ANISOTROPIC VISCOPLASTIC CONSTITUTIVE MODELING FOR METALS AND APPLICATION TO THE TAYLOR IMPACT TEST

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Abstract *An anisotropic viscoplastic constitutive model was developed for the description of the high-strain rate behaviour of zirconium, a HCP metal that exhibits strength differential effects and strong anisotropic strain hardening. The evolution laws for the strength differential parameter and the anisotropy coefficients involved in the expression of the yield criterion were obtained based on numerical tests performed with a crystal plasticity model. The strain rate dependent parameters involved in the formulation were determined from results of dynamic in-plane tests performed on a high purity zirconium plate [9, 10] on a Split Hopkinson Pressure Bar (SHPB). Simulations of the Taylor impact test were conducted using the finite element code ABAQUS with the anisotropic viscoplastic model. The very good agreement between the simulated and experimental post-test geometries of the Taylor specimens in terms of major and minor side profiles and impact interface footprints shows the ability of the phenomenological model to capture the influence of twinning on texture evolution.*

1 Introduction

The importance of an adequate description of plastic anisotropy has been demonstrated in many low strain rate forming applications (e.g., [17,18,14]). Recent efforts in the development of computational models to describe temperature and strain-rate effects on the inelastic response have given rise to robust predictive methods for simulating the anisotropic high strain rate behaviour of body centred (BCC) and face centred (FCC) cubic polycrystals. Maudlin et al. [8, 9] employed a representation of the yield surface from polycrystal calculations to model the mechanical response of a textured BBC tantalum sheet. Anand et al. [1] developed finite element (FE) crystal plasticity models for describing the high strain rate and large deformation behaviour of both BCC and FCC polycrystals. These models describe qualitatively well the important mushrooming of the Taylor impact specimen at the contact interface.

In contrast, the prediction of the geometry changes observed in hexagonal close packed (HCP) specimens remains a challenge [9, 10]. Metallographic investigations have shown that while deformation in HCP materials under quasi-static and dynamic loading includes both slip

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and deformation twinning, the propensity of twinning is higher for deformation at large strain rate and/or low temperature. Therefore, in order to describe with accuracy the dynamic response of HCP materials it is imperative to account for the most important sources of anisotropy in the given material: slip and/or twinning activity, substructure evolution at grain level, and texture development during deformation.

Since in crystal plasticity models the distribution of crystal orientations, the available slip and/or twinning systems, and their activation stress levels are taken into account explicitly, the evolution of anisotropy due to texture development can be characterized by measuring the initial texture and calculating grain reorientation (i.e. updating the texture) using a suitable homogenization scheme. Recently, the development of crystal plasticity models for HCP metals and their implementation into finite element (FE) codes have received much attention [7]. Models that account for both slip and twinning activity and employ full constraint [2] or self-consistent (e.g., [7, 16, 15, 13]) averaging schemes to predict the aggregate behaviour have been proposed. So far, these models have been validated only for quasi-static deformation.

This paper presents a macroscopic viscoplastic description of the dynamic anisotropic plastic response of textured metals. Key in this development is the use of a recently proposed anisotropic yield function [4] (denoted in what follows as CPB06) that is capable of describing simultaneously anisotropy and tension/compression asymmetry. Moreover, if the parameter associated with strength-differential (SD) effects is set to zero, the CPB06 criterion also describes accurately the yielding of metals with tension/compression symmetry.

The outline of the paper is as follows. Section 2 presents an overview of the proposed viscoplastic model. In section 3, an application of the proposed formulation to a high purity clock-rolled zirconium plate sample is presented for quasi-static loading. The parameters involved in the evolution laws are determined on the basis of experimental information and simulation results obtained with the viscoplastic self-consistent (VPSC) polycrystal model of Lebensohn and Tomé [7]. A validation of the anisotropic low strain rate formulation is provided by comparing FE simulations with experimental results. In Section 4, the viscoplastic extension of the model is used in an elasto-viscoplastic FE code to simulate Taylor cylinder impact test for the Zr plate. An application to a BCC textured tantalum plate sample is also briefly discussed. It is shown that the model describes with accuracy the striking difference between the geometries of the Taylor impact post-test specimens of zirconium and tantalum.

