Elastic Nonlinear Response in Granular Media under Resonance Conditions

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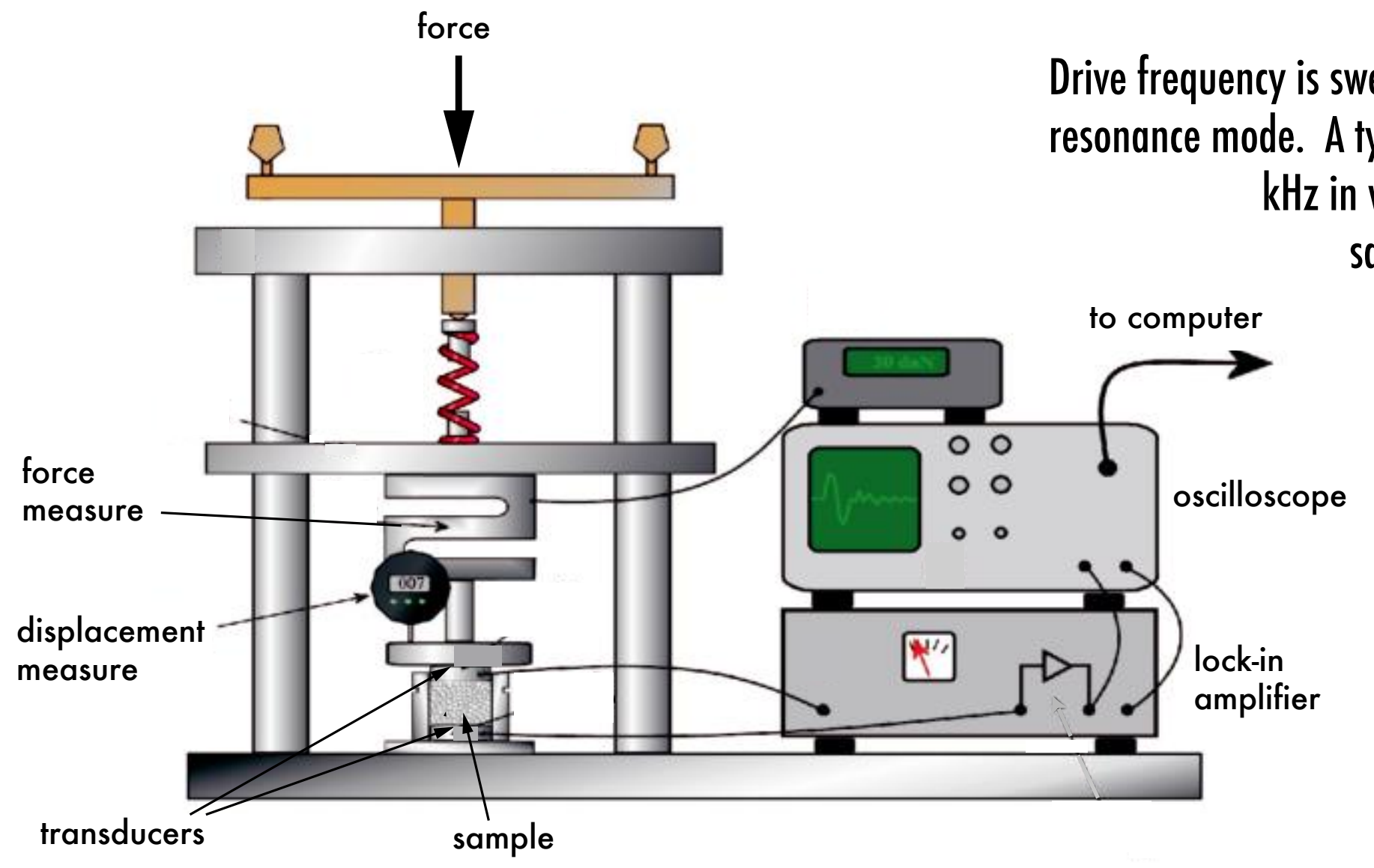
Abstract

We are studying the elastic linear and nonlinear behavior of granular media using dynamic wave methods. In the work presented here, our goal is to quantify the elastic nonlinear response by applying both wave resonance and pulse-mode measurements. Resonance studies are desirable because they provide the means to easily study amplitude dependencies of elastic nonlinear behavior and thus to characterize the physical nature of the elastic nonlinearity. Pulse-mode studies provide information about the time scale of the onset of nonlinear response. This work has implications for a variety of topics, in particular, the in situ nonlinear response of surface sediments and dynamic earthquake triggering. For this work we constructed an experimental cell in which high sensitivity dynamic resonance studies were conducted using granular media under controlled effective pressure. We limit our studies here to bulk modes but have the capability to employ shear waves as well. The granular media are composed of glass beads held under pressure by a piston, while applying resonance waves from transducers as both the excitation and the material probe. The same apparatus is used in pulse-mode experiments.

