Limitations of Preisach Theory: Elastic aftereffect, congruence, and end point memory

K. E. Claytor,[1] J. R. Koby,[1] and J. A. TenCate[1]

Received 10 December 2008; revised 14 February 2009; accepted 25 February 2009; published 26 March 2009.

[1] The earliest reported observations of hysteresis in rocks were published at the beginning of the last century. In analogy with magnetic systems, a Preisach model was adapted in the early 1980s and used to describe the elastic hysteresis in rocks. In spite of its apparent success, as with any model of a physical process, use of the Preisach model has limitations that need to be carefully considered. Several new stress-strain measurements on various sedimentary rocks are reported here to probe the limits of the Preisach model. "Quasistatic" stress-strain measurements shown here explore in detail some of the predictions of this model, namely end point memory and congruence but also demonstrate the impact of elastic aftereffect (or relaxation). It was found, for example, that at certain stress-strain measurement rates for Berea sandstone, elastic after-effect/relaxation effects dominate, hysteresis loops completely vanish, the Preisach model fails, and simple nonlinear behavior remains. **Citation:** Claytor, K. E., J. R. Koby, and J. A. TenCate (2009), Limitations of Preisach Theory: Elastic aftereffect, congruence, and end point memory, *Geophys. Res. Lett.*, *36*, L06304, doi:10.1029/2008GL036978.

1. Introduction

[2] Although hysteresis in rocks has been reported since the turn of the last century, there are many aspects that have not been explored and the actual physics can still only be guessed. In analogy with magnetic systems, Preisach models were developed to describe the elastic hysteresis seen in rocks. Such models predict behavior such as end point or discrete memory and congruence. In magnetic systems, end-point memory and hysteresis are common [see, e.g., *Bertotti*, 1998] and both are fairly easy to observe; even the physics behind the behavior can be easily observed (i.e., flipping magnetic domains). As shown here, this relaxation effect—analogous to magnetic after-effect, i.e., the tendency of magnetic domains to spontaneously relax back to some equilibrium state—can be large enough to make observation of end-point memory impossible and to make hysteresis loops vanish. Elastic after-effect is *not* accounted for in the classical Preisach models developed for rocks.

[3] One of the earliest reported measurements of the hysteresis in rocks that can be found is shown by *Adams and Coker* [1906]. Elastic after-effect, and the resulting problems it caused with laboratory velocity measurements are also mentioned as early as *Nagaoka* [1900]. Careful observations of hysteresis (in a sample of Westerly Granite) were again reported in the mid 1970's by *Zoback and Byerlee* [1975]. *Holcomb* [1978] was perhaps the first to model this behavior with a Preisach-like set of small, phenomenological "sticky" elements. In analogy with the magnetic domains of the Preisach model, a hysteretic rock "domain" or elastic unit opens at one stress and closes at another. *Holcomb* [1981] also performed some of the first experiments to carefully examine hysteresis and to specifically look for the predicted end-point memory—i.e., abrupt changes in slope where minor loops meet and rejoin the major loop. His results were inconclusive due to "relaxation or equipment errors" (see Figure 7 and related discussion). The first qualitative evidence for end-point memory finally came in a trio of papers, [*Boitnott*, 1993; *Hilbert et al.*, 1994; *McCall and Guyer*, 1994]. Even though the data are rather sparse, the experimental evidence was good enough to suggest that a Preisach model was appropriate for a rock. As a result, this model and variations were generally adapted in the rock physics community and are currently accepted to describe the hysteresis and discrete memory [e.g., *Guyer et al.*, 1997; *Guyer*, 2006].

[4] Unfortunately, the existing data are limited and rather noisy. End-point memory can be masked by machine effects and by "relaxation" or elastic after-effect; moreover, mechanical/screw type load frames can exhibit some inherent hysteresis and even show end-point-memory-like behavior, especially on machines where testing can create subtle wear patterns (R. Norton, personal communication, 2004). Furthermore, published stress-strain data are noisy enough that no one has ever looked for congruence of minor loops [*Della Torre*, 1999] in a rock. Congruence—predicted from the classic Preisach model—is the similarity of a pair of inner loops over identical stress ranges. Thus, what is reported here is new, a careful exploration of these aspects of hysteresis.