2 Anisotropic viscoplastic model

For most hexagonal closed packed metals (e.g., Ti, Mg, Zr, etc.), at low temperatures or high strain rates, twinning plays an important role in plastic deformation. The grains cannot accommodate certain shape changes because they lack the necessary deformation systems or because these systems require high activation stresses. Unlike slip, twinning is sensitive to the sign of the applied stress, which is conducive to a strength differential (SD) effect. Furthermore, the strong crystallographic texture displayed by HCP materials leads to a pronounced anisotropy. To describe plastic anisotropy and yield asymmetry, Cazacu et al. [3, 4] proposed two yield functions, one based on the general theory of tensor representation and the other based on a linear transformation of the stress deviator. The latter is based on the principal values $\tilde{S}_k$ of the tensor $\tilde{\mathbf{s}}$ defined as $\tilde{\mathbf{s}} = \mathbf{C}\mathbf{s}$. The fourth order tensor $\mathbf{C}$ contains the anisotropy coefficients, accounts for the macroscopic symmetries of the material, and reduces to the identity for isotropic materials. The yield condition can be written as

$$\phi = \big\| \tilde{S}_1 \big| - k\tilde{S}_1 \big|^a + \big\| \tilde{S}_2 \big| - k\tilde{S}_2 \big|^a + \big\| \tilde{S}_3 \big| - k\tilde{S}_3 \big|^a = \bar{\sigma}^a , \tag{1}$$

where a and k are real constants. This formulation, although pressure insensitive, breaks the tension-compression symmetry through the parameter k, which leads to the following tensile to compressive yield stress ratio for an isotropic material,

$$\frac{\sigma_t}{\sigma_c} = \left\{ \frac{2^a \left(1+k\right)^a + 2\left(1-k\right)^a}{2^a \left(1-k\right)^a + 2\left(1+k\right)^a} \right\}^{1/a} . \tag{2}$$

In the isotropic case, this yield function can reproduce almost perfectly the yield surfaces of randomly oriented FCC and BCC polycrystals deforming solely by twinning and computed either with full constraint [5] or viscoplastic self-consistent [7] crystal plasticity models. Both approaches capture the strength-differential (SD) effect with a ratio of compressive to tensile yield stress respectively larger and smaller than 1 for FCC and BCC materials.