We find significant elastic nonlinearity at all effective pressures studied, manifest by the fundamental-mode resonance curves decreasing progressively, at progressively increasing drive level. This is equivalent to progressive material softening with wave amplitude, meaning the wavespeed and modulus diminish. The wave dissipation simultaneously increases (Johnson and Sutin, 2004). For example, at 0.11 Mpa effective pressure the observed change in resonance frequency of about 2.6% corresponds to a material bulk modulus decrease of about 5.2%. Strain amplitudes are 10^{-7}–10^{-6}. Thus, we would predict that surface sediments should have significant elastic nonlinear response beginning at about 10^{-6} strain amplitude.



Experimental Configuration

High sensitivity dynamic resonance and pulse-mode studies were conducted using granular media, glass beads, under controlled effective pressure (see figure below). The glass beads are held under pressure by a piston, while applying resonance waves from transducers as both the excitation and the material probe. The container is closed with two fitted pistons and a normal load is applied to the granular sample across the top piston. Force and displacement are measured directly.

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Resonant frequency sweeps with frequencies corresponding to the fundamental bulk mode are conducted with the apparatus. The glass beads used in our experiments are of diameter 0.5 mm, deposited in a duralumin cylinder of diameter 30 mm and height of 15 mm. This corresponds to a granular skeleton acoustic wave velocity of 750 m/s under 50 N of force [0.07 Mpa] (Jia, 2004). The loaded system gives fundamental mode resonances in the audio frequency band at approximately half a wavelength.

Plane-wave generating and detecting transducers of diameter 30 mm are placed on axis at the top and bottom of the cylindrical container in direct contact with the glass beads. The wave signals are detected using a lock-in amplifier. Drive frequency is swept from below to above the resonance mode. The resonance frequency at peak amplitude corresponds directly to modulus:

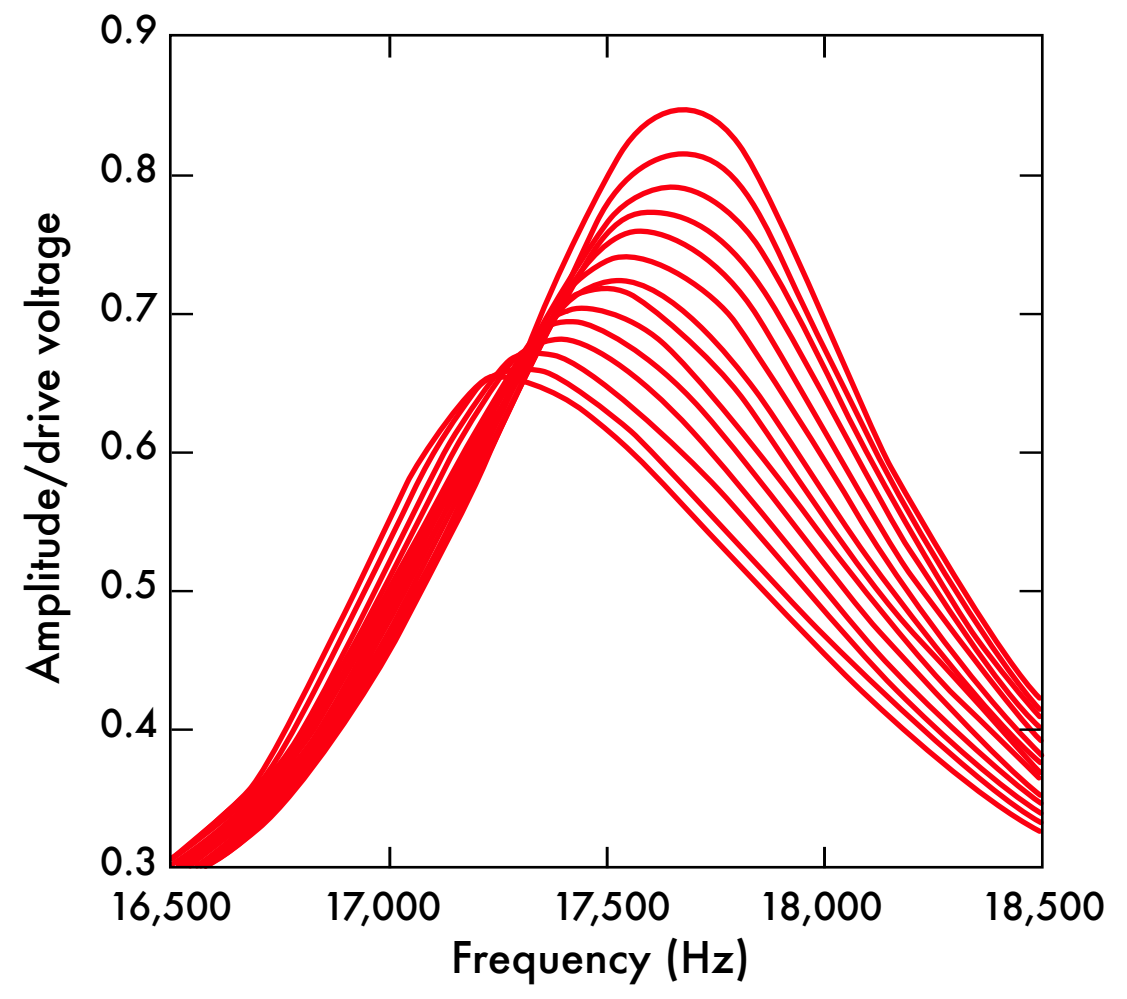
$$f_R \cong \frac{c}{2L} \cong \left(\frac{M + \frac{4}{3}\mu}{\rho}\right)^{\frac{1}{2}} \cdot \frac{1}{2L}$$

