Probing the interior of a solid volume with time reversal and nonlinear elastic wave spectroscopy

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Abstract: A nonlinear scatterer is simulated in the body of a sample and demonstrates a technique to locate and define the elastic nature of the scatterer. Using the principle of time reversal, elastic wave energy is focused at the interface between blocks of optical grade glass and aluminum. Focusing of energy at the interface creates nonlinear wave scattering that can be detected on the sample perimeter with time-reversal mirror elements. The nonlinearly generated scattered signal is bandpass filtered about the nonlinearly generated components, time reversed and broadcast from the same mirror elements, and the signal is focused at the scattering location on the interface.

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1. Introduction

The presence of nonlinear scattering in a solid is a powerful diagnostic indicator that can be used to measure the presence of mechanical damage. Nonlinear scattering is particularly useful for defining irregular damage features; for example, cracks and disbonds induce elastic-nonlinear wave scattering, whereas regular voids in a sample produce only linear wave scattering.[1,2] For this reason, in the last 10 years numerous nonlinear elastic-based techniques have been developed to infer material integrity. Several techniques have been developed to localize sources of nonlinear response. Most of these techniques can only probe the surface of a sample, but the ability to explore a sample's interior and/or nonaccessible surfaces of the sample where localization is required would greatly increase applications of nonlinear imaging techniques. In this paper we describe how one can create a source of nonlinear vibration within a sample and identify and localize the source of the subsequent nonlinear signals generated. We present an experimental method to simulate a nonlinear source inside the body of a material and then localize it; in a more practical version of this method, the localization of the source would be done using a model. The technique we describe here is based on the principle of time reversal (TR). The TR of a classical wave field[3,4] (the pressure in a fluid, the displacement field in a solid, etc.) allows one to re-broadcast a signal that propagates back to the source of the field. Time reversal is extremely attractive for a wide range of applications, including a scheme for lithotripsy,[5] the basis of a secure communication protocol,[6,7] and the ability to create a source within a material, i.e., *virtual source* creation.[4,8] Virtual source creation allows one to direct a wave broadcast at a particular elastic feature in a material and to define the elastic character of that feature, particularly beyond the domain of linear elasticity. Because the elastic nonlinear response is proportional to the wave amplitude and the nonlinear coefficient is proportional to the mechanical damage intensity, it makes TR extremely attractive for nonlinear applications. The greatest power of TR is that it provides the means to focus large amplitude waves at essentially any point in space. In a new method termed time reversal nonlinear elastic wave spectroscopy (TR NEWS), the nonlinearly generated scattered waves (e.g., waves at $2f$, $3f$, etc., where f is the fundamental wave frequency) are detected at the sample perimeter locations known as the time reversal mirror (TRM) elements. The detected signals are then reversed in time and emitted from the same locations; this generates a wavefield that coalesces at the nonlinear scatterer location.[8] The approach has been experimentally demonstrated on objects that contain surficial mechanical damage as well as in numerical simulations.[4,9] Nevertheless, practical application of time reversal theory to define and image the internal features of objects remains a difficult and elusive problem that merits further study. This work represents the first experimental demonstration of the ability of TR to image features in the interior of a solid.

2. Creation of the source of nonlinearity

Here we describe the experimental protocol and results found in exploring the nature of the nonlinearity at the interface of two solids in contact. The sample consists of two blocks, approximately $10 \times 10 \times 10$ cm^3, one of which (optical grade silica glass, density 2200 kg/m^3, longitudinal sound speed 6000 m/s) sits on top of the other (aluminum, density 2700 kg/m^3, longitudinal sound speed 6300 m/s). A laser Doppler vibrometer (Polytec OFV-303 with OFV-3001 controller and VD-02 velocity decoder) looks at the aluminum–silica glass interface through the silica glass. It detects the out-of-plane velocity of the silica glass–aluminum interface (SGA interface) as shown in Fig. 1. The optical quality of the silica glass together with the focal distance of the laser vibrometer ensure that the interface of the two solids is probed. The laser vibrometer is translated stepwise on a 1 mm by 1 mm grid at positions $x_{ij} = \Delta x(i,j)$ ($i = 1,\ldots,N_x = 82$, $j = 1,\ldots,N_y = 52$, $\Delta x = 1$ mm) over an area approximately 8 cm $\times$ 5 cm.

Two sets of receive–broadcast transducers (mirrors) are fixed to the sides of the sample. The transducers are bare piezo-electric disks (PZT) glued on the surface of the sample. When acting as receivers, they are wired to the analog inputs of National Instruments PXI-7852-R cards.

