**Application of nonlinear dynamics to monitoring progressive fatigue damage in human cortical bone**

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In this work, the results of applying nonlinear dynamics to study progressive material fatigue in human bone are described. Material nonlinear dynamical response has been shown to be associated with mechanical damage. The progressive mechanical damage experiments were conducted in cortical bone extracted from a human femur. After each damage step, the material dynamical nonlinear response was measured by applying wave modulation and extracting a nonlinear parameter proportional to the sideband amplitude. The nonlinear parameter increases rapidly with damage step, indicating increased damage after the initial cycling procedure, while the quasistatic stiffness taken from the cycling experiments shows little change. © 2007 American Institute of Physics. [DOI: 10.1063/1.2809565]

Osteoporosis is manifested by progressive bone density loss, simultaneous with microcracking. It is a serious health issue, striking approximately 40% of women and 20% of men over the age of 50, although this is difficult to say with great certainty. The current standard for diagnosing and following the illness progression employs ionizing radiation in the form of x-rays to measure bone mineral density. Linear acoustical methods exist that have been shown to be a good surrogate for bone mineral density. Although bone mineral density remains the best available noninvasive assessment of bone strength in routine clinical practice, many other skeletal characteristics also contribute to bone strength. These include, among other factors, microdamage accumulation. Both x-ray densitometry and linear quantitative ultrasound techniques are relatively insensitive to increased micro-damaging, the physical symptoms of osteoporosis. Nonlinear acoustic/elastic methods, however, have been shown to be very sensitive to damage in a wide variety of materials, including recent measurements in human cortical bone. Developing a nonlinear technique that could potentially be applied in vivo is a challenge. The method and results described herein are from in vitro studies, however, the potential exists for developing an in vivo technique from the principles and analysis presented.

Nearly all acoustical imaging devices currently offer second harmonic imaging capability. Sound intensities are not sufficient for application to imaging bone characteristics, however, and other approaches involving nonlinear dynamics are being explored. While it has been known that nonlinear elastic wave spectroscopy (NEWS) is an extremely sensitive measurement of mechanical damage in materials, it is only now that this technique is being introduced to the medical field as a possible damage diagnostic tool. Application of NEWS for monitoring the increase in mechanical damage, or conversely the decrease (as in bone healing), could provide physicians a tool with the potential to monitor the onset and progression of degenerative bone diseases, such as osteoporosis as well as the development or healing of fractures. The realization of this technique and related nonlinear elastic techniques as medical diagnostic tools is in the future, as many obstacles and considerations must be addressed before an in vivo method can be developed. At the same time, we believe that in vivo studies may commence relatively soon based on the results presented here. Until that time, this tool may provide those investigating mechanical properties of bone more sensitivity to mechanical damage than the currently available methods and safer than ionizing radiation, the state of the art. In the following, we describe the experimental method, results, and conclusions.

Nonlinear methods rely on inducing elastic wave distortion, either locally or volumetrically, in the regions that are elastically soft relative to the surrounding material. A crack in a solid is an example, where a wave encountering the damage will distort as associated harmonics are formed. In the presence of two or more waves propagating with two different frequencies, at the crack, the waves multiply creating sum and difference frequencies (sidebands), e.g., $u_1 \cos(\omega_1 t - kx)u_2 \cos(\omega_2 t - kx)$ produces waves at the sum and difference frequencies proportional in amplitude to the product of the primary wave amplitudes, $u_1u_2 \cos(\omega_1 \pm \omega_2)t$. Higher order nonlinearity can be present as well, producing higher harmonics as well as higher order sidebands.
In this study, we employ a method whereby the vibrational modes of a sample are excited by a mechanical impulse simultaneous with applying a pure frequency continuous wave (cw), in this case 223 kHz. This frequency was selected as a result of optimizing the transducer bandwidth with the attenuation of the system. The vibrational modes mix (i.e., multiply) with the pure tone, producing multiple sidebands. This technique has been applied broadly in industrial materials and geomaterials (e.g., Refs. 6–8).

Here, we investigate fatigue damage induced in a bone sample (Fig. 1) by cyclic uniaxial compression-compression loading, and at steps in the cycling procedure, we measure the elastic nonlinear response from the sideband amplitudes discussed above. We compare the dynamic nonlinear response to a damage parameter extracted from the change in the linear stiffness obtained from quasistatic load-displacement data (Fig. 2) taken during the fatigue cycling. The damage parameter as calculated from the linear stiffness softening as a function of applied damage cycles is a standard measure of progressive failure in mechanics.9

In the experimental protocol applied here, we measured a sample of human cortical bone extracted from the diaphysis of a female aged 70 yr. Before fatigue cycling began, the nonlinear response of the sample was measured by applying the dynamic wave mixing method described above. To realize this, the cw source transducer was bonded to one end of the sample, a small PZT-5 detector (1 cm diameter) was bonded to the outer surface midway between the two ends and the impact was introduced at the end opposite the cw source and along the vertical axis of the specimen. Repeatability tests were performed to ensure the bond quality of the source and receiver transducers.