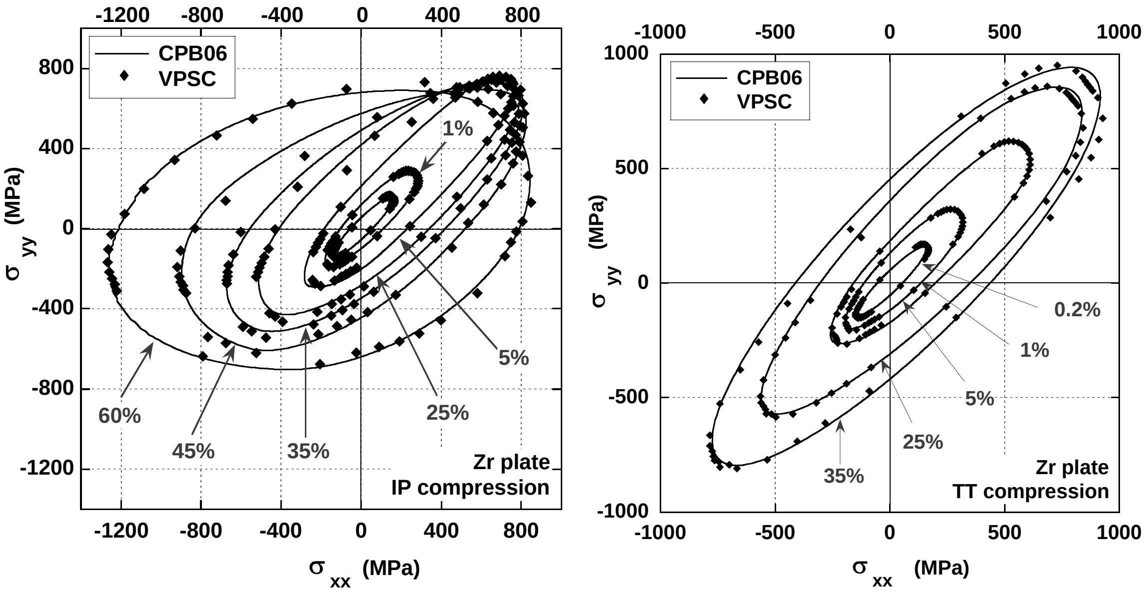

Figure 1: *Theoretical yield surface evolution for a zirconium clock-rolled plate during in-plane (IP, left) and through thickness (TT, right) compression for various levels of pre-strain. Symbols correspond to yield surfaces computed using the VPSC crystal plasticity code [7] to fixed pre-strain levels; Solid lines correspond to CPB06 approximations of the VPSC results.*

Plunkett et al. [12] developed an anisotropic elasto-viscoplastic model based on Perzyna's model [10] for the viscous response. This model is able to take into account the simultaneous influence of strain rate, temperature and anisotropy on the inelastic response of a textured metal for monotonic loading paths. The basic assumption is that the viscous properties become noticeable only after initiation of plastic deformation. Thus, the strain rate can be decomposed additively into elastic and viscoplastic ($\dot{\varepsilon}_{VP}$) parts. The evolution of the viscoplastic strain rate is considered to be given by an overstress type law of the form,

$$\dot{\boldsymbol{\varepsilon}} = \chi \left\langle F(\varphi) \right\rangle \frac{\partial g}{\partial \boldsymbol{\sigma}}, \tag{3}$$

where

$$\left\langle F(\varphi) \right\rangle = \begin{cases} 0 & \text{for } \varphi \le 0 \\ \varphi^m & \text{for } \varphi > 0 \end{cases}. \tag{4}$$

In Eqs. (3) and (4), φ is the quasi-static yield condition expressed in the form

$$\varphi = \frac{\bar{\sigma}}{h\left(\bar{\varepsilon}_{vp}, T\right)} - 1 \tag{5}$$

$g = \varphi$ and $h\left(\bar{\varepsilon}_{vp}, T\right)$ are the quasi-static plastic potential and hardening function, respectively, χ is a viscosity parameter, and m is a strain rate sensitivity constant. The overstress law (3) reduces to the classic plasticity flow rule when the strain rate approaches zero [10]. The quasi-static yield function is the anisotropic version of Eq. (1). The model leads to a yield condition that depends on the accumulated equivalent viscoplastic strain and, possibly, of the temperature [12]. This model was implemented in the commercial finite element (FE) code ABAQUS.

3 Application to static loading

In order to test the ability of the yield function in Eq. (1) to capture plastic anisotropy as well as SD effects, multiaxial yield stresses were generated using a crystal plasticity model for a clock-rolled plate of zirconium, which was studied extensively by Maudlin et al. [8, 9], Kaschner and Gray [6], and Tomé et al. [16]. The visco-plastic self-consistent (VPSC) code [8] was first used to characterize the evolution of the yield surface during in-plane (IP) compression [12]. For this purpose, the polycrystal was pre-strained up to given deformation levels. Figure 1 shows the biaxial stresses of the VPSC yield loci (symbols) corresponding to the different pre-strain levels as indicated in the figure. The experimental initial texture consisted of 377 orientations measured on this clock rolled zirconium plate. The deformation mechanisms operational at room temperature (i.e., prismatic $\langle a \rangle$-slip, pyramidal $\langle c+a \rangle$-slip and tensile twinning) were assumed. The values of the slip and twinning parameters (critical stresses, hardening coefficients, and rate sensitivity exponent) reported by Tomé et al. [16] were used as input.

