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Slow dynamics in the nonlinear elastic response of Berea sandstone

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Abstract. A typical resonance curve—measured acceleration versus drive frequency—made on a thin bar of rock shows peak bending with a softening (nonlinear) modulus as drive levels are increased. The shapes of these nonlinear resonance curves were found in earlier work to depend somewhat on sweep rate; these "slow dynamics" are now examined and quantified. We have measured slow dynamics in a 0.3 m long, 50 mm diameter bar of Berea sandstone under ambient conditions. Peak strain levels during the experiments ranged from 10^{-11} to 10^{-5} at driving frequencies near 4 kHz, the fundamental longitudinal resonance frequency of the bar. Slow dynamics begin to manifest themselves at strain amplitudes above 10^{-6} at ambient conditions and at the onset of nonlinear peak bending. Strains above this value condition the rock, altering its response for minutes to hours after the drive has been turned off.

Introduction

Over the past several years, rocks have been shown to be elastically nonlinear, even at strains as low as 10^{-7}. Static stress-strain curves, e.g., [*Holcomb*, 1981, *Boitnott*, 1993, *Gist*, 1994, *Hilbert et al.*, 1994], nonlinear resonance measurements [*Johnson et al.*, 1996, *Zinszner et al.*, 1996], and nonlinear wave propagation experiments [*Meegan et al.*, 1993, *TenCate et al.*, 1996] have been performed to understand, quantify, and model the mechanisms that produce the observed nonlinearity. Recent theoretical work [*Guyer et al.*, 1995, *Van Den Abeele et al.*, 1996] has applied the Preisach-Mayergoyz (PM) model to describe hysteresis, end-point memory, and other nonlinear elastic properties of rock.

This paper presents resonance curves obtained by measuring the acceleration at the end of a sample bar while sweeping frequency at a fixed drive level. Families of curves were constructed by sweeping up and down at a number of different drive levels. Earlier work [*Johnson et al.*, 1996, *Zinszner et al.*, 1996] with numerous different rock types shows a softening nonlinear peak response, i.e., downward peak shift. The results are similar to those seen in many "simple" nonlinear systems (e.g., a Duffing oscillator), but marked differences remain.

In the resonance experiments noted above, sweep rate and relaxation effects were noted. To avoid these effects, the authors empirically chose a sweep rate that gave them the best repeatability. Additional effects were observed; the response of samples under weak confining pressures took some time (e.g., hours) to stabilize after the pressure on the rock was changed. The work presented here is the first to specifically document the slowly varying time response to periodic excitation (i.e., slow dynamics) of a Berea sandstone sample; moreover, we demonstrate that the slow dynamics produce prominent effects, even at room conditions.

Slow dynamics manifest themselves in a sandstone sample during high intensity longitudinal elastic resonance. We first report the effects of high strain on a sample by comparing "conditioned" resonance curves (e.g., after the sample is driven at high intensity) with a set of fully "recovered" curves (e.g., after the sample is left untouched for a long period). We illustrate that the first resonance curve made on a recovered sample differs remarkably from subsequent curves. We present several results that illustrate aspects of the sample's memory and conclude with a discussion of measurements to determine how the rock recovers and to understand what mechanisms might be at work. *Shamina, et al.*, 1990] suggest that slow dynamics may also play a role in wave propagation.

Experiment

The experimental apparatus used to obtain resonance curves is shown in Fig. 1. The sample is a 0.3 m long, 50 mm diameter bar of Berea sandstone (Cleveland Quarries, Amherst, Ohio). The fundamental longitudinal resonance frequency is around 3.9 kHz at ambient conditions, i.e., temperature between 22-26°C and humidity between 25-30%. A PZT-4 piezoelectric disk having a tantalum inertial backload was epoxied to the end of the rod as a source; it was driven in discrete frequency steps from an HP8904A function generator through a Hafler Pro5000 audio amplifier. A small receiver (a B&K 8309 accelerometer) connected to a B&K 2635 charge amplifier was attached to the opposite end of the bar. A reference signal from the function generator and the signal from the accelerometer/charge amp were both fed into a fully programmable EG&G 5302 lock-in amplifier. The apparatus is capable of measuring voltages corresponding to strains down