where f_R is resonance frequency, c is bulk wave speed, M is bulk modulus, □ is shear modulus, L is sample length, and □ is density.

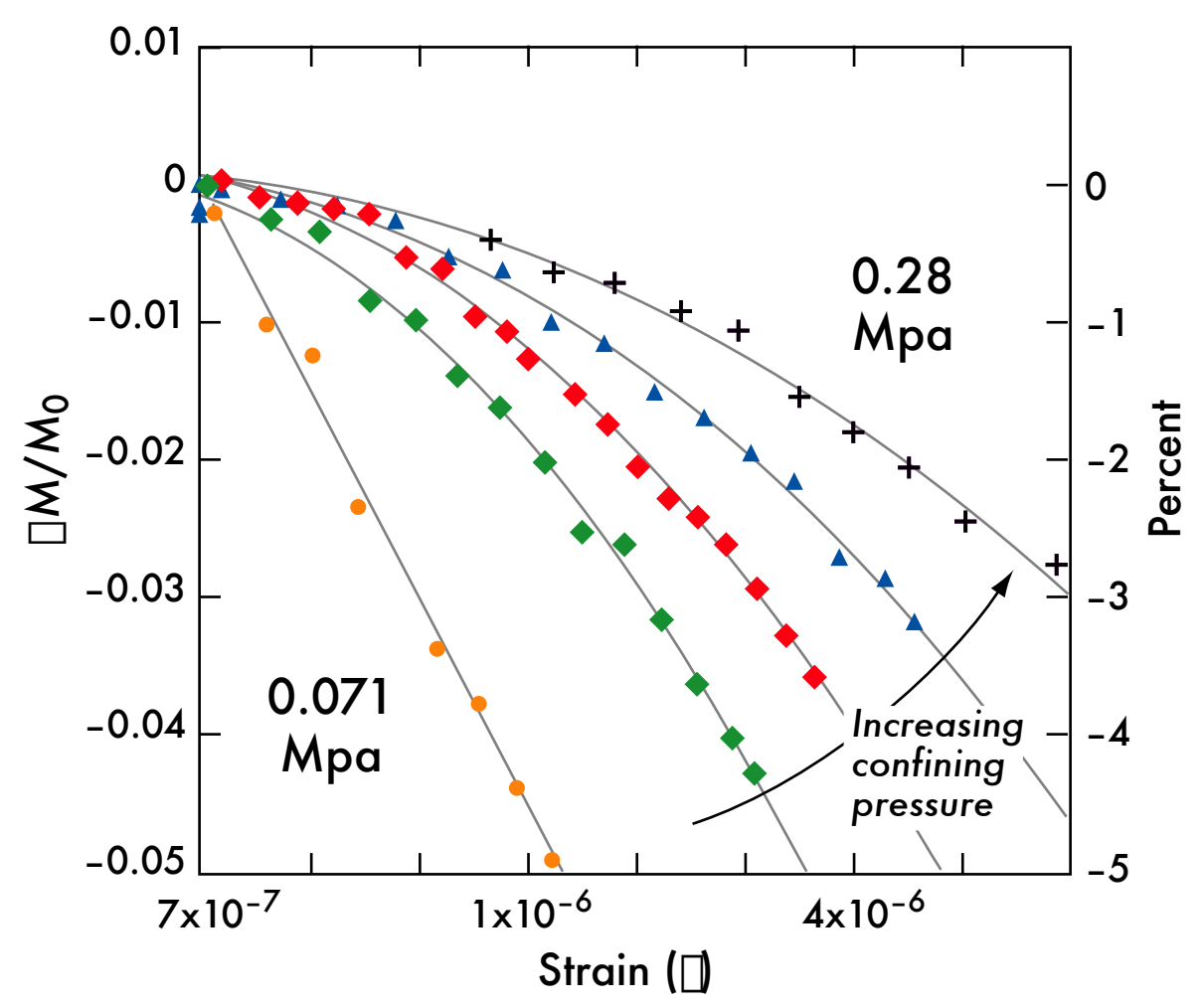
Drive frequency is swept from below to above the resonance mode. A typical frequency sweep is 3 kHz in width with a frequency sampling of 6 Hz. Frequency sweeps are applied at progressively increasing drive voltages to test for nonlinear-dynamical induced modulus softening. The resonance frequency at peak amplitude corresponds directly to modulus.