Next, for each individual pre-strain level, the data points calculated with the VPSC model were approximated with a least square procedure applied to the yield function, Eq. (1) resulting in pre-strain dependent anisotropy coefficient sets. Figure 1 shows that the strong anisotropy as well as SD effects are well predicted by the yield function, Eq. (1). The non-isotropic hardening effect during in-plane (IP) compression is thus adequately captured by the evolution of the anisotropy coefficients, which corresponds to the rapid changes in texture due to twinning as deformation proceeds. Moreover, a linear interpolation technique was used in this work [12] in order to phenomenologically quantify the yield surface (texture) evolution during any proportional loading paths. Figure 1 shows also results for through-thickness (TT) compression tests. By comparing IT and TT compression results, this figure shows that the evolution of the yield surface is strongly influenced by the deformation mode. Moreover, the crystal plasticity results are very well captured by the phenomenological description of anisotropy, Eq. (1).

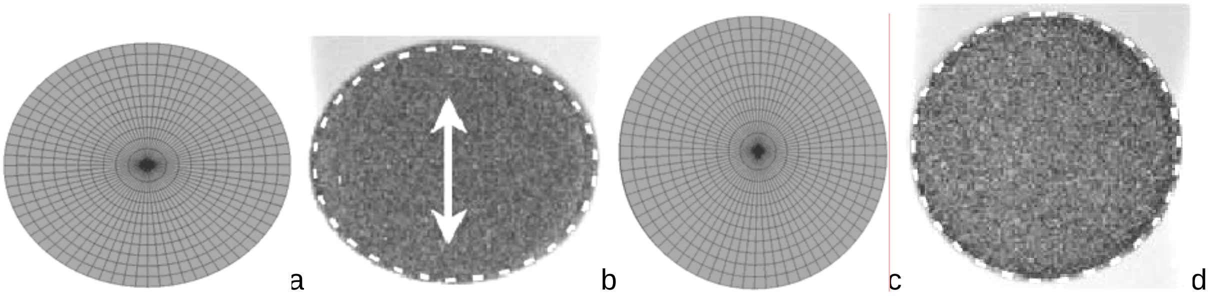

Figure 2: *Comparison of the final cross-sections shapes for quasi-static room temperature cylinders of Zirconium deformed by in-plane (IP) (a, b) and through-thickness (TT) (c, d) compression tests. (a) Final ABAQUS FE mesh obtained with the proposed model; (b) Photograph of the final experimental cross-section and EPIC/VPSC simulation results (dashed lines) reproduced after Tomé et al., [17] (the arrow depicts the direction of the <c> axes). (c) Final ABAQUS FE mesh; (d) Photograph of the final cross-section and EPIC/VPSC simulation results (after Tomé et al. [17]).*

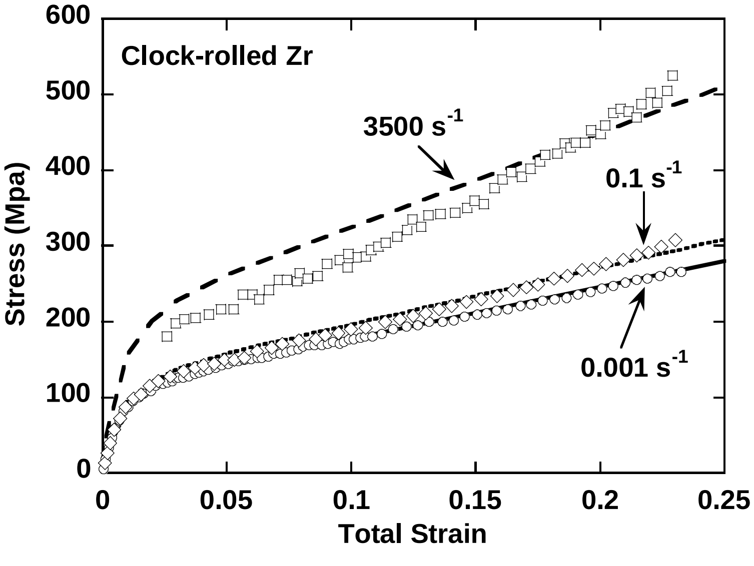

Figure 3: *In-plane compression stress-strain curves for clock-rolled zirconium samples deformed at various strain rates. Symbols: experimental data, after Kaschner and Gray [6]. Lines: simulation results using the proposed viscoplastic model for* $\gamma = 2500\ s^{-1}$ *and* $m = 7.0$.

