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Nonlinear Elastic Wave Experiments: Learning About the Behavior of Rocks and Geomaterials

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Abstract
Nonlinear acoustics was a huge topic of research in the 1960s and 1970s after the creation of the parametric array. (By mixing two high-frequency sound waves together in a nonlinear medium such as water, a very focused beam could be created.) Other applications were also suggested and research in the field exploded. In the mid-1980s a group at Los Alamos began exploring the nonlinearity of the earth with a mind to developing tools such as the parametric array for use in seismic imaging. Initial measurements showed rocks to be highly nonlinear. Yet, attempts at carefully quantifying the dynamic behavior of rocks were frustrating, as were attempts to model the physics. Rocks showed some extremely peculiar behavior, including memory effects (slow dynamics), hysteresis, and end point memory in addition to the expected Landau-type nonlinearity. This chapter traces (historically) the macroscopic experiments that led to our current understanding of the peculiar nonlinearity of not only rocks and geomaterials, but many other materials as well. Results from some recent microscopic measurements where neutron scattering is used to help ascertain the physical origin of the nonlinearity conclude the chapter.

Keywords: Geomaterials, hysteresis, memory effects, nonlinear acoustics, nonlinear elasticity, rocks, slow dynamics

1. Introduction

The study of nonlinearity in rocks was a rather natural outgrowth of the study of nonlinear waves in air and water. After the development of jet engines in the late 1940s, interest in loud (or finite amplitude rather than infinitesimal amplitude) sound boomed. With a rapid increase in submarine fleets and the advent of the Cold War, interest in nonlinear underwater acoustics also grew. Some of the first experiments to study the interaction of nonlinear waves occurred in the late 1950s. The development of the underwater parametric array in the early 1960s—which used the nonlinear mixing of two sound beams to form a narrow difference frequency beam (i.e., an acoustic spotlight)—really drove the field of nonlinear acoustics into a frenzy.

Nonlinearity in air and water can be manifested in various ways. Waves that propagate in a nonlinear fluid such as water distort and develop harmonics with distance. Even in (weakly nonlinear) air, if the wave is intense, distortion will develop and shocks will form, the sonic boom is probably the most well-known example. Wave-mixing effects in the form of intermodulation distortion and concomitant sideband generation may also be observed. Nonlinear effects such as the modulation (and suppression) of sound by sound have been known since the late 1950s.

In seismology, the “acoustic approximation” is frequently used. In fact much of the world’s early seismic imaging is based on acoustic approximation. It was thus only natural to examine the nonlinearity of the earth and earth materials with nonlinear acoustics analogues. These nonlinear experiments were performed to explore and potentially develop techniques commonly used in nonlinear acoustics for seismic imaging applications. Early experiments at Los Alamos were carried out to create a parametric array in the earth (using an array of seismic sources). Although the results were inconclusive, the fact that earth materials were highly nonlinear was unmistakable. Nonlinear research in earth materials was thus scaled down to the laboratory (Johnson et al., 1987, 1989, 1991) and nearly ten years of research into the nonlinearity of earth materials resulted. Over this period, various types of rocks were studied and all were found to be highly nonlinear; notably, sedimentary rocks (which are oil and gas bearing) showed the largest and most interesting nonlinearities.

Two types of experiments in rocks and geomaterials are specifically discussed in this chapter. First, wave propagation experiments are carried out using (mostly) sedimentary solids (large blocks) and in long rods or rock cores. These experiments are analogous to acoustics experiments by D.T. Blackstock and his students at the University of Texas in Austin in the 1970s and 80s in air-filled ducts. Second are resonance experiments on long “thin” rods (or core samples). Although analogous (and mostly unremarkable) resonance experiments were attempted in air- and water-filled tubes, the much larger nonlinearity of rocks made resonance experiments and the effects of their nonlinearity much easier to study. Finally, a note about two-wave interaction experiments. Well-known two-wave nonlinear mixing experiments in water (e.g., modulation of sound by sound, scattering of sound by sound) were also performed in solids (Johnson et al., 1991) and lead somewhat naturally to techniques for nondestructive testing. Such experiments and techniques are discussed elsewhere in this book. This chapter concludes with a discussion of some very recent measurements using neutron scattering to learn about the microscopic behavior of the crystalline components of rocks and how these neutron scattering experiments relate to the peculiar nonlinear behavior that is now discussed.

