Dynamical realization of end-point memory in consolidated materials

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Abstract. Starting with a soft-ratchet model of slow dynamics in nonlinear resonant response of sedimentary rocks we predict the dynamical realization of end-point memory in resonating bar experiments with a cyclic frequency protocol. The effect we describe and simulate is defined as the memory of previous maximum amplitude of alternating stress and manifested in the form of small hysteretic loops inside the big hysteretic loop on the resonance curve. It is most clearly pronounced in the vicinity of bar resonant frequency. These theoretical findings are confirmed experimentally.

INTRODUCTION

Sedimentary rocks are prototypical of a class of materials that exhibit unusual elastic properties. In particular, they possess hysteresis and discrete memory [1-4]. For exploration of their equation of state, both quasi-static [1-3] and dynamic measurements [4, 5] have been used. Dynamic experiments may contain more information than quasi-static measurements. However, a description of the dynamic processes, and in particular, finding the equation of state, is a more difficult problem.

In modeling experiments on the longitudinal vibrational resonance of bar-shaped sedimentary rocks, we have proposed a closed system of equations for describing these processes [6-8]. Moreover, we have predicted the phenomenon of hysteresis with end-point memory in its essentially dynamical hypostasis [7]. In this paper we present recent experimental measurements that confirm above prediction.

PRINCIPAL PHYSICAL FOUNDATIONS OF MODEL

We here restrict the presentation of the theory only by its principal physical foundations and send the reader to our recent papers [6, 7] for details. First of all we believe that the number of intergrain defects determines the strength properties of consolidated medium. On the other hand, during dynamical loading the number of defects in a sample is changed (increased or decreased) relaxing continuously toward its dynamically driven would-be equilibrium value. Arguing a substantial difference between the typical rates of defect creation and defect annihilation, we come to a

CP838, *Innovations in Nonlinear Acoustics: 17th International Symposium on Nonlinear Acoustics*,
edited by A. A. Atchley, V. W. Sparrow, and R. M. Keolian

physical mechanism that breaks the symmetry of system response to an alternating external drive and acts as a sort of soft ratchet or leaky diode. The formalization of these and other basic ideas gives rise to a model system [6-8] that enables us to describe correctly a wide class of experimental facts concerning the unusual dynamical behaviour of such mesoscopically inhomogeneous media as sandstones [4-8], as well as to forecast a new essentially dynamical form of end-point memory [7].

DYNAMICAL REALIZATION OF END-POINT MEMORY: THEORETICAL PREDICTION

The question of whether an effect similar to the end-point (discrete) memory that is observed in quasi-static experiments with a multiply-reversed loading-unloading protocol [1-3] could also be seen in resonating bar experiments with a multiply-reversed frequency protocol has been risen in [7] and firstly was examined

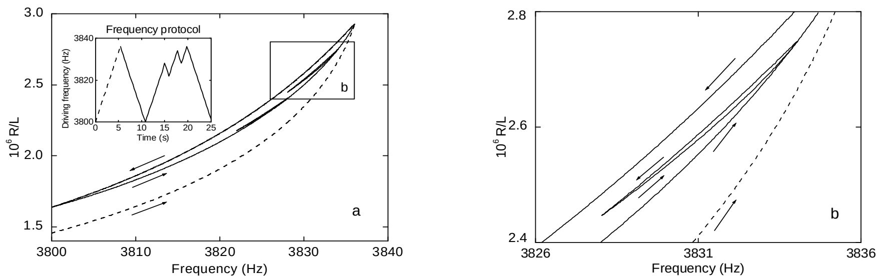

FIGURE 1. Manifestation of end-point memory in dynamic response with a multiply-reversed frequency protocol. R is the response amplitude, L is the length of the bar.

theoretically. The graphical results of this investigation are presented in Figure 1 (see also Fig. 16 in [7], where the model constants are given). One of the features of dynamical end-point memory, defined here as the memory of the previous maximum amplitude of alternating stress, is seen as small loops inside the big loop. The starting and final points of each small loop coincide, which is typical of end-point memory. According to our theory when producing an extremely small inner loop on conditioned (solid-line) curves the chance to find it closed diminishes in proportion to its linear size, being lower on the downward going curve and higher on the upper part of upward going curve. The reason for such behavior is the existence of a threshold stress amplitude (depending on previous history) that must be surmounted in order for the kinetics of the slow subsystem to be switched from defect annihilation at lower amplitudes to defect creation at higher amplitudes. This restriction can be substantially relaxed provided the linear size of the inner loop becomes comparable with that of the big outer loop. In contrast, when dealing with unconditioned (dashed-line) curve, a closed inner loop can be produced anywhere without any restrictions on its smallness (not shown).