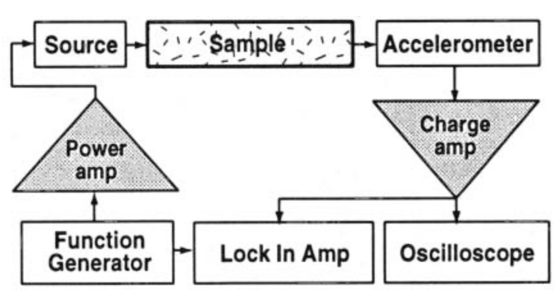

Figure 1. Block diagram of experimental apparatus.

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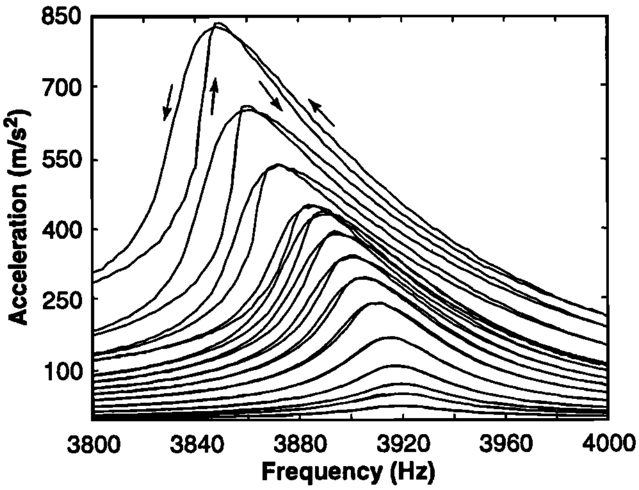

Figure 2. Unconditioned resonance curves measured at successively higher amplitudes. Arrows on highest pair of curves indicate sweep direction. All lower curves are similar. Note sharp peak and asymmetry at highest amplitude.

to 10^{-12}. Sweep rates, step sizes, and data storage are computer controlled.

Resonance curves similar to earlier measurements, e.g., [*Johnson et al.*, 1996] were made. Step size and time increment between steps were 2 Hz and 300 ms, respectively. Frequencies ranged from 3800 to 4000 Hz so total sweep time, ≈2 min, was similar to that of previous work. Measured accelerations shown in Fig. 2 correspond to peak strains between 10^{-9} to 12×10^{-9}. In general, up and down curves are identical up to strains of around 10^{-6}, similar to observations reported earlier.

The first curve made when the rock was in its recovered state differed from subsequent curves. Figure 3 shows a set of conditioned resonance curves obtained after two frequency sweeps were performed at each drive level to achieve repeatable curves. Full sweeps are not necessary for conditioning to occur; high drive levels at a single frequency for a length of time will produce the same effect. Compare Figs. 2 and 3. Notice that conditioning removes many of the "sharp" features seen in Fig. 2 (e.g., the steep rise on the upward sweeps and corners at the top) and generally smooths the results.

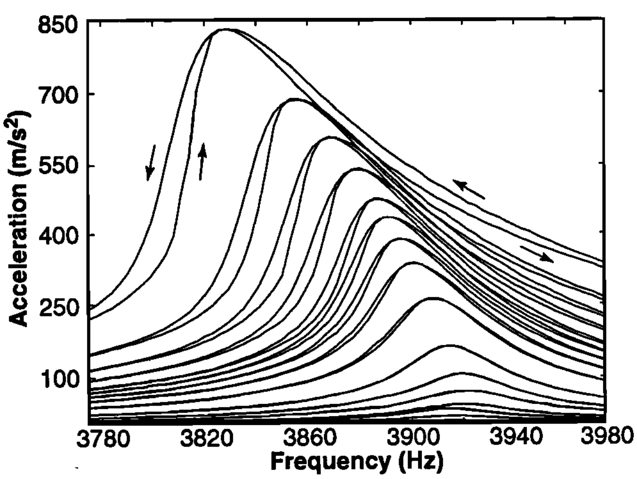

Figure 3. Resonance curves at successively higher amplitudes only after a few up and down sweeps were made first (conditioning). Arrows on highest pair of curves indicate sweep direction.

