Determination of third order elastic constants in a complex solid applying coda wave interferometry

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In this letter we describe the development of coda wave interferometry to determine acoustoelastically derived third order nonlinear coefficients of a highly complex material, concrete. Concrete, a structurally heterogeneous and volumetrically mechanically damaged material, is an example of a class of materials that exhibit strong multiple scattering as well as significant elastic nonlinear response. We show that intense scattering can be applied to robustly determine velocity changes at progressively increasing applied stress using coda wave interferometry, and thereby extract nonlinear coefficients. © *2009 American Institute of Physics*. [DOI: 10.1063/1.3064129]

Acoustoelasticity refers to measurement of material wave speed while progressively increasing stress and is the acoustical analog of photoelasticity in optics. Acoustoelastic-derived nonlinear properties of isotropic homogeneous materials have been obtained for at least half a century. Such measurements provide insight into the nano-to-mesoscale features that determine the elastic nonlinear response and can be used to obtain important physical characteristics, such as material modulus, and to predict material strength. Hughes and Kelly[1] derived expressions for the speeds of elastic waves in a stressed solid using Murnaghan's[2] theory of finite deformations and third order terms in the strain-energy expression. In complex materials, determining the third order constants accurately can be challenging due to significant intrinsic dissipation, as well as heterogeneity leading to strong wave scattering. Most Earth materials fall into this class, known as the nonlinear nonequilibrium class[3] or also the nonlinear mesoscopic class,[4] and an extreme example is concrete. It is highly complex both chemically and mechanically, is porous and permeable, heterogeneous, and highly elastically nonlinear.[5] In typical laboratory acoustic measurements on concrete, frequencies range from 200 kHz to 1 MHz. Associated wavelengths and typical aggregate sizes are equivalent, leading to strong multiple scattering. In this paper, we make use of the information imprinted in the wave form coda generated by multiple scattering. Applying successively larger stresses in combination with coda wave interferometry (CWI) provides the means to obtain velocity as a function of pressure and thereby extract the third order nonlinear coefficients.

The study of multiple scattering in the Earth (termed "coda" originally by Aki[6] more than 50 years ago) has been of interest to the geoscience community for at least 50 years.[7] Poupinet *et al.*[8] developed a method for monitoring velocity variations employing coda, termed "doublets" [referring to successive nearly identical signals from the same earthquake source]. The method was refined in laboratory studies by the addition of monitoring changes in attenuation by applying an active source by Roberts *et al.*,[9] where it was termed the "active doublet method." More recently developments have been aimed at detecting small changes in the scattering field due to modifications in the Earth's crust, including velocity changes induced by thermal stress or stress accumulation in the crust, and source location (earthquake localization). This more recent version of the method has been broadly termed "CWI."[10]

The purpose of this letter is to report the development of CWI in conjunction with incremental changes in the applied stress to a specimen, for determining the third order elastic constants of concrete, a method that can be applied to any solid but is particularly appropriate for complex solids. The essence of the method is to extract velocity and/or attenuation change between two time signals obtained at different stresses, by analyzing wave form coda changes. This is accomplished by cross-correlating moving time windows between time signals captured under different pressure conditions. By inverting results from a number of pressure increments the nonlinear coefficients are calculated. Recently CWI was applied to monitor thermally induced velocity variations in a solid,[11] as well as to observations of velocity changes in a sample of Berea sandstone.[12] In the following, we describe details of how the CWI method is implemented to extract the Murnaghan constants of a concrete sample, and how the CWI results are used to obtain the nonlinear coefficients. This is followed by results and analysis.

In the case of uniaxial loading in the 1 direction (1,2,3 designates an orthonormal basis), the strain-induced velocity variations in an initially isotropic medium can be analyzed to extract the Murnaghan third order elastic constants l, m, and n by a first order approximation assuming small changes in velocities.[13] We define here the acoustoelastic constants L_{ij} as $dV_{ij}/V_{ij}^0 = L_{ij} d\varepsilon$, where $\varepsilon = du/dx$ is strain in the 1 direction, V_{ij} the speed of a wave propagating in the i direction polarized in the j direction, and V_{ij}^0 designates the wave speed in the unstrained state.