Material Softening due to Nonlinear Response under Resonance Conditions

Fundamental-mode resonance-curves taken under 0.11 MPa effective pressure (80 N force; the pore pressure is 1 atm): Resonance frequency of 17,700 Hz progressively decreases to 17,250 Hz with excitation amplitude. Wave dissipation correspondingly and dramatically increases, seen by the decrease in the normed peak-amplitude and the increasing width of the curve.

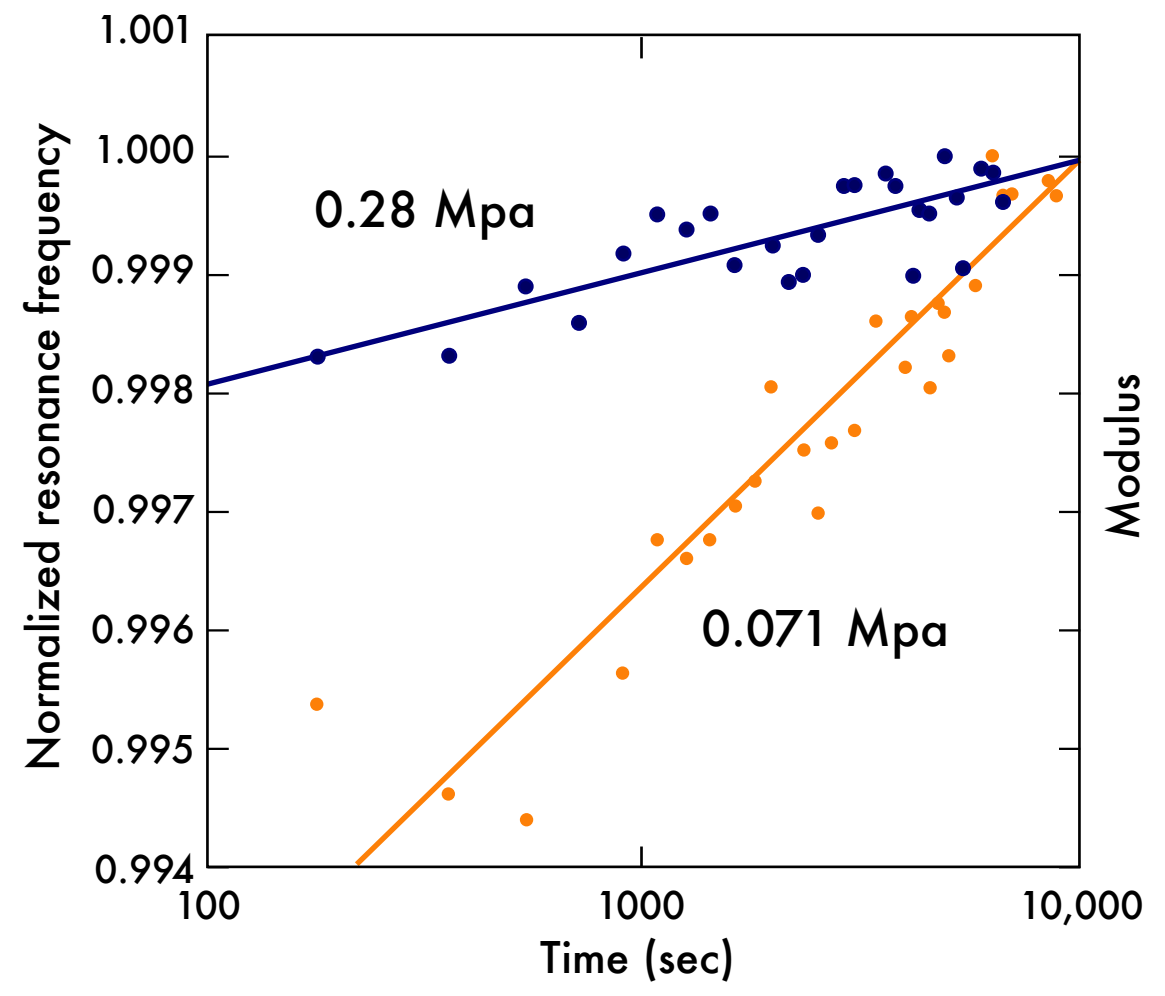


The change in modulus with detected strain-amplitude at five effective pressures noted. The Y-axis shows the change in the bulk modulus (□M = M − M_0) normalized to the equilibrium (elastically linear) value M_0. The right-had Y-axis shows the percent change of the modulus. The X-axis is the wave strain amplitude.