The proposed model was used to simulate the quasi-static uniaxial compression of a right cylinder of circular cross section with its axis parallel to the rolling direction up to 28% longitudinal strain, for comparison with analog experimental results reported by Tomé et al. [16] in Fig. 2. Due to plastic anisotropy, the simulations predict in-plane expansion of 22% and out-of-plane expansion of 6% (or an ovalness long/short radii=1.19), which agrees very well with the experimental results reported in Tomé et al. [16]. It is worth noting that under the assumption of isotropic hardening and for fixed values of the anisotropy coefficients (i.e. texture evolution neglected), the predicted expansions would have been of 27% in-plane and 1% out-of-plane. This clearly implies that a good agreement with the experimental results can be obtained only if the changes in anisotropy induced by texture evolution (in this case, a decrease in plastic anisotropy) are taken into account.

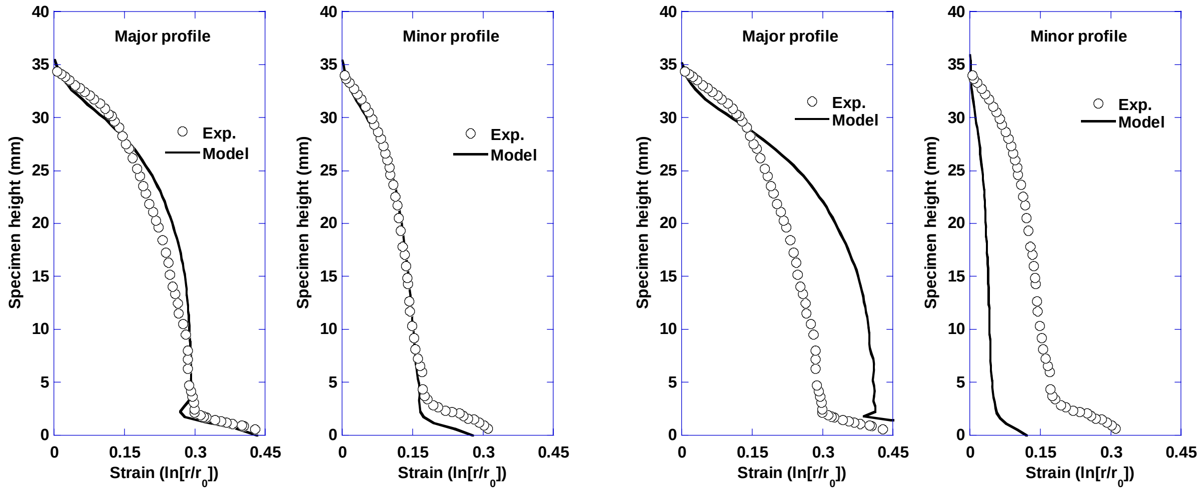

Figure 4. *Comparison of theoretical and experimental (symbols) logarithmic strain profiles for the post test zirconium Taylor impact specimen. Simulations were performed assuming anisotropic (left) and isotropic,(i.e., constant yield surface shape, right) hardening. Data after Maudlin et al. [8, 9].*

Figure 2a shows the final ABAQUS FE mesh. The photograph of the final experimental cross-section shape and the superimposed simulation results (obtained using the FE program EPIC directly linked to the VPSC code) are shown in Fig. 2b (from [16]). The comparison demonstrates that very good agreement with the experimental results can be obtained by using an appropriate initial yield surface representation (CPB06) and a hardening law that accounts for texture evolution. The through-thickness (TT) compression was modeled as well (Fig. 2c&d). Again, a good agreement was observed between FE predicted and experimental results. Note that for this TT orientation, the ovalness of the specimen is not as marked as that of the IP case.

