant
The target–projectile system amalgamate to form a compound state, completely losing their individual identities in the process, existing for a time $\tau$ as a metastable state. This compound state may decay into particles whose species are identical to or distinct from the target–projectile species.
Background/non-resonant vs. resonant

QCD

- QCD degrees-of-freedom: quarks & gluons
- Observables are function(al)s of
  \[ \langle 0 | T\{A_1(x_1) \cdots A_n(x_n)\} | 0 \rangle \]
- Consider the quark “dual diagram”
  - Quarks propagate forward in time – ‘up’
  - Antiquarks propagate backward in time – ‘up’
  - Gluon field is implicit and ubiquitous – imagine gluon field
    describing a membrane spanning quark lines
- “Channels”: a single quark diagram describes several
  processes at the hadronic level
  - s-channel: \( \pi^+\pi^- \rightarrow \rho^0 \rightarrow \pi^+\pi^- \)
  - t-channel: \( \pi^+\pi^- \rightarrow \pi^+\pi^- \rho^0 \rightarrow \pi^+\pi^- \)
- **Fact**: non-resonant and resonant are model dependent concepts
- **Query**: Are these useful concepts? And to what extent?
Background/non-resonant vs. resonant

QCD

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The 1950’s proliferation of strongly interacting particles under the pejorative, “Particle Zoo,” drove some fairly serious folks to humor:

- **J.R. Oppenheimer’s lament**: ‘The Nobel Prize should be given to the physicist who did not discover a particle.’

- **W. Pauli’s other career**: [To Leon Lederman] ’If I could remember the names of these particles I would have gone into botany.’

The apparent chaos of the 100’s of known strongly interacting particles was brought to order by M. Gell-Mann, *The Eightfold Way, Symmetries of Baryons and Mesons*, Phys. Rev. **125**, 1962 without explicit reference to quarks.

- Goldberger-Treiman relation from PCAC: \( \frac{f_\pi g_{\pi NN}}{m_N} = g_A \)

- Adler-Weisberger relation:

\[
g_A^{-2} = 1 + \frac{2m_N^2}{\pi g_{\pi NN}^2} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \left[ \sigma_{\pi^- - p} - \sigma_{\pi^+ + p} \right]
\]
The *Eightfold Way* as an irreducible representation (the octet $8$) of the global symmetry group $SU(3)_{\text{Flavor}}$.

**Generators of $SU(3)_{\text{Flavor}}$**

- $\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
- $\lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
- $\lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
- $\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
- $\lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$
- $\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$
- $\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$
- $\lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$

- Hermitian, traceless, $3 \times 3$ complex matrices
- Why 8 generators?
  - $3 \times 3$ elements
  - $2$ real + imag
  - $9 - 1$ traceless
  - $= 8$
- Top row: isospin!
- First two matrices of each column: raising and lowering operators
- Third column: $\lambda_3$ & $\lambda_8$ diagonal

[D]Cartan subalgebra

Use these to classify states...
The **Eightfold Way** as an **irreducible** representation (the octet \(8\)) of the global symmetry group \(SU(3)_{\text{Flavor}}\).

**Quarks/Antiquarks**

\[
q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} = \text{“}3\text{”}
\]

\[
l_3 = \frac{1}{2} \lambda_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

\[
Y = \frac{1}{\sqrt{3}} \lambda_8 = \frac{1}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}
\]

\[
Q = l_3 + \frac{Y}{2} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}
\]
Group theoretic quark model

The *Eightfold Way* as an *irreducible* representation (the octet $\mathbf{8}$) of the global symmetry group $SU(3)_{\text{Flavor}}$

Mesons

$$M = q \otimes \bar{q} = 3 \otimes 3 = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{3}(2u\bar{u} - d\bar{d} - s\bar{s}) & u\bar{d} & u\bar{s} \\ d\bar{u} & \frac{1}{3}(2d\bar{d} - u\bar{u} - s\bar{s}) & d\bar{s} \\ s\bar{u} & s\bar{d} & \frac{1}{3}(2s\bar{s} - u\bar{u} - d\bar{d}) \end{pmatrix}$$

$$+ \frac{1}{3}(u\bar{u} + d\bar{d} + s\bar{s})$$

singlet

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta \end{pmatrix} + |\text{singlet}\rangle$$

$$= 8 \oplus 1 = 9 \text{ states}$$
The *Eightfold Way* as an irreducible representation (the octet 8) of the global symmetry group $SU(3)_{\text{Flavor}}$