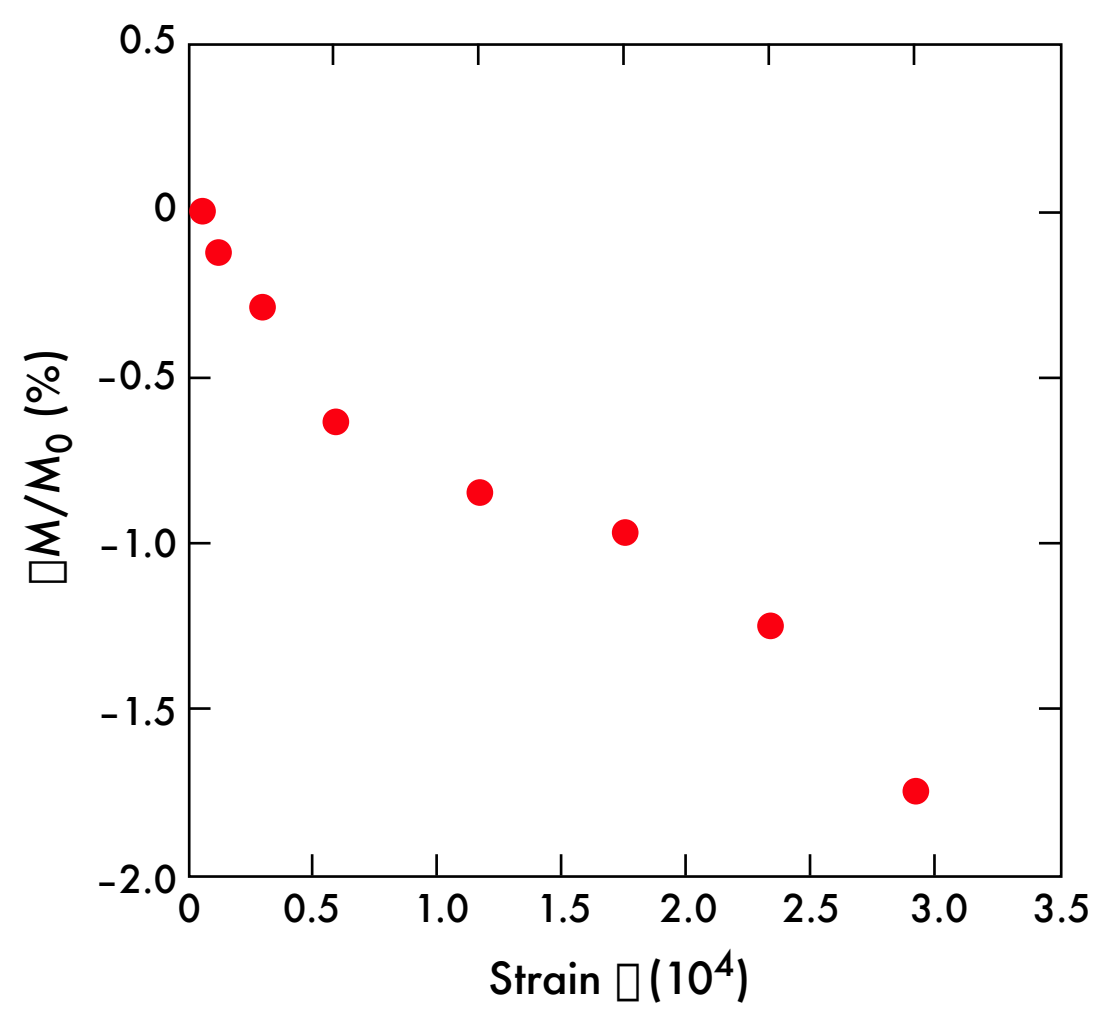
Slow Dynamics in Granular Media

One of the most intriguing aspects of the dynamical nonlinear behavior of granular media is Slow Dynamics, a phenomenon also observed in rock to a lesser degree (see, for example, Guyer and Johnson, 1999). After the wave disturbance, the material modulus and Q do not immediately return to their original, equilibrium values. Recovery takes hours to days depending on the effective pressure and appears to recover with the log of time. The figure below shows the recovery of the modulus under two different effective pressures. This behavior has never before been observed in granular media.



Modulus Decrease with Amplitude in Pulse-Mode

