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Nested Test for Point Sources

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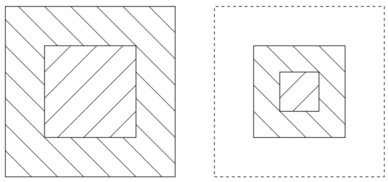
ABSTRACT We describe an extension of a test proposed by Lampton for detecting weak and/or short-lived transient point sources in finite-resolution photon-limited maps of the sky. We test the null hypothesis that the count density is spatially uniform (*i.e.*, no point sources) over a limited region of the sky by decomposing that region into a nested set of source regions and background annuli, and testing whether the count density in each source region differs significantly from the count density in each associated background. The tests are by construction *independent*, so a combined p-value can be rigorously defined.

Our aim is to discriminate real point sources from Poisson fluctuations in the background. The maximum-likelihood formalism prescribes a "matched filter" in which the telescope point-spread function is convolved with the sky map. Points on the convolved map that exceed a background-dependent threshold are identified as sources. If the threshold is set properly (and that is straightforward in the high count limit or whenever the uncertainties are Gaussian), then this technique is demonstrably optimal. The low count regime (in which the statistics are Poisson) is more problematic, though empirical corrections have been proposed [Mar94].

Lampton[Lam94] described a statistical test for point source detection that is in general less powerful than the matched filter, but it does not require a separate estimate of the background, and it is strictly valid even for arbitrarily low counts. In this test, a source region is surrounded by a (typically annular) background region, and the number of photons in each region is counted. If c_S (resp. c_B) is the number in the source (resp. background) region, then $p = I_f(c_S, c_B + 1)$ is the p-value associated with the null hypothesis that the source and background count densities are the same. Here, I_f is the incomplete beta function, and $f = A_S/(A_S + A_B)$ is the ratio of the area of the source region to the sum of the areas of the source and background regions.

In the nested Lampton test, the source region itself is re-divided into source and background regions, and the Lampton test is then applied to these two subregions. This second test assesses whether there are more counts in the center of the source region than there are around the edges. It is important to realize that this second test is strictly independent of the first test. In fact, it is this independence that permits us to reliably and efficiently combine the two p-values into a single measure of confidence.

If we have two p-values, p_1 and p_2, we can always multiply them (let $p_* = p_1 p_2$), but we cannot interpret their product p_* as a p-value, because the product is not uniformly distributed on the unit interval. But it is straightforward to correct for this, and if we let $p = p_*(1 - \log p_*)$, then this "corrected product" can be used as a valid p-value. To combine k independent p-values, use $p = R_k(p_*)$, where $p_* = p_1 p_2 \ldots p_k$ is the product, and $R_k(p_*) = p_* + \int_{p_*}^1 R_{k-1}(p_*/p)\, dp$. It is also possible to use weighted combinations. If we write $p_* = p_1^a p_2^b$, then $a > b$ more heavily weights the first test. Here, the corrected value is given by $p = (a p_*^{1/a} - b p_*^{1/b})/(a-b)$.



src	Median p-values: p_1	p_2	Combined
0	0.5302	0.5716	0.5722
15	0.1403	0.0670	0.0396
25	0.0366	0.0077	0.0017
40	0.0027	0.0002	4.5×10^{-6}

FIGURE 1. The left two figures illustrate the decomposition of the 13×13 region of interest into a 9×9 source region and a background annulus, and the further division of the source region again into a 3×3 source and background. The table on the right shows the results of a Monte-Carlo experiment in which a gaussian point-spread source (with $\sigma = 1.5$ pixels) is placed at the center of this region with low count density background (1 count per pixel), and p-values are computed for the individual and for the combined tests.

Compared to traditional methods (*e.g.,* see Simes [Sim86]), the corrected product rule is not as robust to non-independence of the separate tests, but it does produce a smaller combined p-value in the regime where p_1 and p_2 are both small. This is crucial in large sky maps, because individual point source detections require very high levels of significance.

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1 References

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