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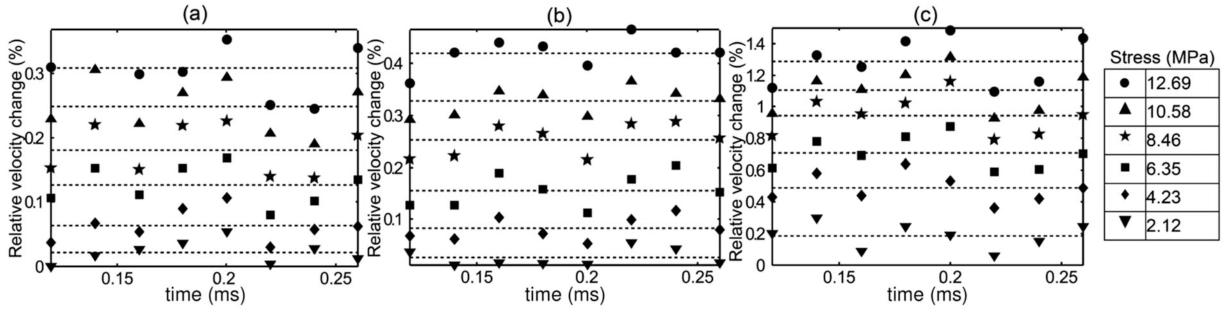

FIG. 1. Result of CWI. Dashed lines are the mean values of dV_{2j}/V_{2j}^0: (a) dV_{23}/V_{23}^0, (b) dV_{22}/V_{22}^0, and (c) dV_{21}/V_{21}^0.

Following a change in the material, by cross-correlating piecewise-in-time two wave forms w and w_0, a sequence of the correlation functions R^{t,t_w} can be determined. In the case of a velocity (modulus) perturbation, the distinct wave packets arrive at the receiver at different times with respect to the corresponding unperturbed packets. It has been demonstrated that the time of the maximum of R^{t,t_w} corresponds to the mean value of these timelags $\langle \tau \rangle_{(t,t_w)}$, where (t,t_w) refer to the center and the half width, of the evaluation time window, respectively, assuming the time window length $\Delta t = 2t_w$ is small and $t \gg t_w$. The relative velocity variation in the entire perturbed signal is then $dV/V^0 = -\langle t/\tau \rangle$, where $\langle \cdots \rangle$ refers to the average over all time windows.[10]

To summarize, in order to obtain the nonlinear coefficients we must measure the second order (linear) coefficients λ and μ and Poisson's ratio ν (as described below). We then must extract the velocity changes dV_{2j}/V_{2j}^0. The quantities dV_{2j}/V_{2j}^0 are obtained from CWI by three separate measures of wave speed, one compressional (dV_{22}), and two polarized shear wave measurements at 90° from each other (dV_{21}, dV_{23}). From these quantities L_{2j} are obtained by the relation $dV_{2j}/V_{2j}^0 = L_{2j} d\varepsilon$. The uniaxial strain ε in the 1 direction is obtained from strain gauges. The third order coefficients l, m, and n are then calculated from Ref. 13. In contrast to classical acoustoelastic measurements, an advantage of this approach is that we do not require a velocity reference in the unstrained state, which is challenging to obtain in a highly scattering medium like concrete.

Our study is performed on a cylindrical concrete sample 160 mm long by 75 mm diameter. Destructive measurements of identical samples to that used in this study yielded a Young's modulus E of 42.39 GPa, an ultimate strength of 76.6 MPa, with a Poisson's ratio ν of 0.21. A hydraulic press (MTS 318.25) was programed to apply a stress protocol of six stress steps from 0 to 13 MPa, and strain gauges were attached to the sample in order to monitor the strain in the direction of loading. The loading protocol is determined so as not to exceed 30% of the ultimate strength in order to remain in the elastic regime.[14] Shear and compressional ultrasonic transducers 25 mm in diameter (500 kHz central frequency) are attached to the bar center using ultrasonic coupling gel and maintained in position applying constant force using springs located in holders. They are oriented facing each other on either side of the sample and shot in the 2 direction. The transducers are driven by a high voltage system using an impulse (Panametrics 5058PR). From these quantities L_{2j} are obtained by the relation $dV_{2j}/V_{2j}^0 = L_{2j} d\varepsilon$. The third order coefficients l, m, and n are then calculated from Ref. 13.

Much empirical evidence[12,15] suggests that computing the intercorrelation function using ten signal periods is optimal. In order to satisfy the assumption $t \gg t_w$, we begin the coda analysis at $t = 10t_w$ ($t = 0.1$ ms, while compressional-mode time of flight is 14 μs.) and continue up to $t = 0.26$ ms. In this manner the relative velocity variation is computed for eight nonoverlapping windows for each stress step. These eight windows provide a robust average of the relative velocity change. Figure 1 presents the results obtained for dV_{2j}/V_{2j}^0. We observe that the relative velocity variation dV_{2j}/V_{2j}^0 is a constant for each stress step and increases with stress. The fact that the velocity is constant over the entire signal duration at a given step indicates that compressional wave coda dominates the measurement for dV_{22} [Fig. 1(b)]. If it did not, the relative velocity variation would evolve to a different value associated with the shear waves. We benefit from the geometry of the sample in this regard. The measurements of P waves are performed by using P transducers as emitter and receiver. In this case we have a geometry such that the P waves are reflected at normal incidence back and forth across the sample multiple times. During this process, energy leaks away to the rest of the sample and is eventually converted to shear. Had we used the full coda, eventually one sees a change in dV_{22} due to the mode conversion.