The source is a sinusoidal, one-cycle pulse composed of 50 kHz, (duration 20 □s), driven at low (linear elastic) to progressively higher (nonlinear elastic) drive amplitudes. The effective pressure is 0.14 Mpa. Modulus is normalized to its (linear-elastic) value. This experiment shows that the modulus decrease is immediate with a pulse, although less significant than under resonance conditions, because wave amplitudes are smaller.



Scaling and Physical Nature of the Nonlinearity

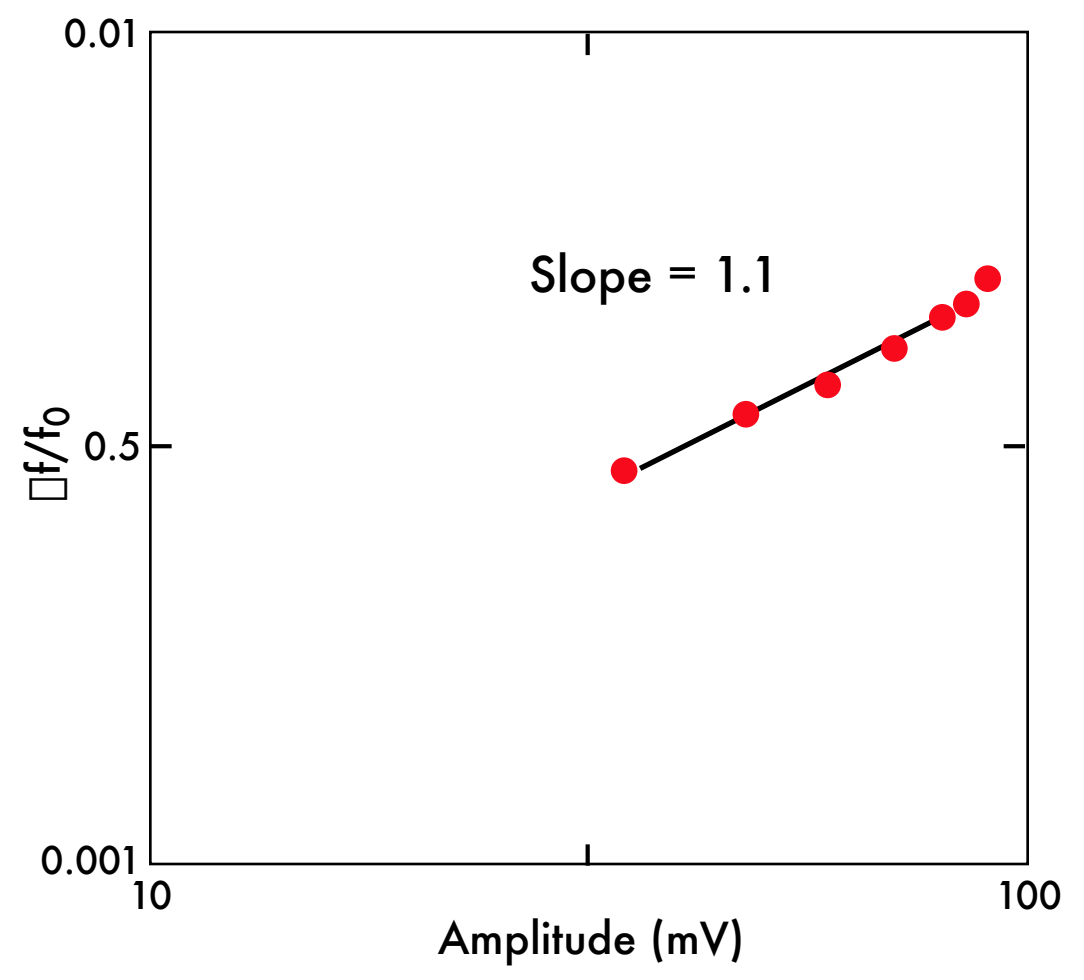
The scaling relation between the normalized change in frequency □f/f_0 and the amplitude provides clues to the physical nature of the elastic nonlinearity, based on existing models. A classical model of solids in one dimension, based on anharmonicity, is described by,

