On zonal jets in oceans

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1 We find that in parameter regimes relevant to the recently observed alternating zonal jets in oceans, the formation of these jets can be explained as due to an arrest of the turbulent inverse-cascade of energy by free Rossby waves (as opposed to Rossby basin modes) and a subsequent redirection of that energy into zonal modes. This mechanism, originally studied in the context of alternating jets in Jovian atmospheres and two dimensional turbulence in zonally-periodic configurations, survives in spite of the presence of the meridional boundaries in the oceanic context. Citation: Nadiga, B. T. (2006), On zonal jets in oceans, Geophys. Res. Lett., 33, LXXXXX, doi:10.1029/2006GL025865.

1. Introduction

2 A proposed explanation of the alternating zonal jets in Jovian atmospheres is that they are due to a tendency of turbulence in thin shells on the surface of a rotating sphere to organize itself into zonal jets [e.g., Vasavada and Showman, 2005; Galperin et al., 2004]. The anisotropic jets result from an interplay between an inverse cascade of energy [Kraichnan, 1967; Charney, 1971] and the latitudinal variation of the vertical component of planetary rotation [e.g., Newell, 1969; Rhines, 1975]. While baroclinic instability and convective processes are thought to be the main sources of small scale energy, classical geostrophic turbulence theory [Charney, 1971] predicts a cascade of this energy (vertically) to larger scales as well in a process that has been termed barotropization. Hence, in the context of this explanation of atmospheric-zonal jets, they have been simulated and studied extensively using the barotropic vorticity equation on either the doubly-periodic or zonally-periodic beta-plane or on the surface of a sphere using forced-dissipative settings. In these settings, the effect of geometry on dynamics is minimized in the sense that the flow in the zonal direction, the direction in which the dynamics of the Rossby waves are highly asymmetric, is homogeneous. Dynamically, the formation of the zonal jets in this homogeneous setting is thought to involve certain kinds of resonant interactions (sideband triad and quartet) of Rossby waves packets whose amplitudes are slowly varying functions of space and time [Newell, 1969].

3 More recently, observational [Maximenko et al., 2005] and computational [Nakano and Hasumi, 2005] evidence point to the occurrence of multiple alternating zonal jets in the world oceans as well. However, the dynamics underlying their formation is not clear.

4 On the one hand, given that the governing equations are the same in the atmospheric and oceanic contexts, it would not be unreasonable to expect, from a turbulence point of view, that the same dynamical mechanism—Rossby wave dispersion arresting the inverse cascade of energy and redirecting it into zonal modes—underlies the phenomenon, be it in the ocean or in the atmosphere. Clearly, unlike the constant stratification of the atmosphere, surface-intensified stratification in the oceans inhibits full barotropization [e.g., Fu and Flierl, 1980]. Nevertheless, the importance of the barotropic mode (with a thermocline depth of 1 km in a 5 km deep ocean) is clearly borne out in Figure 2 and Table 1 given by Fu and Flierl [1980] and other such studies confirm an inverse cascade of barotropic kinetic energy. High vertical coherence of jet structure in models [Nakano and Hasumi, 2005; Maximenko et al., 2005] further suggests the importance of barotropic dynamics.

5 On the other hand, the presence of boundaries can, besides being able to support viscous boundary layers and act as sources/sinks of enstrophy, allow for new (inviscid) mechanisms. For example, in a closed basin, (a) Fofonoff gyres arise as statistical equilibrium solutions of the barotropic vorticity equation, and (b) Rossby basin modes arise, resonant interactions of which have been studied as mechanisms for generating both mean flows [see Pedlosky, 1965] and mesoscale variability [see Harrison and Robinson, 1979]. Such mechanisms could possibly generate alternating zonal jets as well. Interestingly, LaCasce [2002] finds that the arrest of the inverse cascade of energy by basin normal modes is largely isotropic. However, in a recent article studying rectification processes in a three layer quasi-geostrophic beta plane basin, Berloff [2005] concludes that the alternating zonal jets he found in that setting were most likely driven by nonlinear interactions between some meridionally structured baroclinic basin modes and some secondary (i.e., related to finite amplitude background flows) basin modes. If this were to be the most important mechanism for the formation of alternating zonal jets in ocean basins, by involving spatially-nonlocal (basin) modes this mechanism would be fundamentally different from the (spatially) local arguments of turbulence that are usually thought to apply in the atmospheric context.