For the progressive damage tests, the sample was cycled 30 000 times at a rate of 4 Hz (maximum strain ε = 0.5%), thus defining a damage step. This procedure was repeated five consecutive times. Dynamic and static damage parameters were calculated after each damage step, as well as on the undamaged sample. The dynamic measurement $\Gamma$ was obtained by integrating the frequency spectrum around the pure tone from 215 to 231 kHz, in order to include the effects of multiple sidebands. At each damage step, a damage parameter $D = 1 - K_i/K_0$ was calculated,9 where $K_i$ is the sample stiffness measured at successive damage steps, $i = 0–5$, with $i = 0$ referring to the undamaged state. $K = \Delta F/\Delta l$, where $\Delta F$ is the change in applied load and $\Delta l$ is the resulting change in displacement during the fatigue cycling. The overall length change (i.e., the change in length from the undamaged state to the length after damage step 5) was not considered as a measure of damage.

Figure 2 shows selected fatigue loading cycles from the quasistatic measurements for all damage steps. Each curve shows the results from several load/unload cycles just prior to the end of the damage step. There is a clear permanent deformation as a function of damage cycle and the slopes change moderately. There is no sign of hysteresis in the later stages as one might expect; however, the system noise on the displacement measure is relatively high. The only other obvious change over the experiment is the change in slope and the fact that steps 2 and 3 occur out of expected order. This may be a system noise related issue, or due to a difference in temperature from one damage step to the next due to the fact that the five damage steps were conducted over a period of 3 days. Note that at the end of the five cycling steps, the bone sample showed no outward sign of damage despite the clear changes that must have taken place based on the static response.

Figure 3 shows results from the modulation experiments taken at steps 0, 2, and 5, respectively. It is clear that sideband energy increases with damage state and sidebands become visible at 223 ± 4 and 223 ± 8 kHz. To quantify this, we define a nonlinear damage parameter $\Gamma$ such that $\Gamma$ is the area under the spectral curves in the interval of 215–231 kHz (i.e., the interval in which the sidebands are found). The nonlinear parameter $\Gamma$ is shown in Fig. 4 normalized to the intact value at step $i = 0$. The figure also shows the damage parameter $D$ extracted from the quasistatic experiments. It is readily apparent that the dynamic measurement of nonlinearity, which changes by nearly 700%, is far more sensitive than the change in slope from the static experiments which remains roughly the same until the last cycle changing by no more than 10%. It is interesting to note that the values of $D < 0$, indicating the unphysical result of negative damage, may be explained by the porous nature of the bone, in that the small pore structure is initially compacted, thus increasing the material stiffness. Not until after the final

![FIG. 1.](Image) Cross section of the sample, a 6 cm tall (out of the plane of the figure) section of cortical bone taken from a human diaphysis (femur). The average outer diameter of the cross section is approximately 2.5 cm.

![FIG. 2.](Image) Load vs displacement for each damage step. A linear fit has been applied to extract relative changes in Young’s modulus as the fatigue damage progressed. The relative changes are used to calculate a damage parameter $D = 1 - K_i/K_0$, shown in Fig. 4.
damage step has mechanical damage occurred in an amount sufficient to weaken the material or decrease the stiffness.

It is well known that nonlinear dynamical approaches to evaluation of damage are extremely sensitive, perhaps more sensitive than any other method. The results here illustrate the method sensitivity. The results are in accordance with recent measurements of higher order elastic nonlinearity obtained from resonance experiments on a suite of human and bovine bones, which show significant increase in hysteretic dynamic nonlinearity with increasing damage. The results in that work as well as those presented here indicate that the method has great potential for in vitro measurements. We are currently devising manners by which such measurements could be made in vivo. The dynamic strain amplitudes used in this experiment are not damaging to the bone (order 5 × 10^{-6} to 10^{-5}). It remains to be seen if these are reasonable amplitudes for an in vivo application given the increased attenuation of the complete biological system. A major issue to be addressed for both in vitro and in vivo applications is the quantitative relationship between the measured nonlinear response and the microdamage state, as reflected by crack density, length, and orientation; however, we note that the current state of the art, bone density obtained from x ray, does not convey any information at all about microcracking.

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