

Soft-ratchet modeling of slow dynamics in the nonlinear resonant response of sedimentary rocks

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Abstract. We propose a closed-form scheme that reproduces a wide class of nonlinear and hysteretic effects exhibited by sedimentary rocks in longitudinal bar resonance. In particular, we correctly describe: hysteretic behavior of a resonance curve on both its upward and downward slopes; linear softening of resonant frequency with increase of driving level; gradual (almost logarithmic) recovery (increase) of resonance frequency after large dynamical strains; and temporal relaxation of response amplitude at fixed frequency. Further, we are able to describe how water saturation enhances hysteresis and simultaneously decreases quality factor. The basic ingredients of the original bar system are assumed to be two coupled subsystems, namely, an elastic subsystem sensitive to the concentration of intergrain defects, and a kinetic subsystem of intergrain defects supporting an asymmetric response to an alternating internal stress.

INTRODUCTION

Sedimentary rocks, particularly sandstones, are distinguished by their grain structure in which each grain is much harder than the intergrain cementation material [1]. The peculiarities of grain and pore structures give rise to a variety of remarkable nonlinear mechanical properties demonstrated by rocks, both at quasistatic and alternating dynamic loading [1-4]. Thus, the hysteresis earlier established for the stress-strain relation in samples subjected to quasistatic loading-unloading cycles has also been discovered for the relation between acceleration amplitude and driving frequency in bar-shaped samples subjected to an alternating external drive that is frequency-swept through resonance. At strong drive levels there is an unusual, almost linear decrease of resonant frequency with strain amplitude, and there are long-term relaxation phenomena such as nearly logarithmic recovery (increase) of resonant frequency after the large conditioning drive has been removed.

In this paper we present a short sketch of a model [5, 6] for explaining numerous experimental observations seen in forced longitudinal oscillations of sandstone bars. According to our theory a broad set of experimental data can be understood as various aspects of the same internally consistent pattern [5, 6].

SOFT-RATCHET MODEL

A reliable probing method widely applied in resonant bar experiments is to drive a horizontally suspended cylindrical sample with a piezoelectric force transducer cemented between one end of the sample and a massive backload, and to simultaneously measure the sample response with a low-mass accelerometer attached to the opposite end of the bar [2, 4].

The evolution equation for the field of bar longitudinal displacements u as applied to above experimental configuration is assumed to be

$$\rho \frac{\partial^2 u}{\partial t^2} = \frac{\partial \sigma}{\partial x} + \frac{\partial}{\partial x} \left[\frac{\partial \mathfrak{S}}{\partial (\partial^2 u / \partial x \partial t)} \right]. \quad (1)$$

Here we use the Stokes internal friction associated with the dissipative function $\mathfrak{S} = (\gamma/2) [\partial^2 u / \partial x \partial t]^2$. The quantities ρ and γ are, respectively, mean density of sandstone and coefficient of internal friction. The stress-strain relation ($\sigma - \partial u / \partial x$) we adopt in the form

$$\sigma = \frac{E \operatorname{sech} \eta}{(r-a) [\cosh \eta \partial u / \partial x + 1]^{a+1}} - \frac{E \operatorname{sech} \eta}{(r-a) [\cosh \eta \partial u / \partial x + 1]^{r+1}}, \quad (2)$$

which for $r > a > 0$ allows us to suppress the bar compressibility at strains $\partial u / \partial x$ tending toward $+0 - \operatorname{sech} \eta$. Thus, the parameter $\cosh \eta$ is assigned for a typical distance between the centers of neighboring grains divided by the typical thickness of intergrain cementation contact. The indirect effect of strain on Young's modulus, as mediated by the concentration c of ruptured intergrain cohesive bonds, is incorporated in our theory as the main source of all non-trivial phenomena.

We introduce a phenomenological relationship between defect concentration c and Young's modulus E . Intuition suggests that E must be some monotonically decreasing function of c , which can be expanded in a power series with respect to a small deviation of c from its unstrained equilibrium value c_0 . To lowest informative approximation we have

$$E = (1 - c / c_{cr}) E_+. \quad (3)$$

Here c_{cr} and E_+ are the critical concentration of defects and the maximum possible value of Young's modulus, respectively.

The equilibrium concentration of defects c_σ associated with a stress σ is given by

$$c_\sigma = c_0 \exp(v \sigma / kT), \quad (4)$$

where the parameter $v > 0$ characterizes the intensity of dilatation. Although formula (4) should supposedly be applicable to the ensemble of microscopic defects in crystals, it was derived in the framework of continuum thermodynamic theory that does not actually need any specification of either the typical size of an elementary defect or the particular structure of the crystalline matrix. For this reason we believe it should also work for an ensemble of mesoscopic defects in consolidated materials, provided that for a single defect we understand some elementary rupture of intergrain cohesion. The

approximate functional dependence of c_0 on temperature T and water saturation s based on experimental data was treated in [5, 6].

In order to achieve reliable consistency between theory and experiment we have used the concept of blended kinetics, which finds more-or-less natural physical justification in consolidated materials [6]. The idea presents the actual concentration of defects c as some reasonable superposition of constituent concentrations g , where each particular g obeys rather simple kinetics. Specifically, we take the constituent concentration g to be governed by the kinetic equation:

$$\partial g / \partial t = - [\mu \theta(g - g_\sigma) + \nu \theta(g_\sigma - g)](g - g_\sigma). \quad (5)$$

Here $\mu = \mu_0 \exp(-U/kT)$ and $\nu = \nu_0 \exp(-W/kT)$ are the rates of defect annihilation and defect creation, respectively, and $\theta(z)$ designates the Heaviside step function. A huge disparity $\nu_0 \gg \mu_0$ between the priming rates (attack frequencies) ν_0 and μ_0 is assumed, notwithstanding the native cohesive properties of cementation material.