**Baryons**

$$B = q \otimes q \otimes q = 3 \otimes 3 \otimes 3$$

$$= 10 \oplus 8 \oplus 8 \oplus 1 = 27 \text{ states}$$

$$8 = \begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & \Sigma^+ & p \\
\Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & n \\
\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda^0
\end{pmatrix}$$

$$10 = \{ \Delta, \Sigma, \Xi, \Omega \}$$
Quantum chromodynamics

Resonance

Reaction theory

Amplitude parameterization

Modeling

Conclusion

Experiments

Formalism

Outline

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   • Dynamics
   • Results

6 Conclusion
Scattering & reactions

Definitions

- **Target**: a particle (elementary or composite) in the lab rest frame
- **Projectile**: a particle (elementary or composite) which impinges on the target
- **Initial state**: target and projectile at (effectively) infinite separation = non-interacting
- **Final state**: daughter particles (any number) at infinite separation
- **Reaction channel**: $n$ particles where $n \geq 1$ in an initial state; *eg.* $e^- e^-, e^+ N, \pi N, \gamma N, \pi\pi N, \ldots$
Scattering & reactions

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- Reaction channel: \( n \) particles where \( n \geq 1 \) in an initial state; *eg.*
  \( e^- e^-, e^+ N, \pi N, \gamma N, \pi \pi N, \ldots \)
- Elastic: kinetic energy conserved
  - Moller scattering: \( e^- e^- \rightarrow e^- e^- \)
  - Bhabha scattering: \( e^- e^+ \rightarrow e^- e^+ \)
  - Rayleigh/Thompson scattering: \( e^- N \frac{A}{2} (i) \rightarrow e^- N \frac{A}{2} (i) \)
  - Compton scattering: \( \gamma e^- \rightarrow \gamma e^-, \gamma p \rightarrow \gamma p, \gamma A \rightarrow \gamma A \) (nucleus \( A = d, \) \( ^3 \)He, \ldots, \ldots)
  - \( \pi N \) scattering: \( \pi^0 p \rightarrow \pi^0 p \) (neutral), \( \pi^+ p \rightarrow \pi^+ p \) (charged)
  - Gold scattering: \( Au Au \rightarrow Au Au \)
Scattering & reactions

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  - Rayleigh/Thompson scattering: \( e^- N \frac{1}{2} A(i) \rightarrow e^- N \frac{1}{2} A(i) \)
  - Compton scattering: \( \gamma e^- \rightarrow \gamma e^-, \gamma p \rightarrow \gamma p, \gamma A \rightarrow \gamma A \) (nucleus \( A = d, ^3 He, \ldots, \ldots \))
  - \( \pi N \) scattering: \( \pi^0 p \rightarrow \pi^0 p \) (neutral), \( \pi^+ p \rightarrow \pi^+ p \) (charged)
  - Gold scattering: \( Au Au \rightarrow Au Au \)
- **Inelastic**
  - Electron-positron annihilation: \( e^- e^+ \rightarrow \gamma \gamma, \) hadrons
  - Raman scattering: \( e^- N \frac{1}{2} A(i) \rightarrow e^- N \frac{1}{2} A(f), i \neq f \)
  - \( \pi N \) scattering: \( \pi^- p \rightarrow \pi^0 n \) (charge exchange)
  - Meson \( \pi \)-production: \( \pi^- p \rightarrow \eta p, \omega p, \ldots \)
  - Meson photoproduction: \( \gamma p \rightarrow \pi^0 p, \pi^+ n, \eta p, \omega p, \pi^+ \pi^- p, \ldots \)
Scattering & reactions

Experimental setup

Incoming current $j_0$

Interaction region

Current $dj$ scattered into $d\Omega$

Detector aperture $dA$

Scattering angle $\theta$

Outgoing spherical wave

Incoming plane wave

Target

Beam axis

Area $A$
Resonance production

- Photoproduction exhibits strong resonance signature (bumps) in all channels
- Single meson production falls-off $E_\gamma \sim 750$ MeV, $W \sim 1500$ MeV
- Coupled-channel treatment absolute necessity
- Aside: energies
  - $E_\gamma$ photon lab energy [experiment]
  - $W$ total COM energy [calculations]
  - $W = (m_N^2 + 2m_NE_\gamma)^{1/2}$
  - $\approx m_N + E_\gamma - \frac{E_\gamma^2}{4m_N}$
  - $E_\gamma = \frac{W^2 - m_N^2}{2m_N}$
  - $= \frac{1}{2}(1 + \frac{W}{m_N})[W - m_N]$
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M. Paris

