From local to global measurements of nonclassical nonlinear elastic effects in geomaterials

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Abstract: In this letter, the equivalence between local and global measures of nonclassical nonlinear elasticity is established in a slender resonant bar. Nonlinear effects are first measured globally using nonlinear resonance ultrasound spectroscopy (NRUS), which monitors the relative shift of the resonance frequency as a function of the maximum dynamic strain in the sample. Subsequently, nonlinear effects are measured locally at various positions along the sample using dynamic acousto elasticity testing (DAET). After correcting analytically the DAET data for three-dimensional strain effects and integrating numerically these corrected data along the length of the sample, the NRUS global measures are retrieved almost exactly.

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Natural and man-made consolidated granular media like rocks and concrete are known to exhibit interesting properties under dynamic stress that are related to nonequilibrium dynamics and nonlinear elasticity, including material softening and hysteresis.[1] Nonequilibrium dynamics has been first quantified by nonlinear resonant ultrasound spectroscopy (NRUS), in which a slender bar with free boundary conditions, which is representative of a one-dimensional unconstrained system, is vibrated in a steady state with a harmonic signal at one end while the elastic response is recorded at the opposite end.[1–3] The source signal sweeps a frequency range around a resonance frequency with increasing source amplitudes so that the variation of the resonance frequency can be tracked as a function of the maximum strain in the sample. It is very common to use a longitudinal mode of vibration since it is relatively trivial to excite in this kind of system and its linear motion is controlled by only one elastic constant: the Young's modulus *E*. Any mode order can then be selected for the analysis as long as the mode type in unchanged, as recently demonstrated by Remillieux *et al.*[3] In NRUS, the nonlinear elasticity is probed in the frequency domain and globally (i.e., for the entire volume of the sample). Dynamic acoustoelastic testing (DAET) is another technique developed more recently to evidence these effects.[4,5] It is based on a pump and probe scheme in which high-frequency (HF) pulses are used to probe the various phases of a mechanical system set by a low-frequency (LF) pump. Typically, in order to reach sufficiently high strain levels, the sample is excited near a resonance mode of vibration (pump), thus approaching the conditions used in NRUS. In DAET, the nonlinear elasticity is probed in the time domain and locally as a result of positioning the probes at a specific location on the sample (usually, the location of maximum strain). Another local technique named time-reversed elastic nonlinearity diagnostic[6] (TREND) is based on the use of time-reversal mirrors to focus remotely high-amplitude elastic-wave energy on the sample, as demonstrated by Payan *et al.*[7] These local techniques (DAET and TREND)

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have been compared to NRUS,[7,8] which is still considered to be a reference technique for quantifying nonequilibrium dynamics. However, moving from one experimental procedure to another affects the *nonlinear parameters* by up to an order of magnitude without a consistent explanation. For instance, in the work of Payan *et al.*,[7] the *nonlinear parameter* α measured on a sample of concrete with TR follows the same trend as that measured with NRUS but differs in magnitude by more than a factor of 10. In the work of Renaud *et al.*,[8] the change in elastic modulus caused by nonequilibrium dynamics varies with the probing direction (i.e., axial or radial) used in DAET and is assumed to originate from the anisotropy of the acoustoelastic effect.

In this letter, the equivalence between DAET and NRUS is established formally for the first time. As part of this study, it is also shown that the so-called anisotropy of acoustoelasticity in nonequilibrium dynamics is in fact a linear effect stemming from three-dimensional elasticity (i.e., the consequence of working with a tensor) that can be corrected for analytically. Additionally, it is demonstrated that the change in resonance frequency observed in NRUS as a function of the strain amplitude is caused mostly by nonequilibrium dynamics, since this is the only effect included in the analysis (i.e., classical nonlinearity is ignored).

Experiments were conducted on a sample of Berea sandstone (Cleveland Quarries, Amherst, OH) with a diameter of 25.8 mm, a length of 305.5 mm, a mass density of 2054 kg/m^3, and a nominal permeability ranging between 500 and 1000 mD. The sample was instrumented with a piezoelectric disk (a PZT-5H ceramic with a diameter of 25.54 mm and a thickness of 6.35 mm) epoxied on one of its flat ends and an accelerometer (PCB Piezotronics 352A60) glued on the opposite end. The linear elastic properties of this sample were characterized in previous work using resonant ultrasound spectroscopy (RUS).[9] It was found that the sample is well described by a homogeneous and isotropic material with a Young's modulus $E = 9.9$ GPa and a Poisson's ratio $\nu = 0.068$. A schematic representation of the experimental setup is depicted in Fig. 1(a). The sample is suspended on strings to enforce free boundary conditions and symmetry of the strain field with respect to the center of the sample.