These cards have a 16 bit resolution over ± 10 V dynamic range. When used for generating waves inside the sample, the PZTs are supplied by the analog outputs of the cards with the same specifications via amplifiers from Artann Laboratories. These amplifiers provide amplification by a factor of 50, which allows the experimental apparatus to reach the necessary amplitude to generate nonlinear signals. The first set of transducers, called TRM1 and having elements labeled $n = 1$ through $N = 30$, is used to create a displacement field on the SGA interface. The second set, called TRM2 and having elements labeled $m = 1$ through $M = 8$, is used to detect broadcasts stimulated by this nonlinear displacement field.

A space–time local displacement field, or source, is created at $x_0 = x_{63,30}$ on the SGA interface using the following procedure. A signal composed of a few cycles of sinusoid at $f_0 = 150$ kHz for $0 \le t \le 150$ μs centered at $t = T = 3.27$ ms from TRM1 element $n = 1$ is broadcast, detected with the laser vibrometer, and stored as $v_2(x_0,1;t)$ for the time $0 \le t \le 6.54$ ms ($v_z = \dot{u}_z$). This procedure is carried out for all of the elements in TRM1, i.e., we collect the signal set $v_z(x_0,n;t)$, $n = 1,\ldots,N$. The time reversed signal set $v_z(x_0, n;$ 6.54 ms$-t)$, $n = 1,\ldots,N$ is then formed and the signals are simultaneously broadcast from the respective TRM1 elements for $0 \le t \le 10$ ms. This procedure produces a *virtual source*, the coherent focusing of the broadcast signals at x_0 near time

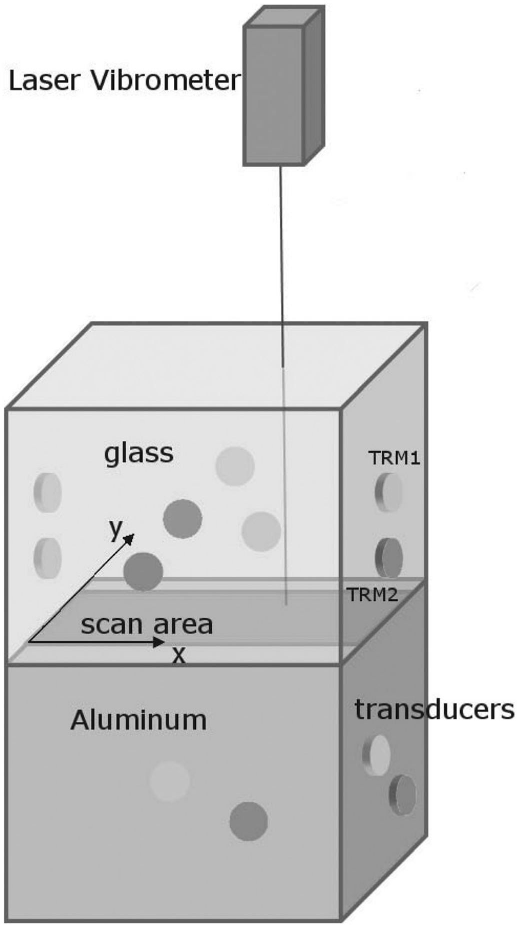

Fig. 1. Schematic drawing of the experimental configuration. An array of 30 piezo-transducers is used to create a wavefield that is recorded at the laser Dopler vibrometer spot location. The 30 transducers are used to emit the time-reversed, detected signal. These focus at the laser spot location with very large amplitudes creating elastic nonlinear scattering. A second set of eight piezo-transducers record the scattered nonlinear waves. The source of nonlinearity is then localized using time reversal and the second set of transducers.

$t = 10$ ms T. This signal, $V_z(x_0,t)$ in Fig. 2(a), is sharp in time and carries the frequency signature of the initial broadcasts [Fig. 2(a), inset].

The virtual source is also focused in space. To characterize its spatial structure we repeat the time reversed broadcasts with the laser vibrometer focused on each of the $N_x \times N_y = 4\,264$ points in the scan area on the SGA interface. From the resulting velocity field map, $V_z(i,j;t)$, $i = 1,\ldots,N_x$, $j = 1,\ldots,N_y$, we form the displacement field map $U_z(i,j;t) = V_z(i,j;t)/\omega_0$ ($\omega_0 = 2\pi f_0$) and then the *energy flux* image.[10] This image of the spatial structure of the virtual source is formed using the discrete version of the *membrane energy flux*,

$$Q(\mathbf{x},t) = \oint d\mathbf{l} \frac{\partial U_z(\mathbf{x},t)}{\partial z} \frac{\partial U_z(\mathbf{x},t)}{\partial t}, \qquad (1)$$

where $d\mathbf{l}$ is a differential element of perimeter pointing outward from the associated area.