QCD, $N^*$, & $\mathcal{R}-\mathcal{H}$
Formalism

Scattering matrix $S$

- In/Out states & Scattering matrix
  \[
  \psi_{\alpha}^{\pm} = \begin{cases} 
  \text{In state } \alpha \text{ before scattering} \\
  \text{Out state } \alpha \text{ after scattering}
  \end{cases}
  \]

- Time-translation invariance (倾向于 Poincaré)

- Inverting $[E_{\alpha} - H_0] \rightarrow [E_{\alpha} - H_0 \pm i\epsilon]^{-1}$ with boundary conditions

Generalized Schrödinger equation

Relativistic Lippmann-Schwinger equation

\[
E_{\alpha} \psi_{\alpha}^{\pm} = H \psi_{\alpha}^{\pm} = [H_0 + H_{\text{int}}] \psi_{\alpha}^{\pm} \quad [E_{\alpha} - H_0] \psi_{\alpha}^{\pm} = H_{\text{int}} \psi_{\alpha}^{\pm}
\]

\[
\psi_{\alpha}^{\pm} = \Phi_{\alpha} + \frac{1}{E_{\alpha} - H_0 \pm i\epsilon} H_{\text{int}} \psi_{\alpha}^{\pm}
\]

\[
H_0 \Phi_{\alpha} = E_{\alpha} \Phi_{\alpha}
\]
Formalism

Lippmann-Schwinger equation

- L-S equation
  \[ \psi^\pm_\alpha = \Phi_\alpha + G_0(E_\alpha) V \psi^\pm_\alpha \]
- Interaction mechanisms \( \pi N \rightarrow \pi N \)
- Iteration
  \[ \psi^\pm_\alpha = \Phi_\alpha + G_0(E_\alpha) V \Phi_\alpha + G_0(E_\alpha) V \Phi_\alpha + G_0(E_\alpha) V \Phi_\alpha + \cdots \]

Definitions:
- \( \psi^\pm_\alpha \) exact w.f.
- \( \Phi_\alpha \) homogeneous w.f.
- \( G_0(E_\alpha) = \frac{1}{E_\alpha - H_0 \pm i\epsilon} \) propagator
- \( V \equiv H_{\text{int}} \) interaction

M. Paris  QCD, \( N^* \), \& \( R - H \)
Define **free propagator** \( G_0 \) and **exact propagator** \( G \) which have singularities (denominator \( \to \) zero) in the spectrum of \( H_0 \) or \( H \)

\[
G_0^{-1}(E_\alpha) = E_\alpha - H_0 \pm i\epsilon \\
G^{-1}(E_\alpha) = E_\alpha - H \pm i\epsilon \\
G^{-1} = G_0^{-1} - V \\
G = G_0 + G_0 VG
\]

Rewrite L-S

\[
\Psi_\pm^\alpha = [1 + \ G^\pm V] \Phi_\alpha \\
\Psi_\alpha^- = \Psi_\alpha^+ + (G^- - G^+)V\Phi_\alpha
\]

**S** matrix

\[
S_{\alpha\beta} = (\Psi_\alpha^-, \Psi_\beta^+) \\
= (\Psi_\alpha^+, \Psi_\beta^+) + ([G^- - G^+]V\Phi_\alpha, \Psi_\beta^+) \\
= \delta_{\alpha\beta} + 2\pi i\delta(E_\alpha - E_\beta)(\Phi_\alpha, V\Psi_\beta^+) \mathcal{R-H}!! \\
= \delta_{\alpha\beta} + 2\pi i\delta(E_\alpha - E_\beta) T^+_{\alpha\beta} \\
T^+_{\alpha\beta} = -(\Phi_\alpha, V\Psi_\beta^+)
\]
Differential cross section $1 + 2 \rightarrow 1' + 2'$ (exclusive)

\[
d\sigma \over d\Omega = \frac{\# \text{ particles scattered into } (\theta, \phi)}{\text{unit time} \cdot \text{incident flux}} \\
= \frac{(4\pi)^2}{k^2} \rho_{1'2'}(k')\rho_{12}(k) \left| T_{\lambda_1, \lambda_2', \lambda_1 \lambda_2}(k', k; W) \right|^2
\]

- Complete set of measurements: \# ampls. = $\prod_i N(\lambda_i)$
- Need twice ($\mathbb{C} \rightarrow \mathbb{R}$) number of observables, modulo symmetries ($C, P, T$) & discrete ambiguities
- Polarized particles
- New experiments (FROST, HD-ICE)
- Upcoming complete measurement $\gamma \vec{p} \rightarrow K^+ \bar{\Lambda}$
- Unitarity requires *multi-channel* data, *eg.*
  $\gamma N \rightarrow \pi N, \gamma N \rightarrow \pi \pi N, \gamma N \rightarrow \eta N, \ldots$
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The analytic continuation
The miracle of complex numbers