$$\square = \square\square(1 + \square\square + \square\square^2 + \ldots)$$

where □ is stress, □ is average strain, and □,□ are higher order nonlinear contributions. This model, derived by Landau and Lifshitz (1986) for instance, tells us that the scaling between □f/f_0 and the amplitude goes as strain squared. This is not what we observe here, and that is sensible because Landau theory is derived for atomic-level elastic nonlinearity. Several phenomenological models containing hysteresis in the stress-strain relation (see next column) do describe the scalings we observe, and result in an equation such as,

$$\square = \square\square(1 + \square\square + \square\square^2) + \square\square$$

where □ describes the hysteresis in the material (see, e.g., Guyer and Johnson (1999). The equation predicts a scaling of □f/f_0 and amplitude that is linear, as we (approximately) observe in the granular media below.

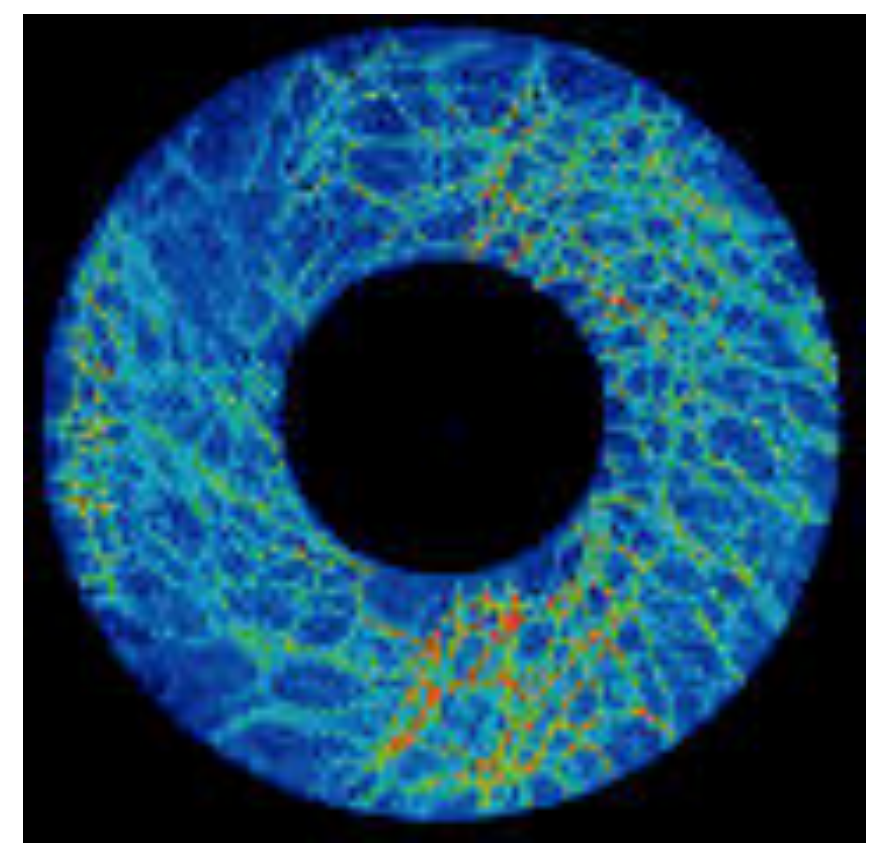


On the Physical Origin of the Behavior

As noted, we have a phenomenological description of the elastic behavior. The actual physics has its origin in the grain-to-grain interactions that can be described by Hertz-Mindlin theory (see, e.g., Ostrovsky and Johnson, 2001) for instance. Hertz-Mindlin theory describes the the forces between interacting spheres, where hysteretic effects arise from contained fluids, fluid films and shearing during pressure oscillation.