As we are primarily interested in the shape of the focus, we have set the elastic constant in the equation for the energy flux equal to 1. In Figs. 3(a)–3(c) we show

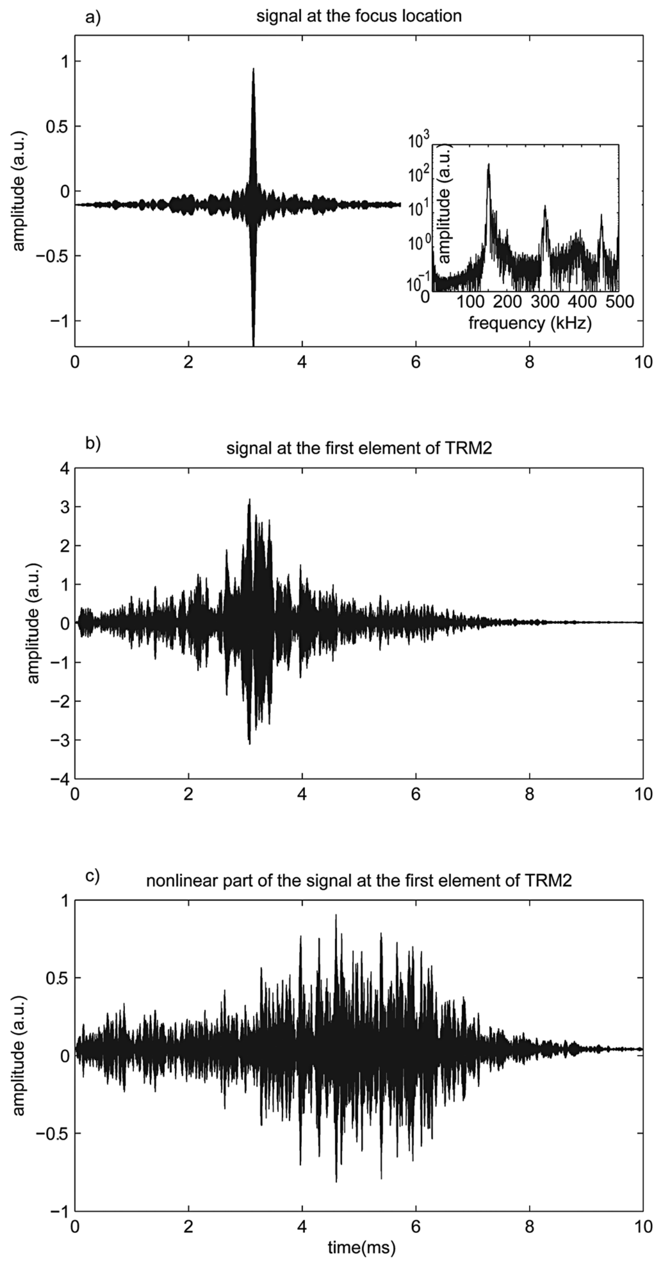

Fig. 2. Signals recorded while broadcasting from TRM1 (a) at the focus location, (b) on the edge of the sample, and (c) on the edge of the sample after filtering out the linear portion.

$Q(\mathbf{x})$ at a time just before that of energy focus $t=T$. Energy flows into the region around x_0 at $t<T$ and out of this region at $t>T$.

When the signal set $v_z(x_0, n; 6.54\ \text{ms}-t)$, $n=1,\ldots,N$ is broadcast at high amplitude we expect the virtual source to drive the nonlinear character of the elastic structure at (near) x_0, i.e., the asperity set that supports SGA contact. To look for initial evidence of nonlinearity, we high pass filter the signals recorded at one of the TRM1 receivers [Fig. 2(b)] using $f>1.5f_0$, and find the time train shown in Fig. 2(c). Note the marked increase in nonlinear amplitude beyond time $t>T$.

3. Localization of a source of nonlinearity

The virtual source procedure creates a signal localized in time and space at x_0 on the SGA interface that drives the nonlinear character of elasticity there, which in turn produces a nonlinear source. Our primary interest in this study is the detection and use of the broadcast from this source. To detect this broadcast we employ the mirror set TRM2.