The results of Figs. 2 and 3 would look different had the first sweep been down from 4000 Hz instead of up from 3800 Hz. As mentioned above, the first resonance curve made on a "recovered" sample differed from the following curves. Figures 4(a) and (b) show these initial curves (indicated on each figure) and several successive up/down or down/up sweeps. Figure 4(a) represents a set of curves beginning at 3800 Hz and sweeping up, Fig. 4(b) represents a set starting at 4000 Hz and going down. Although done on separate days with slightly different environmental conditions, both sets of curves were made after the rock had recovered overnight. If the first sweep was made at a high level, it altered the response of the rock; repeated cycling conditions the sample to repeatable behavior. Amplitudes of the repeatable curves all lie above the initial curve at frequencies below the resonance frequency 4(a); they lie below the initial curve for downward sweeps at frequencies above the resonance frequency 4(b).

Once conditioning was achieved, the dependence of resonance curves on sweep rate was examined more carefully. Because of time considerations (e.g, stability of laboratory ambient conditions for long periods

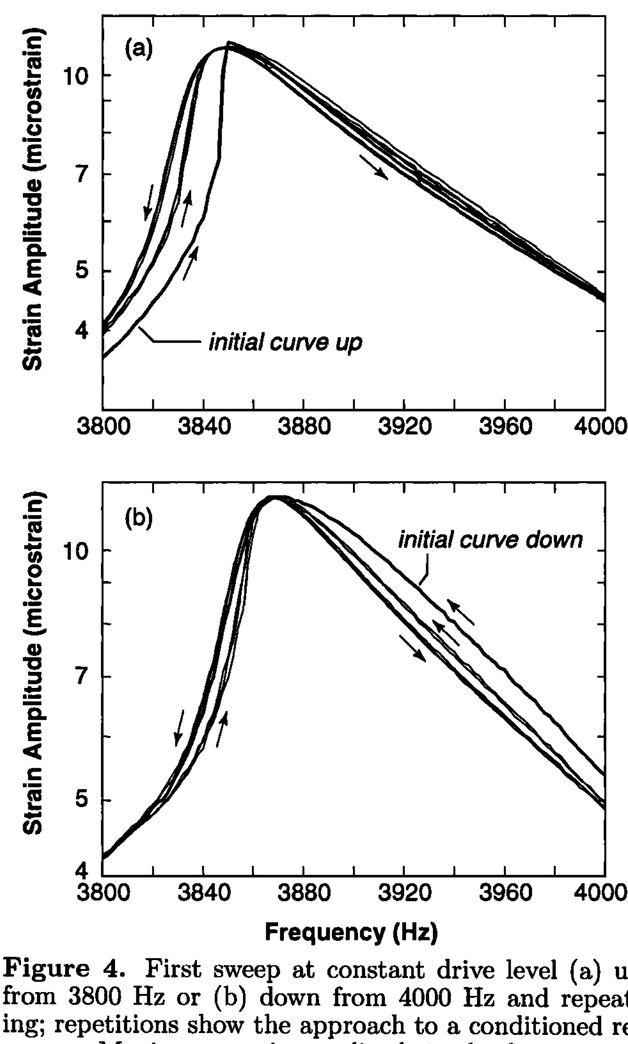

Figure 4. First sweep at constant drive level (a) up from 3800 Hz or (b) down from 4000 Hz and repeating; repetitions show the approach to a conditioned response. Maximum strain amplitude in the figure corresponds to an acceleration of 600 m/s^2.