Force Chains in Granular Media

The two figures below show how, under modest pressures, granular media form "force chains" (e.g., Jaeger et al., 1996). Force chains are the network that literally carry the load, and the paths by which sound travels. In the blue figure, the media is placed in torsion, forced from the center. In the second figure, the material is under hydrostatic load; in both cases the bright regions show the chains. The nonlinear response of the material would arise primarily in the force chains in our view, rather than in the entire bulk. This must be rigorously demonstrated however.



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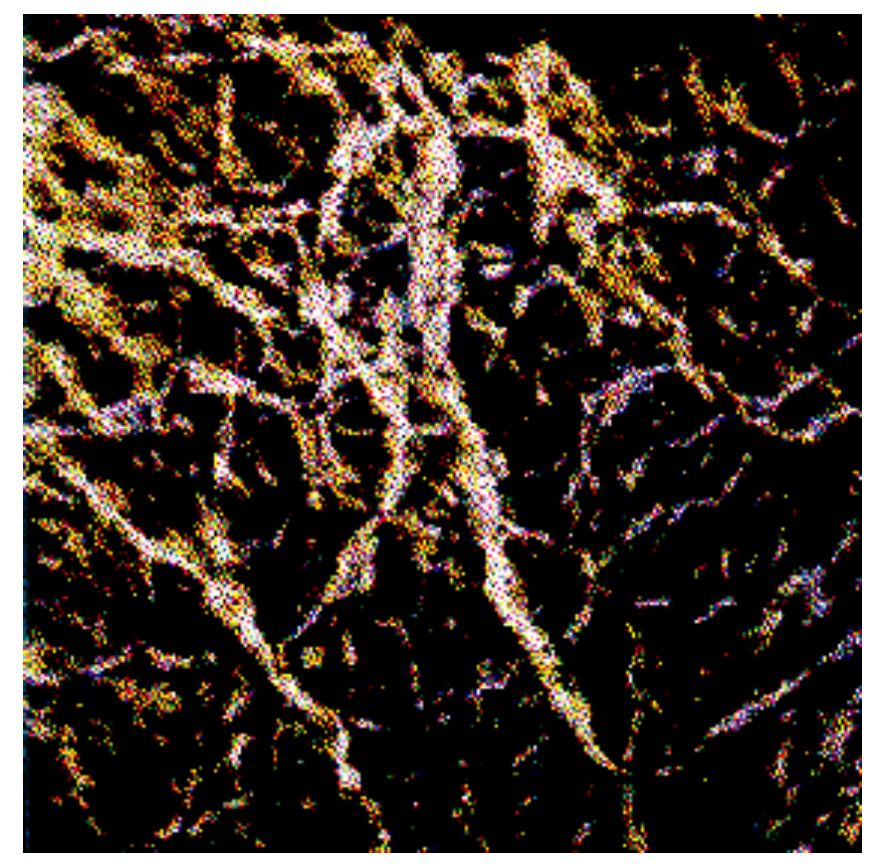

jfi.uchicago.edu/.../ granular2/force_chains.html

Summary and Conclusions

The physics of sand piles is an area of intense research (e.g., Jaeger et al. 1996). We have described dynamical measurements in granular media under resonance and pulse-mode conditions. The measurements indicate that granular media is highly nonlinear and hysteretic. Granualr media also exhibit the intriguing phenomenon called slow dynamics, a creep-like phenomenon where, after dynamic wave excitation, the material modulus takes hours to days to return back to its original state. We have also shown that, as effective pressure increases, the nonlinear response decreases.

In addition to the intrinsic interest of basic research, regarding the physics of granular media, studies of it's nonlinear behavior have import on understanding soil behavior during strong ground motion and of the behavior of fault gouge, especially in regards to dynamic earthquake triggering.

Presentations on these topics are offered during this meeting. Please see:

Abstract Reference Number: 9919
Abstract Title: Nonlinear Soil Response Induced in Situ by an Active Source at Garner Valley, Paper Number: S42A-04, presented Thursday, 16 December, Location: 3006, Starting time: 11:15.

Abstract Reference Number: 7055
Abstract Title: Nonlinear Dynamics, Granular Media and Dynamic Earthquake Triggering
Paper Number: S24A-06
Presentation Date: Tuesday, 14 December
Location: 3004, Starting time: 17:15

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