[5] The practical impact of this work is primarily in rock physics. For example, if moduli measurements are being proposed to monitor CO_2 content or the amount of cementation in a sample with aftereffect, determination of these moduli from stress-strain data may yield very different values depending on measurement rate. Moreover, the nonlinear behavior of many rocks can be masked by nonequilibrium effects like slow dynamics [*TenCate and Shankland*, 1996] or elastic aftereffect. Only at slow rates (where the Preisach model actually fails), can the actual intrinsic nonlinearity of a sample be determined. The interested reader can learn more about applications of the Preisach model in geophysics and many other fields by starting with the article by *Guyer* [2006] in *The Science of Hysteresis*.

[6] In this paper we present new measurements showing end-point memory, congruence, and the impact elastic after-

[1]Geophysics, Earth and Environmental Sciences, Los Alamos National Laboratory, Los Alamos, New Mexico, USA.

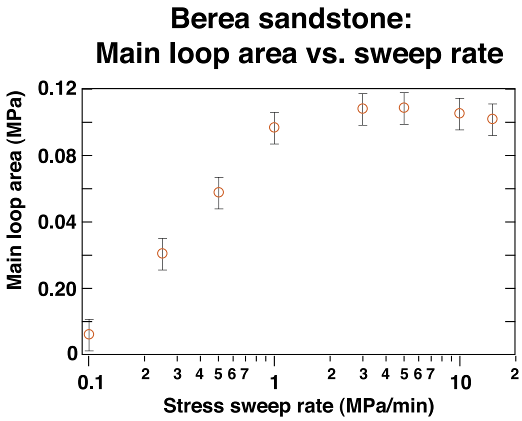

Figure 1. Main loop area as a function of measurement sweep rate. Area is essentially constant above 3 MPa/min.

effect can have on quantitatively observing and describing the hysteretic behavior of rocks. We show that, for certain rates and samples, the Preisach model is actually a good description for the hysteresis. However, we also show how elastic after-effect can make the observation of end point memory impossible and hysteretic loops which vanish with slow enough measurement rates. We also show, for the first time, congruence of minor inner loops for a rock with appropriate experimental rates. Finally, we conclude by pointing out the inadequacies of the current models.

2. Experiments

[7] A new Instron 5569 uniaxial lead screw type load frame together with an Instron dynamic clip-on extensometer (25 mm gauge length, #2620-602) was used for the primary source of strain data; this extensometer has an absolute displacement resolution of 0.025 mm. The frame extension encoder information (from the leadscrew and corrected for the frame compliance) was also used as an additional measure of strain. This particular load frame is capable of forces up to 50 kN and a calibrated load cell used was capable of measuring forces of up to 30 kN (#2525-810).

[8] The samples for this work had diameters from 22 to 26 mm and ranged from a recommended 2:1 to 3:1 height-to-width ratio [see *Jaeger and Cook*, 1979] allowing us to apply stresses from 0 to approximately 100 MPa. The samples were two different Berea sandstones (from Cleveland Quarries, Amherst OH), two samples of Fontainebleau sandstone (different nearby quarries), and a sample of Meule sandstone (a green Vosges sandstone) all from France. A 6061 Aluminum sample was used as a standard to validate the system and our measurements. Unless otherwise specified, the data reported here are from measurements on the Berea sandstone samples. Results from the other samples will appear in detail in a more comprehensive paper (in preparation).

[9] As has been done historically for these kinds of measurements, the test protocol was controlled by the compressive force being applied to the sample, i.e., stress was the independent variable. Our plots thus have stress on the horizontal axis (to reflect this protocol), rather than strain. The stress was calculated from the amount of force applied across the cross sectional area of the sample; the strain here is change in sample length over initial sample length. In all cases the sample was measured pre and post cycling, and the change in length due to permanent deformation was negligible, within measurement error.

3. Results

[10] As pointed out by *Holcomb* [1981], stress-strain measurements on rocks can be rate dependent. An example of the impact that rate effects can have on Berea sandstone is shown in Figure 1. At stress sweep rates slower than about 3 MPa/min (0.5 millstrain/min) the area of the major hysteretic loop on this particular sample decreased to zero as the rate of applied stress with time got slower. At very slow rates the sample is *only* nonlinear and *not* hysteretic at all (i.e., a manifestation of elastic after-effect [e.g., *Mack*, 1946]). This result could be repeated for all our other samples of Berea sandstone. (We note that the Meule and Fontainebleau sandstone samples did not show such behavior over the rates tested; longer rates were impossible due to long term temperature stability issues.) Thus, when making a "quasistatic" measurement on a rock, choice of stress-strain rate can be extremely important, certainly in the case of Berea sandstone samples. We note that for the granite sample used by Holcomb, "relaxation" effects foiled his attempts to observe end-point memory. We also expect similar problems at slow measurement rates with Berea sandstone. To avoid elastic after-effect in the following experiments, all measurements were taken at a rate of 10 MPa/min, a rate at which the area of the major loop was stable and repeatable. This rate is close to reported rates used by Boitnott [*Guyer et al.*, 1995].