2. Wave Propagation Experiments

Some of the first experiments on wave propagation in rocks (sandstone cores) were done at Los Alamos in the 1990s. Meegan et al. (1993) showed some of the very first wave propagation and harmonic measurements made in a long rock core of Berea sandstone. However, potential issues with receiver site effects due to bonding (a common problem in seismology), lack of strict environmental controls, and new modeling efforts led TenCate et al. (1996) to carefully repeat and expand Meegan’s results in a more carefully controlled environment.

Both sets of experiments yielded data that strongly suggested rocks were more complex than expected. Quasistatic stress–strain loops on sandstones were known to be hysteretic and highly unusual since the 1900s (Adams and Coker, 1906) and quasistatic measurements by many others (see Guyer et al., 1995) suggested the need to include hysteresis in models developed at that time. Even so, propagation time waveforms didn't match simple expectations (Kadish et al., 1996) and the prediction of harmonic levels, even with ad hoc improvements (e.g., including hysteresis) in Landau theory (Van den Abeele et al., 1997), were not very encouraging. A new type of experiment was needed.

3. Resonance Experiments

The examination of a particular resonance mode at increasing excitation levels is a common experiment and often used to study nonlinear oscillators. Softening or hardening nonlinearity (with increasing drive amplitude) produces an easy-to-identify family of resonance curves. With a softening nonlinearity, the resonance frequency drops with increasing amplitude; with hardening, the resonance frequency rises. The Duffing oscillator (which includes an additional cubic nonlinearity in the spring constant) is perhaps one of the most well-known and frequently studied nonlinear oscillators. Noticeable peak shifts and jumps are possible and can be quite common. Experiments done in the early 1970s (Cruikshank, 1972) were performed to see if an air-filled tube showed any of the behavior typical of a nonlinear oscillator. Results were positive but at the same time disappointing. Air is simply not very nonlinear.

On the other hand, similar experiments on long (thin) core samples of various rocks produced dramatic sets of nonlinear resonance curves. Johnson et al. (1996) showed such results for a wide variety of rocks. Resonance frequencies of the samples they examined always softened with increasing drive level; in one particular Fontainebleau sandstone the frequency shift they observed was nearly 10%! Moreover, resonance curves obtained by sweeping frequencies upward while watching the sample's response differed from curves obtained sweeping downward. The results were highly reminiscent of curves one might obtain from a Duffing oscillator. As mentioned before, sedimentary rocks (especially clean sandstones) showed some of the most dramatic nonlinear peak shifts with increasing drive levels.

Efforts to describe the nonlinear resonance curves obtained on rocks with Duffing-like theoretical treatments failed, sometimes miserably, so additional experiments were performed. TenCate and Shankland (1996) discovered that the different up and down response curves obtained as a rock was swept through a resonance are repeatable, but only after the rock was "conditioned" first. Moreover, once the rock was given suitable time to rest or "recover," the whole resonance behavior was completely reproducible. In the case of one sandstone sample, the behavior of the rock was repeatable for hundreds of experiments; that is, the rock's macroscopic behavior was unchanged during these experiments. The authors dubbed this behavior "slow dynamics" (discussed in many places throughout this book). Fortuitously, the time scales of the slow dynamics in rocks were on the order of tens of minutes which made them very easy to study.