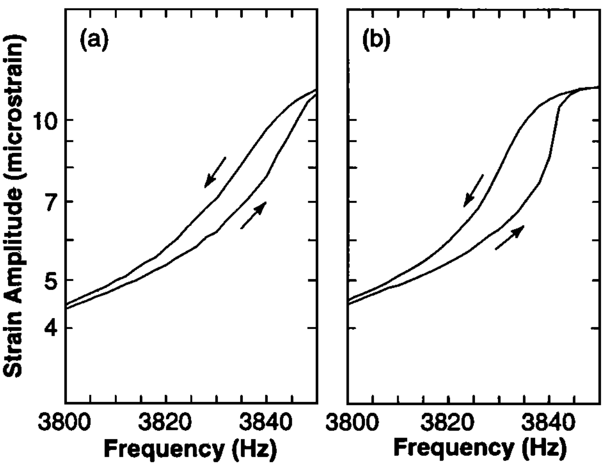

Figure 5. Time dependence on sweep rate. Only low frequency sides of resonance curves are shown. Time between frequency steps is (a) 300 ms and (b) 30 s.

of time and worry about damaging the rock or transducer bond at these levels), only the low frequency sides of the resonance curves were measured (from 3800 to 3850 Hz). Figure 5(a) shows curves made with a step size of 2 Hz and time between steps of 300 ms, identical to the timing used to record the curves shown in Fig. 4(a). The curves shown in Fig. 5(b) were made the same way except with a much longer 30 s wait between steps. In both cases the rock was first conditioned by driving the rock for 2 min at the resonance peak for this drive level. The result is perhaps surprising. Extremely slow sweep rate measurements might be expected to trace the same path up and down. However, the measurements suggest that during the slow sweep downward, "memory" of the highest strain amplitude still persists. Note that the upward sweep of Fig. 5(b) resembles the first upward sweep of Fig. 4 for a recovered sample that has "forgotten" being at high strain.

Still slower sweep rates will yield results different from those shown Fig. 5(b). While making a repeatable up/down curve, we stopped the sweep (drive still on); acceleration was observed and recorded for nearly 10 minutes. As shown in Fig. 6, measured acceleration gradually decreased for the experiments where the stopping frequency was lower than the resonance frequency, and increased when the stopping frequency was higher (i.e., the rock gradually lost memory of the highest strain). After several minutes, both levels approached the same long term level. Thus, from these measurements, we assume that a sweep time of a few days in carefully controlled conditions would produce the same up and down curves for this sample.

To examine another facet of the rock's recovery time we varied the previous experiment by sweeping down from resonance and stopping the sweep (drive turned off this time) for several seconds. In a relatively short time (10's of seconds), the rock's memory of the high strain amplitude it had experienced at resonance diminished, far more quickly than when the drive was left on. Figures 7(a) and (b) show the faster response for both sides of the resonance curve. In both cases we first conditioned the sample by driving at the resonance frequency for this strain level (3850 Hz) for 2 min. We then swept downward from resonance to 3825 Hz, Fig. 7(a),

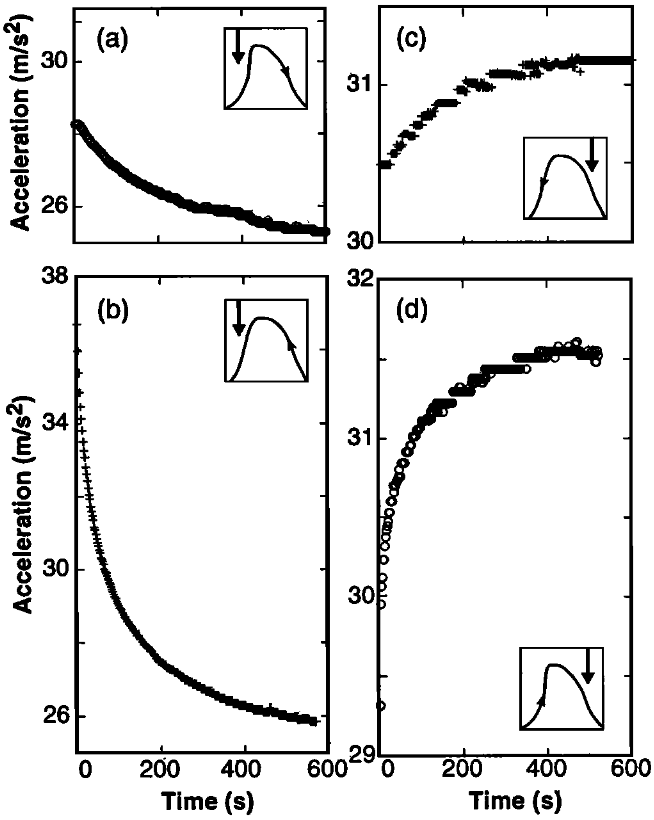

Figure 6. Measurement of acceleration decay at fixed frequency. Frequencies are 3825 Hz, upward going (a) or downward going (b), and 3900 Hz downward going (c) or upward going (d). Note that (a) and (b) end up at nearly the same acceleration; a similar observation holds for (c) and (d).