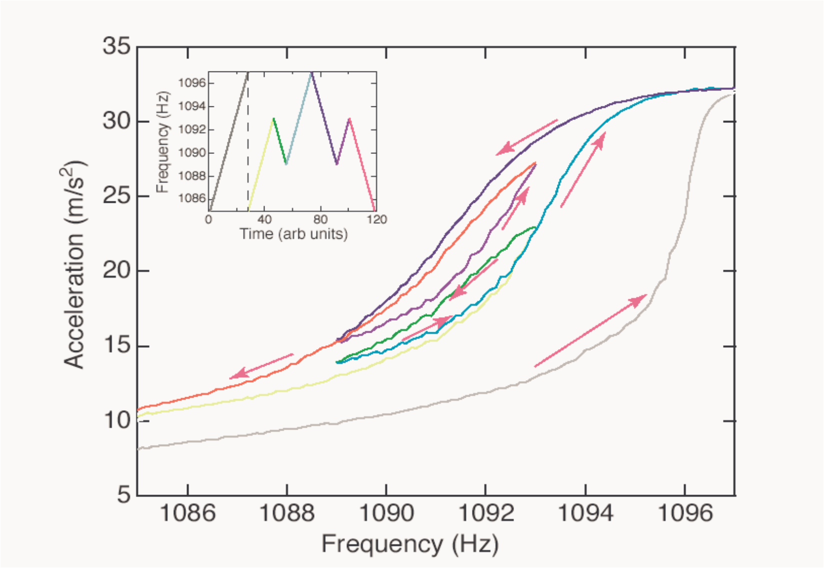

FIGURE 2. The low frequency sides of experimental resonance curves for Fontainebleau sandstone.

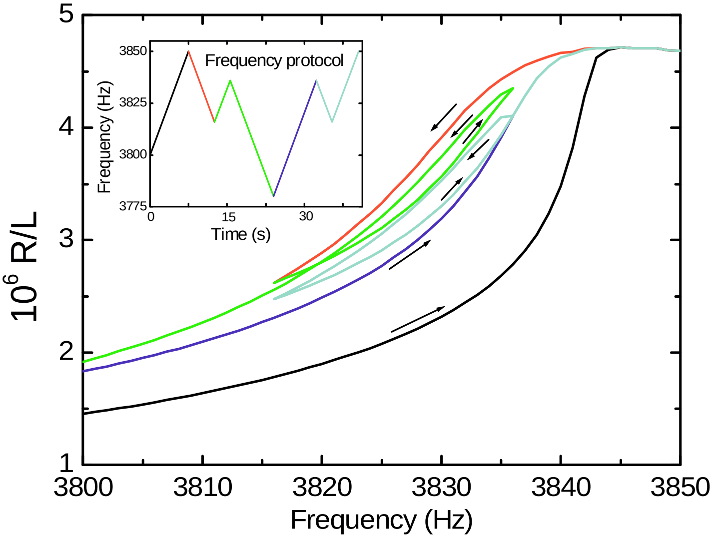

FIGURE 3. The low frequency sides of the resonance curves calculated for Berea sandstone.

DYNAMICAL REALIZATION OF END-POINT MEMORY: EXPERIMENTAL CONFIRMATION

Following the theoretical results, shown in Fig. 1, we performed experimental measurements to verify our prediction. The sample bar was a Fontainebleau sandstone and the drive level produced a calculated strain of about $2 \cdot 10^{-6}$ at the peak. Figure 2 shows the low frequency sides of resonance curves that correspond to the frequency protocol given on inset of Fig. 2. We clearly see that the beginning and end of each inner loop coincide, i.e., a major feature of end-point memory.

The experimental results for the Fontainebleau sandstone shown in Fig. 2 were simulated by using existing model equations (including a state equation) [6-8] and corrected (as compared with Fig. 1) constants for Berea sandstone. We note the good qualitative agreement between the experimental (Fig. 2) and the theoretical (Fig. 3) curves suggesting that our physical model is appropriate for both sandstones.

ACKNOWLEDGMENTS

This research was supported by the Science and Technology Center in Ukraine under Grant No. 1747. O.O.V. acknowledges support from the National Academy of Sciences of Ukraine, Grant No. 0102U002332. J.A.T. and T.J.S. thank the Geosciences Research Program, Office of Basic Energy Sciences of the US Department of Energy for sustained assistance.

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