4 Application to dynamic loading

In addition to the deformation induced yield surface evolution (see Fig. 1 above), the viscosity coefficient c and the strain rate sensitivity parameter m in Eqs. (3) and (4) were determined (see Fig. 3). These parameters were found to be $c = 2500 s^{-1}$ and $m = 7$ from results of dynamic in-plane tests performed on the clock-rolled Zr material. These tests were conducted and reported by Kaschner and Gray [6] on a Split Hopkinson Pressure Bar (SHPB) at strain rates ranging from $1000s^{-1}$ to $3500s^{-1}$. The constitutive behaviour for the in-plane compression corresponding to strains beyond the range of the experimental data was estimated based on polycrystal calculations. The proposed viscoplastic model was then applied in FE simulations of the Taylor impact test on the clock-rolled Zr at room temperature. The predicted results were compared with experimental data reported by Maudlin et al. [8, 9]. The predicted anisotropic behavior, characterized by the major and minor side profiles (Fig. 4) was in excellent agreement with the experiments. It is worth noting on this figure the influence of the hardening assumption used in the simulations.

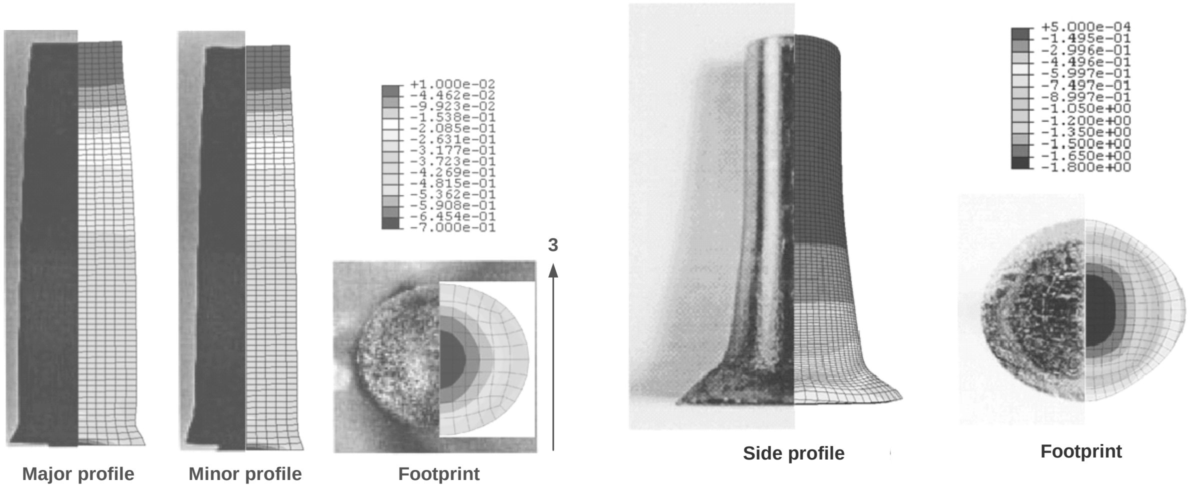

Figure 5: *Comparison of the simulated (with contours of axial plastic strain) and experimental cross-sections of the post-test Taylor impact experiments for the major profile, the minor profile, and the footprint. (Zirconium left) and tantalum (right). Photographs of the post-test specimen after Maudlin et al. [8, 9].*

A constitutive description of the same type was also conducted for a BCC Tantalum plate. The CPB06 criterion can also be used to represent the yielding behavior of a material with cubic crystallographic structure deforming solely by slip by simply setting the strength-differential coefficient k to zero. Fig. 5 shows the mesh and a photograph of the major and minor profiles, as well as the impact footprint, of the specimens after high strain rate impact for both Zr and Ta plates. The simulated and experimental post-test specimen shapes were found to be in good agreement. All the results presented in this paper validate the anisotropic viscoplastic model described by Eqs. (1) to (5).

5 Conclusions

A macroscopic anisotropic viscoplastic model that captures the influence of evolving texture on the mechanical response of textured metals for both quasi-static and dynamic loading conditions was proposed. This model was able to predit the experimental post-test geometries of Taylor impact test specimens in terms of major and minor side profiles and impact footprints for HCP zirconium and BCC tantalum plates.

6 References

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