(Other materials of interest for nondestructive testing applications showed slow dynamics on much shorter time scales.) Finally, TenCate et al. (2000) discovered that the recovery back to the original state of many rocks went as the logarithm of time. One other notable recovery process can be described with a log(t) behavior, creep back to equilibrium. Slow dynamics, however, is not necessarily related to creep; slow dynamics is induced with an ac (acoustic) drive; creep is induced with a dc driving force. Rocks are peculiar solids.

During the above experiments, it was discovered that rocks (sedimentary rocks and concretes in particular) and their concomitant response were also highly susceptible to humidity and temperature; however, slow dynamics always remained an identifying feature of the rock's response unless the rock was fully saturated. Thus, great care and extreme measures were taken to be sure measurements were made in carefully controlled environments. As a consequence, an isolation chamber was built; careful measurements made in this chamber showed that there was a threshold above which slow dynamics became dominant; below that drive threshold, the rock behaved as a weakly nonlinear Duffing oscillator. It was also shown that different rocks have different thresholds. For more details, see Chapter 26 by D. Pasqualini in this book.

4. Microscopic Measurements—Neutron Scattering

Within the last few years, several neutron diffraction experiments were carried out by Darling et al. (2004a,b) on intact samples of rock. In these experiments the authors and their colleagues took simultaneous neutron diffraction data while performing quasistatic stress–strain loops, and while doing conditioning and recovery experiments. In this way, information on the atomic (crystalline lattice) scale was obtained at the same time as some of the classic nonlinear macroscopic measurements were made. In addition, neutron diffraction was recently used to determine how much of the rocks was amorphous and how much was crystalline; some fascinating hints at mechanisms for nonlinearity have been identified. Three sets of experiments are described in this final section.

Quasistatic stress–strain measurements on rocks show hysteresis loops as well as nonlinearity. As with many quasistatic stress–strain measurements there is an initial conditioning cycle followed by a repeatable banana-shaped loop. Neutron diffraction measurements (Darling et al., 2004), however, show that the crystalline lattice always behaves in a completely reversible and linear fashion. In fact, the authors estimate that only a few percent of the volume of the rock must contribute to the nonlinearity and hysteresis seen in the macroscopic measurements. They conclude that it is likely in the bond structure of the rock where all the peculiar nonlinearities occur. Placing the origin of the nonlinearity with the bond structure was not a new idea; however, these are the first compelling experiments that support that hypothesis.

Recent measurements (Page et al., 2004) show another interesting aspect of sandstones. When a pair distribution function technique is applied to the diffraction pattern obtained from a pure quartz sandstone (Fontainebleau sandstone in their particular case), it was found that there were an excess number of Si–O and O–O bonds not belonging to any long-range crystalline structure in the rock. The authors suggest there may be an amorphous phase (glass?) within the rock. The idea is appealing. Glassy dynamics is certainly reminiscent of many of the peculiar behaviors seen in rocks.

Finally, recent neutron diffraction measurements by TenCate et al. (2005) were taken while abruptly changing temperature and also while applying and removing a conditioning acoustic drive. Both sets of macroscopic measurements show abrupt changes in the state of the rock (i.e., initial drop of modulus) and then slow, log(time) recovery back to the original (or a new) equilibrium state. Neutron diffraction, on the other hand, suggests that the bulk of the crystalline material behaves as expected during the temperature changes (a dc "driving" force), with no unusual nonlinear behavior whatsoever. The acoustic (ac "driving" force) experiment has yet to be analyzed. Work on this topic is nearly complete and another publication is in preparation.

5. Summary

Rocks (especially sandstones and other sedimentary rocks) have been shown to have very peculiar nonlinearities. On the other hand, they are also easy to study, and their nonlinear properties have proven helpful for studies of a host of other materials that display rocklike behavior. Much has been learned but very careful measurements were necessary. Although it has long been suspected that most of the interesting nonlinear behavior seen in rocks lies in the way the rock is put together (the bond system), recent neutron measurements confirm what was long suspected. Applications of this work include better concretes, understanding more about the strength and durability of buildings made of stone, and numerous nondestructive testing applications.

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