An NRUS experiment is first conducted using a sequence of sinusoidal bursts, each with a duration of 55 ms, sweeping a frequency range of 3.1 to 3.3 kHz in steps of 1 Hz. This frequency range encompasses the first mode of longitudinal vibration. The vibration signal measured by the accelerometer on the flat end opposite to the source is conditioned (PCB Piezotronics 482C) and digitized with a sampling rate of 10 MHz (National Instrument PXI-4122). For each burst, only the last 40 ms of the excitation were considered for further processing, thus ensuring that a steady-state elastic response had been reached by the sample. Vibrational spectra were constructed from the harmonic responses, using heterodyne processing for 40 excitation amplitudes, as shown Fig. 1(b). Material softening, which corresponds to a drop of the Young's modulus for a longitudinal mode, as a function of dynamic strain is well evidenced in this figure as the resonance frequency of the sample decreases for increasing excitation amplitudes.

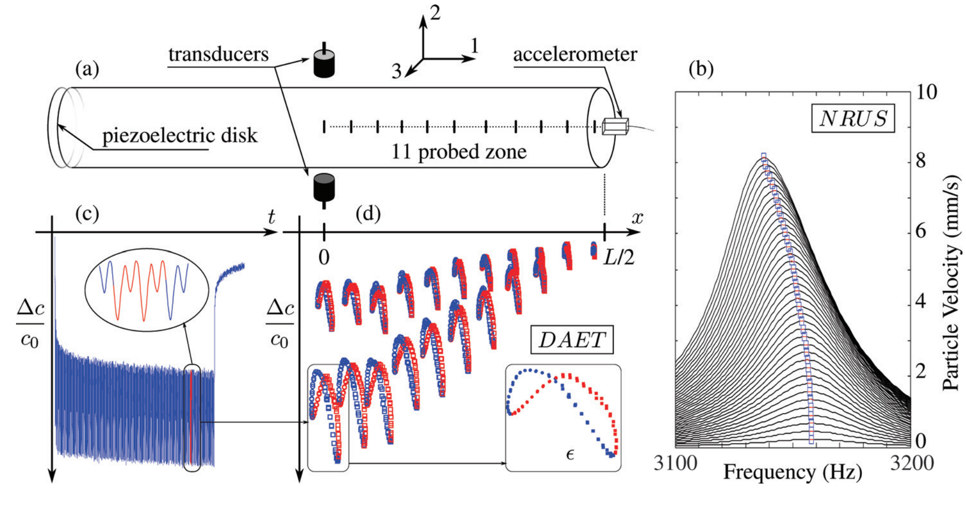

Fig. 1. (Color online) (a) Schematic representation of the experimental setup. The transducers generate high-frequency pulses (probe) propagating across the diameter of the sample while a piezoelectric disk is generating a sinusoidal signal with a duration of 55 ms at a resonance frequency of the sample (pump). (b) *Global* changes in resonance frequency for 40 excitation amplitudes. (c) DAET as a function of time during the vibration of the sample. (d) *Local* changes in speed of sound are presented for 11 positions along the sample and for two driving amplitudes where the blue and red symbols indicate that the strain is increasing and decreasing, respectively.