Typical resonant response experiments [1, 2, 4] correspond to forced longitudinal vibration of a bar, which we associate with the boundary conditions:

$$u(x=0|t) = D(t) \cos(\varphi + \int_0^t d\tau \omega(\tau)), \quad \sigma(x=L|t) + \gamma \frac{\partial^2 u}{\partial x \partial t}(x=L|t) = 0, \quad (6)$$

where L is sample length, and $D(t)$ is driving amplitude. The initial conditions are

$$u(x|t=0) = 0, \quad \frac{\partial u}{\partial t}(x|t=0) = 0, \quad g(x|t=0) = c_0. \quad (7)$$

COMPUTERIZED REPLICAS OF ACTUAL EXPERIMENTS

Computer modeling of nonlinear and slow dynamics effects was performed in the vicinity of the resonance frequency $f_0(2)$, which we choose to be the second frequency ($l=2$) in the fundamental set,

$$f_0(l) = \frac{2l-1}{4L} \sqrt{(1-c_0/c_{cr})E_+/\rho} \quad (l=1, 2, 3, \dots). \quad (8)$$

Figure 1 shows typical resonance curves, i.e., dependences of response amplitudes R (calculated at $x=L$) on drive frequency $f = \omega/2\pi$, at successively higher drive amplitudes D . Solid lines correspond to conditioned resonance curves calculated after two frequency sweeps were performed at each driving level in order to achieve repeatable hysteretic curves. The dashed line illustrates an unconditioned curve obtained without any preliminary conditioning. Arrows on the three highest curves indicate sweep directions. To improve the illustration, results of the computer simulations were adapted to experimental conditions appropriate to the data obtained by TenCate and Shankland for Berea sandstone [2]. In particular, $L=0.3\text{m}$, $f_0(2) = 3920\text{Hz}$, $\nu E_+ / k \cosh \eta = 275\text{K}$, $\cosh \eta = 2300$, $r=4$, $a=2$.

The shift of resonance frequency as a function of drive amplitude D was found to follow the almost linear dependence typical of materials with nonclassical nonlinear response, i.e., materials that possess all the basic features of slow dynamics (see [5, 6] for more details).

Figure 2 shows the gradual recovery of resonance frequency f_r to its maximum limiting value f_0 after the bar has been subjected to high amplitude conditioning and conditioning was stopped. We clearly see the very wide time interval $10 \leq (t - t_c)/t_0 \leq 1000$ of logarithmic recovery of the resonant frequency, in complete agreement with experimental results [4]. Here t_c is the moment when conditioning switches off and $t_0 = 1\text{s}$ is the time scaling constant. Curves $j = 1, 2, 3$ on Figure 2 correspond to successively high water saturations $s_j = 0.05(2j - 1)$.

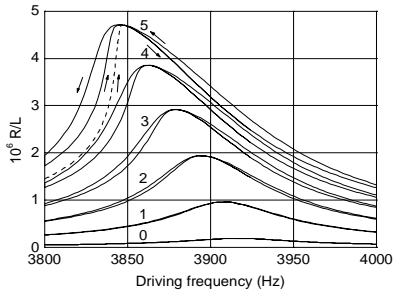


FIGURE 1. Resonance curves $j = 0, 1, 2, 3, 4, 5$ at successively higher driving amplitudes $D_j = 3.8(j + 0.2\delta_{j0})10^{-8}L$. The time to sweep back and forth within the frequency interval 3700–4100Hz is chosen to be 120s.

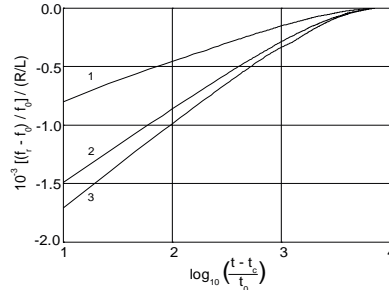


FIGURE 2. Time-dependent recovery of resonant frequency f_r to its asymptotic value f_0 . The frequency shift $f_r - f_0$ is normalized by both the asymptotic frequency f_0 and the unitless response amplitude R/L attained at conditioning resonance.

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REFERENCES

1. R. A. Guyer, and P. A. Johnson, *Phys. Today* **52**, 30 (1999).
2. J. A. TenCate, and T. J. Shankland, *Geophys. Res. Lett.* **23**, 3019 (1996).
3. K. E.-A. Van Den Abeele, J. Carmeliet, P. A. Johnson, and B. Zinszner, *J. Geophys. Res., [Solid Earth]* **107**, 2121 (2002).
4. J. A. TenCate, E. Smith, and R. A. Guyer, *Phys. Rev. Lett.* **85**, 1020 (2000).
5. O. O. Vakhnenko, V. O. Vakhnenko, T. J. Shankland, and J. A. TenCate, *Phys. Rev. E* **70**, 015602(R) (2004).
6. O. O. Vakhnenko, V. O. Vakhnenko, and T. J. Shankland, *Phys. Rev. B* **71**, 174103 (2005).