- Complex analytic functions (*holomorphic, regular*)
  - All derivatives exist everywhere in open domain, $\mathcal{D}$
  - Derivatives independent of direction
    (*Cauchy-Riemann eqs.*)
  - Harmonicity: $u_{xx} + u_{yy} = 0$ and $v_{xx} + v_{yy} = 0$ [Laplace]

- Analytic continuation - AC
  - Analytic function in $\mathcal{D}$ uniquely determined by values on a domain or along an 1-dim curve
  - $f_1(z)$ analytic in $\mathcal{D}_1$ and $f_1(z) = f_2(z)$ in $\mathcal{D}_1 \cap \mathcal{D}_2 \Rightarrow$, then there *may be* $f_2(z)$ analytic in $\mathcal{D}_2$; if so, **unique**.

- Contrast with real functions
  - Analytic $f_1(x)$ on $a < x < b$ and $f_1(x) = f_2(x)$ doesn’t imply $f_2(x)$ is unique (if it exists)

- Cauchy-Gorsat [Green’s/Stoke’s theorem+C-R]
  \[
  \oint_C dz f(z) = 0 \quad \left[ \oint_C d\mathbf{l} \cdot \mathbf{A}(x) = \int d^2 \mathbf{S} \cdot \nabla \times \mathbf{A}(x) \right]
  \]
Poles & resonances
'Toy' model: 1-D scattering

Scattering from a finite square well

\[
V(x) = \begin{cases} 
0 & x \geq \frac{a}{2} \\
-V_0 & -\frac{a}{2} \leq x \leq \frac{a}{2} \\
0 & x \leq -\frac{a}{2}
\end{cases}
\]

\[
\psi_1(x) = e^{ipx} + Re^{-ipx}
\]

\[
\psi_2(x) = Ae^{ipx} + Be^{-ipx}
\]

\[
\psi_3(x) = Se^{ipx}
\]

\[
p = \sqrt{2mW} \quad \bar{p} = \sqrt{2m(W + V_0)} \quad W > 0
\]

\[
S(E)e^{ipa} = \frac{1}{\cos \bar{p}a - \frac{i}{2} \left[ \frac{p}{\bar{p}} + \frac{\bar{p}}{p} \right] \sin \bar{p}a}
\]

\[
T(E) = |S(W)|^2 = \frac{1}{1 + \frac{V_0^2}{4E(E + V_0)} \sin^2 \bar{p}a}
\]
Analytic structure of $S$

Bound states, resonances, & poles

Bound states: $W < 0$

$$
\begin{align*}
\psi_1(x) &= e^{\kappa x} \\
\psi_2(x) &= A \left( \frac{\cos \bar{p} x}{\sin \bar{p} x} \right) \\
\psi_3(x) &= \pm e^{-\kappa x}
\end{align*}
$$

$x < -a/2$  
$-a/2 \leq x \leq a/2$  
$a/2 < x$

$$
\kappa = \sqrt{-2mW} > 0, \ W \leq 0
$$

$$
S(E)e^{ip\alpha} = \frac{1}{\cos \bar{p} a - \frac{i}{2} \left[ \frac{1}{\bar{p}} + \bar{p} \right] \sin \bar{p} a}
$$

Denominator zeros → bound states when $W < 0$

$$
\begin{align*}
\tan \frac{\bar{p} a}{2} &= \frac{\kappa}{\bar{p}} \\
\tan \frac{\bar{p} a}{2} &= -\frac{\bar{p}}{\kappa}
\end{align*}
$$

$p = i\kappa.$
Riemann surface

\[ p = \sqrt{2mW} \quad W \in \mathbb{C} \]

\[ \sqrt{W} = |W|^{1/2} e^{i\theta/2} \]

\[ \theta = \begin{cases} 
0 \leq \theta < 2\pi & \text{‘upper’ sheet} \\
2\pi \leq \theta < 4\pi & \text{‘lower’ sheet} 
\end{cases} \]

Disc \( p \equiv p(W + i\epsilon) - p(W - i\epsilon) \)

\[ = \sqrt{2m|W|[e^{i\cdot0/2} - e^{i\cdot2\pi/2}] = 2\sqrt{2m|W}|} \]

Riemann surface representation of the function \( \sqrt{W} \).