[11] Figure 2 shows several repeated up-down stress-strain curves obtained for a Berea sandstone sample, both major loops and several thin minor/inner loops. After the initial conditioning stress protocol was applied (0 to 25 to 0 MPa), the consecutive major hysteretic loops are remarkably repeatable. Elastic after-effect is negligible. The stress

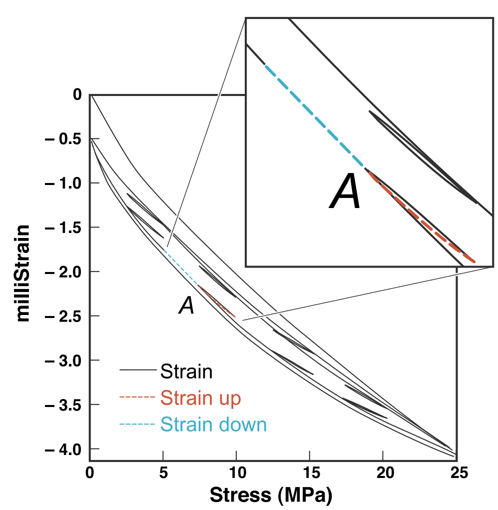

Figure 2. The stress-strain curve of a Berea sandstone sample. The red highlight ("up") marks the return from the inner loop onto the main loop. The main loop immediately following the inner loop is marked by a blue line ("down"). A discontinuity at the endpoint (a sharp/sudden change in the slope) can be seen.

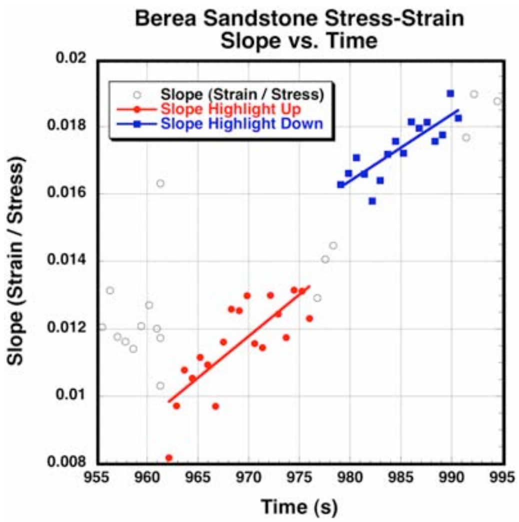

Figure 3. The local slopes of the Berea stress-strain curve in Figure 2. The large spike at 961 s is from the load frame changing direction. Red line shows the slopes of the end of the inner loop, the blue line shows slopes on the main loop. The R values for the fits are 0.792 (up) and 0.842 (down).

protocol for this experiment was simple: two linearly increasing and decreasing cycles, a triangle protocol, were performed to eliminate conditioning effects and to establish the major outer loop, followed by a similar triangular protocol with 8 small minor loop triangles superposed within. The sign convention here is that compressive stresses are positive and, since stress is the independent variable it is plotted on the abscissa. The red and blue colored sections of the curves in Figure 2, on the inner loop and then back on the major loop respectively, show a region of data we now examine carefully.

[12] Of interest is exactly how the inner loop approaches and then leaves the endpoint "A" shown in Figure 2. *Holcomb* [1981] shows data approaching an endpoint by smoothly curving and gently rejoining the major stress-strain loop. True classical Preisach model behavior, on the other hand, predicts that the minor loop should reach the endpoint, abruptly change slope, and then continue along the major loop. To test this, the portions of the stress-strain curve where the inner loop rejoins the major loop were examined in detail (inset Figure 2). Specifically the local slopes—calculated and plotted from successive ($i - 1$)th and (i)th data points—were found along the minor loop as it approached the major loop (red, labeled "Strain Up") and then on the major loop (blue, labeled "Strain Down"), immediately after reaching the endpoint A (see Figure 3). As can be seen, there is a large jump in the value of the slope as the curve progresses onto the major loop. This discontinuity is most easily detected at the lower stress value loops where the inner loop forms a sharper angle with the major loop. The two line segments in Figure 3 are best straight-line fits to the local slopes before and after point A.