For dV_{21} and dV_{23} shear waves are inputted into the sample, and shear wave coda dominates. The large difference

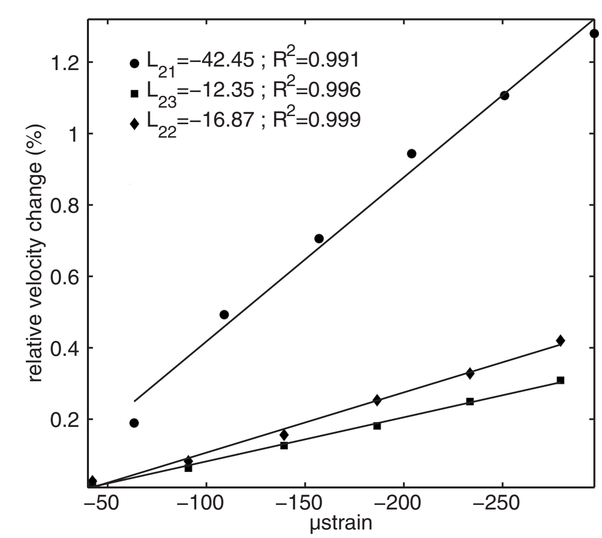

FIG. 2. Relative velocity changes vs quasistatic strain used for inversion of the third order constant (see Table I). Solid lines are the fits of experimental values. R^2 is the determination coefficient of the linear regression.

TABLE I. Third order elastic constants and the nonlinear parameter β in concrete.

l (GPa)	m (GPa)	n (GPa)	β
$-3007 \pm 2.8\%$	$-2283 \pm 1.2\%$	$-1813 \pm 3.4\%$	$-157 \pm 1.9\%$

between these two polarized shear waves implies that coda waves carry polarization information due to stress-induced birefringence. They carry it for the full coda used in the study. The relative uncertainty in dV_{2j}/V^0_{2j} decreases from 12% to 2.3% with increasing stress. These values are similar to those reported in literature.[12] The repetitiveness in the pattern of the data scatter for each successive stress step suggests that the waves follow the same path. We posit that the scatter is due to local stress field inhomogeneities due to inclusions.

The fits of dV_{2j}/V_{2j} as a function of the measured strain for each applied load for the 2 direction are given in Fig. 2. The slopes of the fits are the acoustoelastic constants L_{ij} used for calculation of l, m, and n. The associated third order constants are shown in Table I. As in less complex materials such as iron or glass,[1] we observe that the most sensitive waves to stress are those which have particle displacement in the 1 direction, i.e., V_{21}.

The nonlinear behavior is not characterized by the absolute values of l, m, and n but by the ratio of second and third order elastic constants. A nonlinear one dimensional stress-strain relationship can be written as $\sigma = E\varepsilon(1+\beta\varepsilon)$, where the nonlinear β parameter can be written as a combination of Murnaghan's and Lame's elastics coefficients[16] by $\beta = 3/2 + (l+2m)/(\lambda+2\mu)$. The large negatives values reported in Table I are similar to those found in rocks such as quartz-rich sandstone, marble, and granite,[17–19] and are around two orders of magnitude greater than an ordinary nonlinear material such as steel or iron and pyrex glass.[1] For comparison, the nonlinear parameter β can be extracted from quasistatic measurements. Our results are in agreement with those reported in literature[20] for a concrete sample of similar composition.

We have presented a robust method by which to extract the third order nonlinear parameters, the Murnaghan coefficients, of concrete. CWI and acoustoelasticity have been combined to extract nonlinear parameters, and illustrate yet another of the rich applications of CWI. A unique aspect of the reported work is the development of a method by which to link the measured velocity changes dV/V^0 to L_{ij} in order to calculate the Murnaghan coefficients l, m, and n. The method is highly accurate and precise for determining the third order constants in any material, and is especially appropriate for materials exhibiting strong scattering. The approach overcomes much of the difficulty in acoustoelastic measurements of shear waves in particular, resulting in more accurate calculation of l, m, and n. We note that the method should work well for homogeneous materials with little internal scattering as long as there is sufficient scattering from the free surfaces.

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