The complex–W plane is horizontal. The vertical axis gives the imaginary part of the function.
Analytic structure of $S$

Bound states, resonances, & poles

Given $T(W) = \text{Re} \, T(W) + i \text{Im} \, T(W)$, for $W > 0$ consider AC in $z = W + i \text{Im} \, z$

$$S(E) e^{i \rho a} = \frac{1}{\cos \rho a - \frac{i}{2} \left[ \frac{p}{p} + \bar{p} \right] \sin \rho a}$$

- Denominator zeros on the second sheet $\rightarrow$ resonances
Analytic structure of $S$

Bound states, resonances, & poles

Given $T(W) = \text{Re} T(W) + i \text{Im} T(W)$, for $W > 0$ consider AC in $z = W + i \text{Im} z$

$$S(E)e^{ip\alpha} = \frac{1}{\cos \bar{\alpha} - \frac{i}{2} \left( \frac{\bar{p}}{p} + \frac{p}{\bar{p}} \right) \sin \bar{\alpha}}$$

$$T(E) = |S(W)|^2 = \frac{1}{1 + \frac{V_0^2}{4E(E+V_0)} \sin^2 \bar{\alpha}}$$

$$\bar{\alpha} = n\pi \rightarrow E_n = n^2 \frac{\pi^2}{2m^2a^2} - V_0$$

- Denominator zeros on the second sheet $\rightarrow$ resonances
Analytic structure of $S$
Bound states, resonances, & poles

Given $T(W) = \text{Re} \, T(W) + i \text{Im} \, T(W)$, for $W > 0$ consider AC in $z = W + i \text{Im} \, z$

$$S(E) e^{i\bar{p}a} = \frac{1}{\cos \bar{p}a - \frac{i}{2} \left[ \frac{p}{\bar{p}} + \frac{\bar{p}}{p} \right] \sin \bar{p}a}$$

$$T(E) = |S(W)|^2 = \frac{1}{1 + \frac{V_0^2}{4E(E+V_0)} \sin^2 \bar{p}a}$$

$$\bar{p}a = n\pi \rightarrow E_n = n^2 \frac{\pi^2}{2ma^2} - V_0$$

- Denominator zeros on the second sheet $\rightarrow$ resonances
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The Riemann–Hilbert problem

Properly: ‘The scalar $\mathcal{R}–\mathcal{H}$ method’

- Reconstruction of complex *sectionally holomorphic* function given boundary data
- Diverse applications in math . . .
  - Find $f(z) = u(z) + iv(z)$ given $\alpha(z(t))u(z(t)) + \beta(z(t))v(z(t)) = \gamma(z(t))$ on a curve $C$
  - [Poisson problem on circle $\alpha = 1, \beta = 0$]
  - Solve (singular) linear integral equations
  - Solve partial differential equations
  - Integral transforms (generalized Fourier transforms)
  - Solve “Fuchsian” system diff. eqs. via representation of monodromy group on the punctured Riemann sphere
  - . . .

- . . . & physics
  - Elasticity: Laplace boundary value prob. on $D^+$
  - Hydrodynamics: non-linear Korteweg-deVries (KdV) equation, $u_t + u_{xxx} + uu_x = 0$ shallow water *soliton* waves
  - Electrostatics: find surface density on $C \Rightarrow$ constant potential
  - Hadronic physics: discontinuity data from unitarity
  - Renormalization group: *Connes & Kreimer* showed that renorm. is equivalent to solving an $\mathcal{R}–\mathcal{H}$ problem
  - . . .
Unitarity

- Unitarity $\leftrightarrow$ Conservation of probability

$$|\Psi^+_{\beta}\rangle = \sum_{\alpha} |\Psi^-_{\alpha}\rangle \langle\Psi^-_{\alpha} | \Psi^+_{\beta}\rangle$$

$$= \sum_{\alpha} |\Psi^-_{\alpha}\rangle S_{\alpha\beta}$$

- Unitarity constraint on $T$

$$S^\dagger S = SS^\dagger = 1 \quad \text{and} \quad S = 1 + 2i\rho T$$

$$T^+ - T^- = 2iT^- \rho T^+$$

$$T^{-1} - T^+ = 2i\rho$$

$$\text{Disc } T^{-1} = -2i\rho$$
Unitarity ↔ analytic structure

\[ \langle \alpha \left| \left\{ T^+ - T^- = 2iT^+ \rho T^- \right\} \right| \beta \rangle \rightarrow T^+_{\alpha \beta} - T^-_{\alpha \beta} = 2i \sum_{\sigma} T^+_{\alpha \sigma} \rho_{\sigma}(W) T^-_{\sigma \beta} \]

\[ \rightarrow \text{Im } T(W) = 2i \sum_{\sigma} T^+_{\alpha \sigma}(W) \rho_{\sigma}(W) T^-_{\sigma \beta}(W) \]

\[ \rho_{\sigma}^{(2)} = \theta(W - (m_{\sigma,1} + m_{\sigma,2}))K_2 \]

\[ \rho_{\sigma}^{(3)} = \theta(W - (m_{\sigma,1} + m_{\sigma,2} + m_{\sigma,3}))K_3 \]

\[ \vdots \]

\[ \rho_{\sigma}^{(n)} = \cdots \]