[13] Another objective of this work was to determine if inner loops taken between equal stress ranges, regardless of the stress history of the rock, differ only in that they are offset in strain. If so, these pairs of minor loops are said to be "congruent" [*Della Torre*, 1999]. Preisach models must have congruency and, while congruence has been discussed [*Guyer*, 2006], it has never been demonstrated for rocks. (It is not present in magnetic systems.) To ascertain whether congruence is truly present we devised a simple quantitative test. Two pairs of inner loop data were measured, one set between 2.5 and 5 MPa and one set between 17.5 and 20 MPa (see inset Figure 4). A point-by-point subtraction was then performed with data from the low stress to high stress portion of one inner loop subtracted from the corresponding portion of the other inner loop. The same was done for the high stress to low stress value. If the two loops are congruent, the subtraction should yield two line segments of equal value (two horizontal lines). This is indeed what we see when we do the subtraction and plot the results (Figure 4, the red and gray data points are the inner loop subtraction). The x-axis units are arbitrary, representing the index of the differences of the data points (they can be interpreted as time). For comparison, we also performed the same subtraction procedure for a portion of the curve on the major hysteresis loop (Figure 4, blue data points); a straight line does not result.

[14] Error bars are not shown on Figures 3 and 4 for clarity. The quoted absolute error for the Instron extensometer is large on the scale of this plot but absolute errors are not relevant here; an estimate of point-to-point (relative) error was made instead and was found to be about 3 millistrain. Thus, within the errors bars, we obtain essentially four straight-line segments which nicely demonstrates the congruence property for these samples.

4. Modeling

[15] For the Berea sandstones of this study, elastic aftereffect can be ignored with careful choice of rates and can be fit with a classical Preisach model. The Preisach-Mayergoyz (PM) description [*McCall and Guyer*, 1994; *Guyer et al.*, 1995, 1997] was used as a basis to develop a

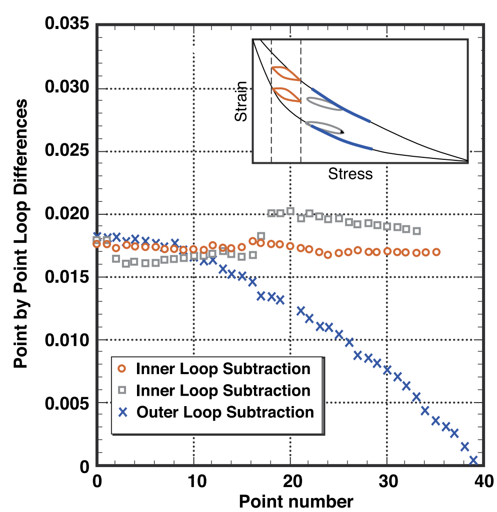

Figure 4. Congruence of inner loops. This figure shows the difference between the upper and lower loops for a low stress (2.5–5.0 MPa) and a high stress (17.5–20.0 MPa). The horizontal segments indicate that the inner loops are identical except for a shift along the strain axis. The sudden jumps correlate with the times when the load frame changes direction.

simple computer solver in MATLAB to model the experimental results and to explore the limits of the model. The original simulated annealing approach took several hours with a starting population of around 500–1000 PM elements. The approach taken here is a simpler Monte-Carlo like method, able to run on a small personal computer. Results of one such inversion on the Berea sandstone sample are typical of those given by *Guyer et al.* [1995]. The result took ~6 hours with 5000 elements on a 1.67 MHz laptop.

[16] However, experience with this model has shown that there are significant problems with PM inversions such as these. PM elements may well represent real physical features (e.g., sticky cracks [*Pecorari*, 2004]) but the model is really phenomenological. Adaptations of the classic model to include elastic aftereffect are often done in a purely ad hoc way [see *Scalerandi et al.*, 2003]. Finally, an interesting aspect we experienced running our simple model—one which is seldom pointed out—is that getting a PM distribution to match actual inner loop data is extremely difficult. *Bertotti* [1998] sums up all these problems best: "One soon realizes that naïve approaches, based on some empirical classifications of material properties and on the use of limited phenomenological models developed on purpose, are largely inadequate for gaining convincing insight into the origin of the phenomena observed, or some significant power to predict them under various conditions."