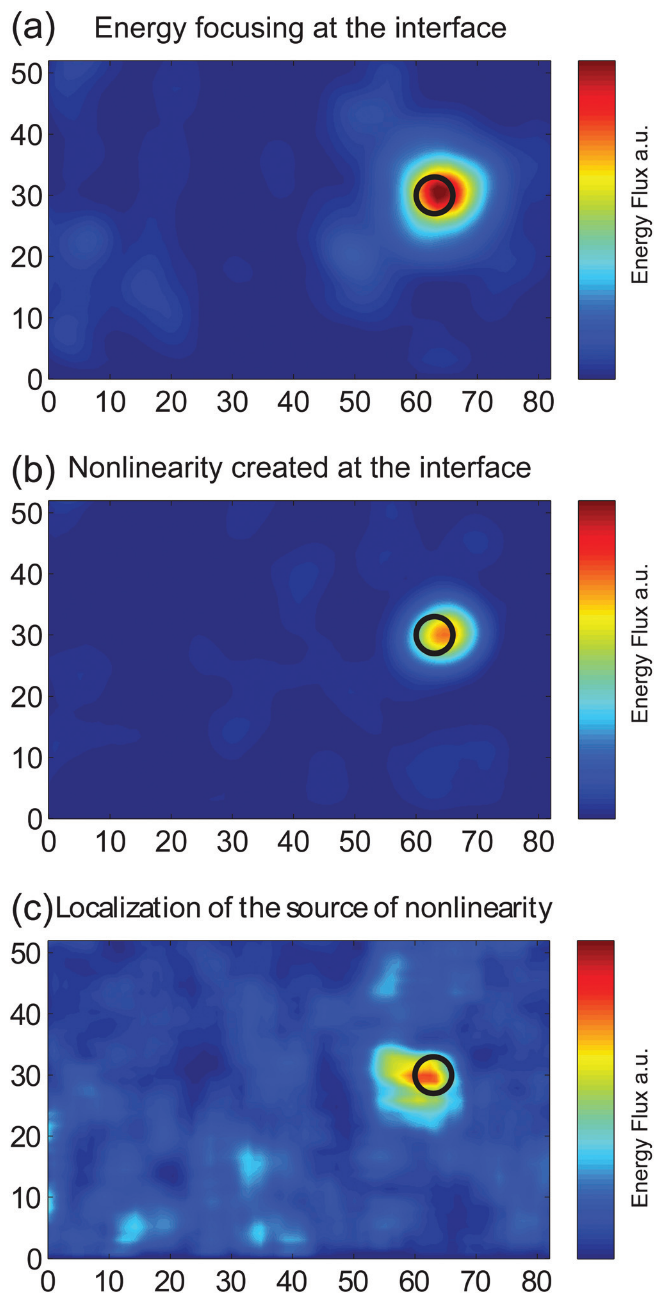

Fig. 3. (a) Norm of the energy current during the creation of the scattering source. The wavefield coalesces at one point thanks to the time reversal principle. (b) Norm of the energy current of the source but filtered to contain only the nonlinear portion (obtained by applying a high pass filter). (c) Norm of the energy current during the localization of the nonlinear source: The nonlinear part of the signal is time reversed and broadcast within the sample creating a wavefield that coalesces at the point of origin of the nonlinearity.

We perform a second time reversal step to establish the spatial location of the nonlinear source detected by the mirror set TRM2. During the virtual source creation step, $0 \leq t \leq 2T$, we record the signals $W(m,x_0,t)$, $m = 1,\ldots,M$ on the TRM2 mirror set. We filter these signals at $f > 1.5f_0$, forming the signal set $W_F(m,x_0,t)$, $m = 1,\ldots,M$. The signals $W_f(m,x_0,t)$ are time reversed around $2T$, producing the signal set $W_f(m, x_0, 2T-t)$, $m = 1,\ldots,M$, and later simultaneously broadcast from the respective TRM2 elements. During the broadcast, the laser vibrometer records the signal $w_z(i,j;t)$ received at pixel (i,j) on the scan area of the SGA interface. The broadcast is repeated $N_x \times N_y$ times, at each pixel on the SGA scan surface, and the $w_z(i,j;t)$ map created. We take this map through the above-described membrane energy flux scheme and produce the energy flux image shown in Fig. 3(c). We see that the source of the nonlinear broadcast is very nearly x_0, as it should be.

4. Conclusion

In this paper we have shown experimentally how to locate a nonlinear elastic scatterer inside a transparent solid using TR NEWS. For application to imaging opaque solids, the method requires combining experimental procedure with modeling. There are a number of ways to accomplish this. One can, for example, use the following three steps. (1) A velocity model of an object is created applying standard elastic wave tomography. (2) Large wave amplitudes are input into the sample in order to excite features that are elastically nonlinear. All scattering, linear and nonlinear, is detected at TRM elements on the sample perimeter. (3) These signals are time reversed, (a) filtered and backpropagated in the model to locate nonlinear scatterers; or (b) left unfiltered and backpropagated in the model to locate all scatterers. Actual location of scatterers in the volume requires an imaging procedure similar to Eq. (1), but needs to be applied with an energy flux in three dimensions. We discern linear from nonlinear scatters by comparing the results of steps (3a) and (3b). We believe that these observations illustrate a powerful nondestructive evaluation technique, and can perhaps even be used to image the Earth, and generate other applications yet to be developed.

Acknowledgments

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