- ‘Kinks’ due to Heaviside-\( \theta \) function, due to \( \delta(E - H) \)
- Non-analytic function? \([Eden (1952)]\)
- Violation of Cauchy-Riemann conditions → branch points
Unitarity $\leftrightarrow$ analytic structure

$$\langle \alpha | \{ T^+ - T^- \} | \beta \rangle \rightarrow T^+_{\alpha \beta} - T^-_{\alpha \beta} = 2i \sum_{\sigma} T^+_{\alpha \sigma} \rho_{\sigma}(W) T^-_{\sigma \beta}$$

$$\rightarrow \text{Im } T(W) = 2i \sum_{\sigma} T^+_{\alpha \sigma}(W) \rho_{\sigma}(W) T^-_{\sigma \beta}(W)$$

$$\rho^{(2)}_{\sigma} = \theta(W - (m_{\sigma,1} + m_{\sigma,2})) \mathcal{K}_2$$

$$\rho^{(3)}_{\sigma} = \theta(W - (m_{\sigma,1} + m_{\sigma,2} + m_{\sigma,3})) \mathcal{K}_3$$

$$\vdots$$

$$\rho^{(n)}_{\sigma} = \ldots$$

- ‘Kinks’ due to Heaviside-$\theta$ function, due to $\delta(E - H)$
- Non-analytic function? [Eden (1952)]
- Violation of Cauchy-Riemann conditions $\rightarrow$ branch points

Threshold behaviour in quantum field theory

By R. J. Ellis*, Pembroke College, University of Cambridge

(Communicated by P. A. M. Dirac, F.R.S. — Received 31 July 1951 — Revised 17 September 1951)

The elements of the $S$ matrix are functions of the energy and momenta of a set of hadrons. For sufficiently high relative energies of the hadron particles the matrix is analytic and single valued inside the threshold. At the thresholds for each reaction process, the $S$ matrix will have a complicated behaviour. This behaviour is investigated when the $S$ matrix...
Riemann-Hilbert

First blush

\[
([G^- - G^+] V\Phi_\alpha, \Psi_\beta^+) = (\Phi_\alpha, V^+[G^+ - G^-]\Psi_\beta^+)
\]

Plemelj Formula:

\[
G^\pm = \frac{1}{E_\alpha - H \pm i\epsilon}
\]

\[
= \frac{1}{E_\alpha - H} \mp i \lim_{\epsilon \to 0^+} \frac{\epsilon}{(E_\alpha - H)^2 + \epsilon^2}
\]

\[
[\Phi_\alpha, V^+[G^+ - G^-]\Psi_\beta^+] = -2\pi i\delta(E_\alpha - E_\beta)\Psi_\beta^+
\]

\[
\rightarrow S_{\alpha\beta} = \delta_{\alpha\beta} + 2\pi i\delta(E_\alpha - E_\beta) T^+_{\alpha\beta}, \quad T^+_{\alpha\beta} = -(\Phi_\alpha, V\Psi_\beta^+)
\]

- The scattering amplitude is proportional to the discontinuity in $G$ across the real energy axis $E_\alpha$: Disc $G = G^+ - G^- = 2\pi i\delta(E_\alpha - H)$
- Plemelj formula $\Rightarrow$ imaginary part gives coupling to the continuum
- Sectionally holomorphic function
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Chew-Mandelstam approach

Discontinuity data from unitarity: \( \text{Disc} \ T^{-1}(W) = \Im T(W) = -\rho(W) \)

- Direct approach: Cauchy-integral representation or ‘dispersion relation’ \([W \in \mathbb{C}]\)
  
  \[
  T(W) = \frac{1}{2\pi i} \oint_C dW' \frac{T(W')}{W' - W} \\
  T(W) = \int_{W_t}^{\infty} \frac{dW'}{\pi} \frac{\Im T(W')}{W' - W}
  \]

- Alternate approach: Chew-Mandelstam
  
  - Use Heitler \( K \) matrix
    
    \[
    T^{-1} = \Re T^{-1} + \Im T^{-1} = K^{-1} - i\rho \\
    T = K + iK\rho T
    \]