[17] **Acknowledgments.** The authors are grateful to Robert Guyer, Donatella Pasqualini and Salman Habib for constructive comments and motivation, and for institutional funding support from the Los Alamos LDRD program.

References

Adams, F. D., and E. G. Coker (1906), An investigation into the elastic constants of rocks, more especially with reference to cubic compressibility, *Carnegie Inst. Washington Publ.*, *46*, 69 pp.

Bertotti, G. (1998), *Hysteresis in Magnetism*, Academic, San Diego, Calif.

Boitnott, G. N. (1993), Proceedings of the Numerical Modeling for Underground Nuclear Test Monitoring Symposium, edited by S. R. Taylor and J. R. Kamm, *Los Alamos Nat. Lab. Rep. LA-UR 93-3839*, Los Alamos Natl. Lab., Los Alamos. N. M.

Della Torre, E. (1999), *Magnetic Hysteresis*, IEEE Press, Piscataway, N. J.

Guyer, R. A. (2006), *The Science of Hysteresis*, vol. 3, edited by G. Bertotti and I. D. Mayergoyz, Academic, New York.

Guyer, R. A., K. R. McCall, and G. N. Boitnott (1995), Hysteresis, discrete memory, and nonlinear wave propagation in rock: A new paradigm, *Phys. Rev. Lett.*, *74*, 3491–3494.

Guyer, R. A., K. R. McCall, G. N. Boitnott, L. B. Hilbert Jr., and T. J. Plona (1997), Quantitative use of Preisach-Mayergoyz space to find static and dynamic elastic moduli in rock, *J. Geophys. Res.*, *102*, 5281–5293.

Hilbert, L. B., T. K. Hwong, N. G. W. Cook, K. T. Nihei, and L. R. Myer (1994), Effects of strain amplitude on the static and dynamic nonlinear deformation of Berea sandstone, in *Proceedings of the 1st North American Rock Mechanics Symposium*, edited by P. P. Nelson and S. E. Lauback, pp. 473–478, Balkema, Rotterdam, Netherlands.

Holcomb, D. J. (1978), A quantitative model of dilantancy in dry rock and its application to Westerly Granite, *J. Geophys. Res.*, *83*, 4941–4950.

Holcomb, D. J. (1981), Memory, relaxation, and microfracturing in dilatant rock, *J. Geophys. Res.*, *86*, 6235–6248.

Jaeger, J. C., and N. G. W. Cook (1979), *Fundamentals of Rock Mechanics*, 3rd ed., Chapman and Hall, London.

Mack, C. (1946), Plastic flow, creep, and stress relaxation, *J. Appl. Phys*, *17*, 1101–1107.

McCall, K. R., and R. A. Guyer (1994), Equation of state and wave propagation in hysteretic nonlinear elastic materials, *J. Geophys. Res.*, *99*, 23,887–23,897.

Nagaoka, H. (1900), Elastic constants of rocks and the velocity of seismic waves, *Publ. Earthquake Invest. Commit. Foreign Lang.*, *2*, 47–67.

Pecorari, C. (2004), Adhesion and nonlinear scattering by rough surfaces in contact: Beyond the phenomenology of the Preisach-Mayergoyz framework, *J. Acoust. Soc. Am.*, *116*, 1938–1947.

Scalerandi, M., P. P. Delsanto, and P. A. Johnson (2003), Stress induced conditioning and thermal relaxation in quasi static compression experiments, *J. Phys. D Appl. Phys.*, *36*, 288–293.

TenCate, J. A., and T. J. Shankland (1996), Slow dynamics in the nonlinear elastic response of Berea sandstone, *Geophys. Res. Lett.*, *23*, 3019–3022.

Zoback, M. D., and J. D. Byerlee (1975), The effect of cyclic differential stress on dilatancy in westerly granite under uniaxial and triaxial conditions, *J. Geophys. Res.*, *80*, 1526–1530.

K. E. Claytor, J. R. Koby, and J. A. TenCate, Geophysics, Earth and Environmental Sciences, Los Alamos National Laboratory, Mail Stop D443, Los Alamos, NM 87545, USA. (tencate@lanl.gov)