and upwards from resonance to 3910 Hz, Fig. 7(b). The drive was shut off for only 30 s, turned back on, and the remainder of the curve recorded. Results are clearly seen as discontinuities on the curves. At frequencies lower than resonance, the response quickly drops down

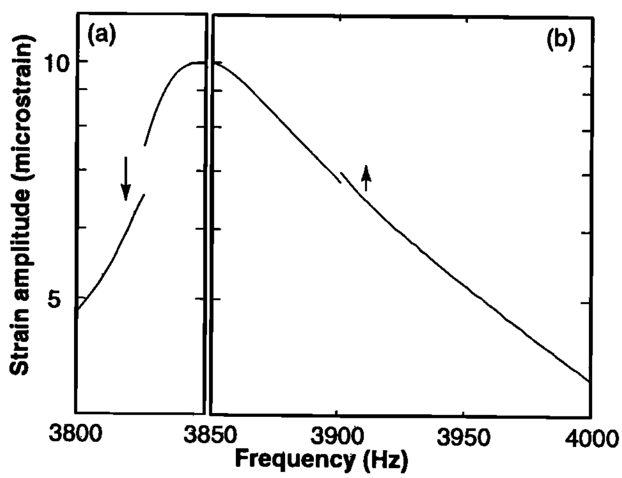

Figure 7. Stopping resonance curve sweep, turning off the drive for 30 s and then continuing the sweep. Stopping frequencies are 3825 Hz and 3900 Hz. The arrows indicate the direction the measured amplitude jumps that take place during the time the drive is off.

toward the first (recovered) up-going curve shown in Fig. 4(a). At frequencies higher than resonance the response jumps up towards the first (recovered) curve shown in Fig. 4(b). Compare the recovery times in Figs. 6 and 7. Memory of the highest strain is lost much more quickly—seconds instead of minutes—if the drive is turned off.

A qualitative view of these jumps can come from the nonlinear change of bar modulus E with amplitude. After a period of high intensity, the rock's resonance curve shifts downward in frequency (i.e., the modulus decreases). If the drive is then turned off, the resonance curve moves back (i.e., the modulus increases) as memory of the high strain is lost.

Summary and discussion

This work documents unusual time-dependent effects in rocks driven in resonance into the nonlinear regime of visible peak shifts. In this regime resonance curves differ depending on whether the sweep was started above resonance or below. Even starting from one side of the peak, each direction of sweep differs from the other. Both effects imply memory of strain history. There are two asymptotic states of the rock, fully recovered and conditioned. The former state is fully achieved after several hours of no excitation; the latter possesses reproducibility on successive sweeps.

When the drive frequency paused while descending the flanks of the resonance curve, amplitude slowly moved toward a level equivalent to a slow sweep starting from the recovered state. This implies that over several minutes the rock memory of peak amplitude is lost. On the other hand, when the drive is turned off while descending the flanks of the resonance curve and then restored, amplitude rapidly (in seconds) approached the level of a slow sweep started from the recovered state. The direction of recovery depends on the side of the resonance peak, but in both cases recovery is down on the low frequency side and up on the high frequency side. These time-dependent dynamics can be broadly thought of as "memory" of excitation to a condition of nonlinear-dynamic strain and rate-dependent loss of that condition at lower amplitudes.

As bulk heating ($1/2\ Ee^2 f/Q$ or 33 $\mathrm{mW/cm^3}$) is immeasurable small, we believe that the physical origins of slow dynamics lies in nonlinear effects at the micro structural level of cracks, pores, and interstitial clays. Further work is examining environmental effects on conditioning and recovery as a means of relating them to physical properties and microtexture of the rock.

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