  - Account for the cuts \textbf{exactly} . . .
    
    \[
    T^{-1} = K^{-1} - i\rho = K_{CM} - C \\
    \Im C = -\rho
    \]

  - . . . and parameterize \( K_{CM} \)
    
    \[
    K_{CM} = \sum_n c_n [W - W_t]^n
    \]

- Parameters are fixed by fitting \textbf{scattering observables} (unpolarized diff. x-sec., pol. asymmetries, . . .)
Chi-squared per datum compared with model calculations

\[ \chi^2(p) = \frac{1}{N_{data}} \sum_{i=1}^{N_{data}} \left[ \frac{\Phi_n(i) y_i(p) - Y_i}{\Delta Y_i} \right]^2 + \frac{1}{N_{exp}} \sum_{n=1}^{N_{exp}} \left[ \frac{\Phi_n - 1}{\Delta \Phi_n} \right]^2 \]

<table>
<thead>
<tr>
<th>( \chi^2/\text{Data} )</th>
<th>SP06</th>
<th>FA02</th>
<th>KA84</th>
<th>EBAC</th>
<th>Gießen</th>
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<td>Norm</td>
<td>UnNorm</td>
<td>Norm</td>
<td>UnNorm</td>
<td>Norm</td>
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<tr>
<td>( \pi^+p \rightarrow \pi^+p )</td>
<td>2.0</td>
<td>6.1</td>
<td>2.1</td>
<td>8.8</td>
<td>5.0</td>
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<tr>
<td>( \pi^-p \rightarrow \pi^-p )</td>
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<td>6.2</td>
<td>2.0</td>
<td>6.6</td>
<td>9.1</td>
</tr>
<tr>
<td>( \pi^-p \rightarrow \pi^0n )</td>
<td>2.0</td>
<td>4.0</td>
<td>1.9</td>
<td>5.9</td>
<td>4.4</td>
</tr>
<tr>
<td>( \pi^-p \rightarrow \eta n )</td>
<td>2.5</td>
<td>9.6</td>
<td>2.5</td>
<td>10.5</td>
<td></td>
</tr>
</tbody>
</table>

\[ \pi N \rightarrow \pi N \]

Analytic continuation

Spectroscopic notation: \( L_{2I,2J} - L \): rel. \( \pi N \) orb. ang. mom.; \( I \): isospin; \( J \): total intrinsic ang. mom.
$\pi N \rightarrow \pi N$ dispersion relations

$\pi NN$ coupling; $\sigma$ term

- The fit supplement with dispersion relation (DR) 'pseudo-data'
- Solution method
  - Fit data via $K_{CM}$-matrix parameters
  - Evaluate forward/fixed-$t$ DR’s, evaluate subtraction constants and include deviations from average as pseudo-data
  - Adjust real part of invariant amplitudes (and $K_{CM}$ pars.) to minimize $\chi^2$

**Fixed-$t$ DR**

\[
\begin{align*}
(\nu_B \pm \nu) \left\{ \mp \Re B_{\mp} (\nu, t) \right\} \\
\pm \frac{\nu}{\pi} \int_{\nu_{th}}^{\infty} \frac{d\nu'}{\nu'} \left[ \frac{\Im B_+}{\nu' \mp \nu} + \frac{\Im B_-}{\nu' \pm \nu} \right] \\
= \frac{g^2}{M} + \tilde{B}(0, t)(\nu_B \pm \nu)
\end{align*}
\]

\[g = 13.69 \pm 0.07\]  \[f = 0.0757 \pm 0.0004\]
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Effective field theory

Local, relativistic fields + canonical commutation relations → correct analytics poss.

Hadronic interactions:
\( \pi, \eta, N, \Delta: \)

\[
L_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi}_N \gamma_\mu \vec{\tau} \psi_N \cdot \partial^\mu \bar{\phi}_N,
\]
\[
L_{\pi NA} = -\frac{f_{\pi NA}}{m_\pi} \bar{\psi}_N \gamma_5 \hat{T} \psi_N \cdot \partial_\mu \bar{\phi}_N,
\]
\[
L_{\pi AA} = \frac{f_{\pi AA}}{m_\pi} \bar{\psi}_N \gamma_\mu \vec{\tau} \gamma_5 \hat{T} \bar{\psi}_N \cdot \partial_\nu \bar{\phi}_N,
\]
\[
L_{\eta NN} = -\frac{f_{\eta NN}}{m_\eta} \bar{\psi}_N \gamma_\mu \vec{\tau} \psi_N \gamma_5 \partial^\mu \bar{\phi}_N.
\]