Subsequently, a DAET experiment is conducted by following the procedure described by Rivière *et al.*[5] and Renaud *et al.*[8] Relative changes in the speed of sound in the "2"-axis (radial) direction of the sample are monitored with two Olympus V303 transducers (probe). One transducer is used to send short pulses centered at 1 MHz that are generated by an arbitrary waveform generator (National Instrument PXI-5412). Once the pulses have been propagated across the diameter of the sample, they are received by the second transducer and digitized at 50 MHz (National Instrument PXI-4122). The probe system is activated while the sample is excited at the frequency of the first resonance mode of longitudinal vibration (pump), i.e., the mode used for the NRUS experiment. Relative changes in the speed of sound are obtained by cross-correlating the modulated transmitted pulses with the one transmitted before the pump activation. The pulses are sent with a repetition frequency corresponding to 1.02 times the period of the low-frequency pump signal. Provided that the response of the sample has reached a steady state, the relative changes in the speed of sound induced by one cycle of the pump can be fully captured over 50 cycles of the pump, with a regular spacing equal to 0.02 times the period of the low-frequency harmonic signal. A representative example of the measured DAET data is shown in Fig. 1(c). In the figure, c denotes the speed of sound in the "2"-axis direction. When the pump is activated, the speed of sound drops rapidly and oscillates in a near steady-state regime, a process referred to as conditioning.[10] This regime never actually reaches a steady state, as shown in Fig. 1(c), but for all practical purposes a steady state can be assumed for the 50 cycles during which the stroboscopic probe is analyzed. When the pump is stopped, the equilibrium value of c is recovered smoothly, a process referred to as relaxation.[10] DAET experiments were conducted at 11 positions along the sample, from its center to its end, and repeated at four amplitudes of the pump. DAET data measured at two pump amplitudes along the sample are shown in Fig. 1(d).

The first step toward establishing a link between the DAET and NRUS experiments consists of averaging the quantity $\Delta c/c_0$ measured in the near steady-state regime with a low-pass filter. The averaging process eliminates the hysteretic effects, i.e., the fact that the mechanical path is not the same for increasing and decreasing strains [see Fig. 1(d)], the effects from classical nonlinearity, and retains only the contribution from conditioning. Figure 2 shows the variation of these averaged quantities as a function of the position on the sample. Data are well fitted by the following expression,

$$\frac{\Delta c(x)}{c_0} = \left\langle \frac{\Delta c(x=0)}{2c_0} \right\rangle_T \left[\cos\left(\frac{2\pi x}{L}\right) + 1 \right], \qquad (1)$$

where the brackets and the subscript "T" denote the time average over one cycle of the low-frequency pump. This is not exactly an image of the theoretical linear strain distribution for the first longitudinal mode (i.e., linear strain has a non-zero slope at the ends of the sample) suggesting that conditioning is not activated at the lowest strain amplitudes (near the ends of the sample) and that there is a nonlinear relationship between strain amplitude and changes in elastic properties.

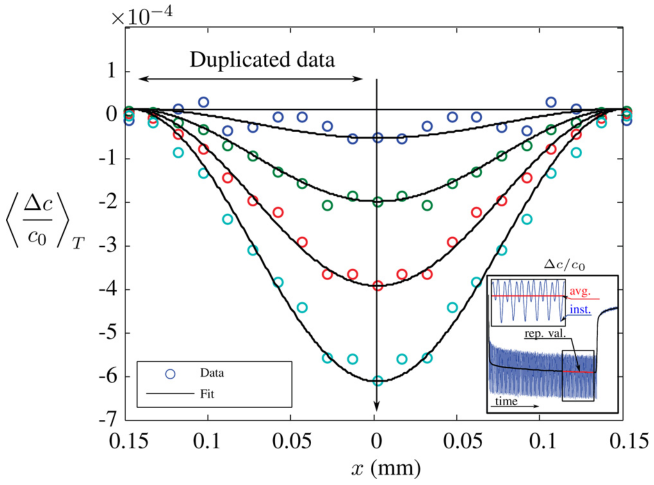

Fig. 2. (Color online) Averaged values of the relative changes in the speed of sound in the "2"-axis (radial) direction for the four pump amplitudes as a function of the position along the sample. By symmetry, the values are duplicated to the second half of the bar (from $x = -L/2$ to $x = 0$) and fitted with Eq. (1).

In the work of Renaud *et al.*,[8] it is shown that the relative changes in the elastic modulus observed with NRUS is an order of magnitude larger than the relative changes in the speed of sound observed in DAET when probing in the radial direction. Such difference may be explained by the strain induced anistotropy.[11,12] In other words, NRUS measures a change in elasticity in the "1"-axis (axial) direction whereas DAET measures a change in the speed of sound in the "2"-axis (radial) direction. The latter is essentially related to the former by the Poisson effect. DAET data can be corrected for this anisotropy using the work of Lott *et al.*[13] In this work, the relationship between the relative change in the speed of sound $\Delta c/c_0$ measured by DAET in the radial direction and the relative change in the Young's modulus $\Delta E/E_0$ measured by NRUS is expressed as

$$\left\langle \frac{\Delta c(x)}{c_0} \right\rangle_T = \frac{\nu}{2} \left\langle \frac{\Delta E(x)}{E_0} \right\rangle_T . \tag{2}$$