\( \rho: \)

\[
L_{\rho NN} = g_{\rho NN} \bar{\psi}_N \left[ \gamma_\mu - \frac{\kappa_\rho}{2m_N} \sigma_{\mu\nu} \partial^\nu \right] \vec{\rho}^\mu \cdot \vec{r} \bar{\psi}_N,
\]
\[
L_{\rho NA} = -i \frac{f_{\rho NA}}{m_\rho} \bar{\psi}_N \gamma^\mu \gamma_5 \vec{r} \cdot [\partial_\mu \rho_N - \partial_\mu \vec{r} \times \partial_\nu \psi_N] + [h.c.],
\]
\[
L_{\rho AA} = g_{\rho AA} \bar{\psi}_N \left[ \gamma_\mu - \frac{\kappa_{\rho AA}}{2m_A} \sigma_{\mu\nu} \partial^\nu \right] \vec{r} \bar{\psi}_A,
\]
\[
L_{\rho NN} = g_{\rho NN} \bar{\psi}_N \partial_\mu \phi_\rho \partial^\mu \bar{\phi}_N \phi_\sigma.
\]

\( \omega: \)

\[
L_{\omega NN} = g_{\omega NN} \bar{\psi}_N \left[ \gamma_\mu - \frac{\kappa_{\omega}}{2m_N} \sigma_{\mu\nu} \partial^\nu \right] \omega^\mu \psi_N,
\]
\[
L_{\omega pp} = -\frac{g_{\omega pp}}{m_\omega} \epsilon_{\mu\nu\lambda} \bar{\omega}^\mu \partial_\nu \phi_\rho \partial_\lambda \bar{\phi}_N \phi_\sigma.
\]

\( \sigma: \)

\[
L_{\sigma NN} = g_{\sigma NN} \bar{\psi}_N \psi_N \phi_\sigma,
\]
\[
L_{\sigma pp} = -\frac{g_{\sigma pp}}{m_\sigma} \partial^\mu \phi_\rho \partial_\mu \bar{\phi}_N \phi_\sigma.
\]
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Lagrangian density of preceding page → Hamiltonian density

\[ H = \int d^3x \mathcal{H}(x) = H_0 + H_{\text{int}} \]
\[ H_{\text{int}} = \sum_{M, B, B'} \Gamma_{MB, B'} + \sum_{M, M', M''} \Gamma_{MM', M''} \]

Dynamical Lippmann-Schwinger equation

\[ T = V + TG_0 V \]
Hadronic $\pi$ and $\omega$ production

$\pi N \rightarrow \pi N, \omega N$

Real part, isospin 1/2
Hadronic $\pi$ and $\omega$ production

$\pi N \rightarrow \pi N, \omega N$

Imag part, isospin 1/2
Hadronic $\pi$ and $\omega$ production

$\pi N \rightarrow \pi N, \omega N$

Real part, isospin 3/2
Hadronic $\pi$ and $\omega$ production
$\pi N \rightarrow \pi N, \omega N$

Imag part, isospin 3/2
Photoproduction of $\pi$ and $\omega$ production

$\gamma N \rightarrow \pi N, \omega N$

\[
\frac{d\sigma}{d\Omega} \gamma p \rightarrow \pi^0 p
\]
Photoproduction of $\pi$ and $\omega$ production

$\gamma N \rightarrow \pi N, \omega N$

$\frac{d\sigma}{d\Omega} \frac{\gamma p}{\pi^+ n}$

M. Paris
Photoproduction of $\pi$ and $\omega$ production

$\gamma N \rightarrow \pi N, \omega N$

$\Sigma(W) \gamma p \rightarrow \pi^0 p$
Photoproduction of $\pi$ and $\omega$ production

$\gamma N \rightarrow \pi N, \omega N$

$\Sigma(W)\gamma p \rightarrow \pi^+ n$
Conclusion

- Non-perturbative QCD
  - Problem of mass in QCD requires detailed understanding of the hadronic spectrum
- Resonance
  - Signals onset of complex dynamics
- Scattering & reaction amplitudes
  - Comprehensive reaction theory required to make contact between theory and experiment
- Phenomenology
  - Provides an indispensable bridge between measured and calculated quantities
- Modeling
  - Necessarily challenging endeavor of \textit{ab initio} calculations guided by/informs phenomenology
Dedication

To the memory of our friend and colleague, Dick Arndt, GWU Research Professor and Virginia Tech Emeritus Professor, who passed Saturday, April 10, 2010.