After measuring the conditioning locally and correcting analytically these data for the Poisson's effect, the last step of the analysis is to determine what the resonance frequency of the sample is for a given conditioning state. This step relies on numerical analysis. The Helmholtz equation with spatially varying coefficients (i.e., elastic modulus varies with the position on the sample) is projected onto a one-dimensional finite-element space using linear edge elements, eventually leading to the following linear system of ordinary differential equations,

$$\omega^2 \mathbf{M}\mathbf{u} = \mathbf{K}\mathbf{u}, \tag{3}$$

where $\mathbf{M}$ is the global mass matrix, $\mathbf{K}$ is the global stiffness matrix, and $\mathbf{u}$ is the vector of the nodal displacements. At the ith node of the domain, the conditioned elastic properties (i.e., elastic modulus varies with position on the sample according the DAET measurements) are specified as

$$E(x_i) = E_0 - \frac{2}{\nu} \left\langle \frac{\Delta c(x_i)}{c_0} \right\rangle_T , \tag{4}$$

where the relative changes in the speed of sound are obtained from the fitted data depicted in Fig. 2 and $\nu = 0.068$, as measured in previous work with resonant ultrasound spectroscopy.[9] A numerical simulation is conducted for each of the four amplitudes of the pump. An additional numerical simulation is conducted without conditioning (i.e., elastic modulus is independent of the position on the sample and is equal to E_0) and taken as the reference linear solution to compute the relative change in resonance frequency of the sample. Comparison between NRUS, corrected DAET data, and uncorrected DAET data are shown in Fig. 3. Once the DAET data have

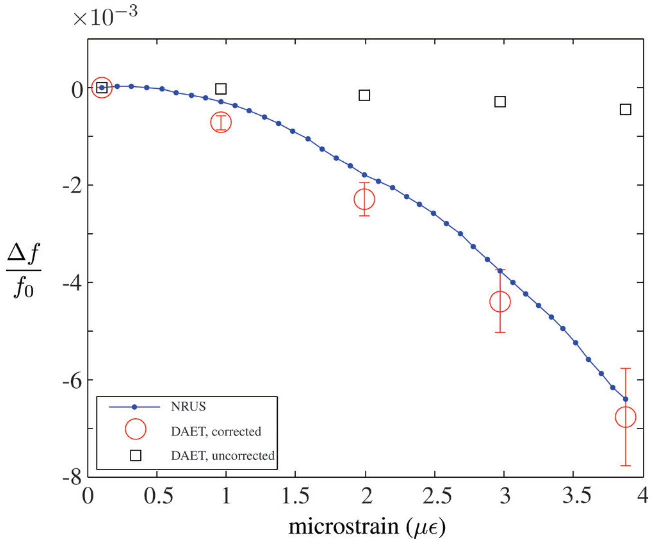

Fig. 3. (Color online) Relative frequency shift as a function of the maximum strain in the sample measured by NRUS (line with solid circles), simulated with finite elements based on DAET data corrected with $\nu = 0.068$ (hollow circles), and simulated with finite elements based on uncorrected DAET data or equivalently corrected with $\nu = 1$ (hollow squares). The error bars are obtained by assuming an error of up to 15% on the linear measurement of the Poisson's ratio.

been corrected analytically for the Poisson's effect and integrated numerically along the sample, the agreement between NRUS and DAET is excellent. On the other hand, if DAET data are not corrected for the Poisson's effect, the relative change in resonance frequency predicted by DAET is an order of magnitude smaller than that predicted by NRUS, as found by Renaud *et al.*[8]

In this letter, DAET was conducted at different positions along a sample vibrated at resonance while in most studies DAET is conducted at the position of maximum strain. Through appropriate analytical corrections of the conditioning effect and numerical simulations of the shift in resonance frequency due to the conditioning, it was demonstrated that nonequilibrium dynamics is quantified almost identically by DAET and NRUS. This work not only highlights the importance of three-dimensional effects in nonlinear elasticity but also represents an important step toward unifying nonlinear elasticity through the techniques that measure it. Conducting DAET at different positions along the sample also revealed that the strain needs to reach a threshold value to activate conditioning and more generally the mechanisms of nonequilibrium dynamics. This is an important result that will support the development of analytical models describing the complex elastic behavior of geomaterials.

Acknowledgments

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