6 In this letter, we demonstrate that in parameter regimes relevant to alternating zonal jets in the oceans, such jets can be formed by free Rossby waves (as opposed to Rossby basin modes) arresting the inverse-cascade of energy. We then go on to show that the jet width scales well with Rhines’ scale. This suggests that the dynamics of alternating zonal jets in oceans are likely local and in this sense similar to those in previously studied atmospheric contexts. We suggest that the nonlinear resonant-interaction-of-basin-modes mechanism becomes more important at larger values of turbulent kinetic energy (TKE). Curiously, such jets can be formed by free Rossby waves (as opposed to Rossby basin modes) arresting the inverse-cascade of energy. We then go on to show that the jet width scales well with Rhines’ scale. This suggests that the dynamics of alternating zonal jets in oceans are likely local and in this sense similar to those in previously studied atmospheric contexts. We suggest that the nonlinear resonant-interaction-of-basin-modes mechanism becomes more important at larger values of turbulent kinetic energy (TKE). Curiously,
only the latter regime has been investigated before within
the framework of the barotropic vorticity equation
[LaCasce, 2002], and as far as we know this is the first
time that alternating zonal jets have been obtained in a
closed basin using the barotropic vorticity equation.

The rest of the letter is structured as follows: The next
section briefly describes the modelling approach, and the
following one presents computational results. As a matter of
convenience, and with no loss of generality, these two
sections consider the governing equations and present
results in a nondimensional form. The final section
establishes the correspondence between the nondimensional
paramerter values considered and their dimensional counter-
parts in actual ocean settings.

2. Modeling Approach

We consider the barotropic vorticity equation

$$\frac{\partial \psi}{\partial t} + J(\psi, q) = F + D$$

for the evolution of barotropic potential vorticity $q = \zeta + \beta y =
\nabla^2 \psi + \beta y$, where $\zeta$ is relative vorticity, $\psi$ is velocity
streamfunction, $F$ is forcing, $D$ is dissipation and $J(q)$ is the
Jacobian operator given by

$$J(q) = -\frac{\partial \psi}{\partial x} \frac{\partial q}{\partial y} - \frac{\partial \psi}{\partial y} \frac{\partial q}{\partial x}.$$  

The above equation is considered on a midlatitude beta
plane with a latitudinal gradient of the vertical component
of rotation of $\beta$; $y$-coordinate increases northward and $x$-coordinate eastward in a closed square basin, $2\pi$ on a
side, discretized into $1024 \times 1024$ cells. An energy and
enstrophy conserving finite-differencing is used with
Runge-Kutta timestepping [Greathatch and Nigada, 2000].

Given the inverse-cascade of energy of 2D turbulence,
forcing $F$ is concentrated around a high wavenumber $k_h$, as a combination of sines and cosines consistent with the
boundary conditions used. Their amplitudes $\sigma$, drawn
randomly from a Gaussian distribution, are delta-correlated
in time resulting in $F = \sigma(t) \sqrt{\delta f(k_h, t)}$ with an energy input
rate $\epsilon$ of $\sigma^2 \int \int f^2 \delta f dx dy$ and an enstrophy input rate $\eta$ of
$k_h^2 \epsilon$. In all the computations presented, given the domain size
of $2\pi \times 2\pi$ discretized into $1024 \times 1024$ cells, $k_{max}$ is
512 and $k_f$ is between 128 and 129.

Dissipation $D = -\nu_p \nabla^2 \psi - \nu_0 \dot{\psi}$, consisting of a
small-scale-selective component to dissipate the (largely)
downscale-cascading enstrophy input at the forcing scale, and
Rayleigh friction component that mainly acts to dissipa-
ate the (largely) upscale-cascading energy. At lateral
boundaries, besides no through-flow, we use superslip
boundary conditions. The coefficient $\nu_p$ is diagnosed
dynamically in terms of the enstrophy input rate as $\nu_p =
C_k \epsilon \Delta x^2$, using Kolmogorov-like ideas and a Kolmogorov
scale of $\Delta x$.

[11] Given the above setup, the problem consists of three
important parameters: $\beta$, $\epsilon$ and $v_0$. We briefly recall a few
relevant spatial scales in terms of these parameters. First, in
purely two-dimensional turbulence, a Kolmogorov scale for
the dissipation of energy may be obtained using the usual
arguments as

$$k_0 = \left( \frac{3 \epsilon}{\nu_p / \Delta x^2} \right)^{1/5} \approx 50 \left( \frac{\nu_p}{\epsilon} \right)^{1/5}$$

(2)

Figure 1. Meridional plot of the instantaneous zonally-
averaged zonal-velocity and vorticity.

Table 1. Basic Parameters, Derived Scales and the Jet-Width Wavenumber for the Simulations Considered

<table>
<thead>
<tr>
<th>Case</th>
<th>$\beta$</th>
<th>$\epsilon$</th>
<th>$v_0$</th>
<th>$k_h$</th>
<th>$k_f$</th>
<th>$k_{max}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>15.9</td>
<td>0.50</td>
<td>0.1</td>
<td>15.9</td>
<td>0.50</td>
<td>15.9</td>
</tr>
<tr>
<td>B</td>
<td>22.2</td>
<td>0.50</td>
<td>0.1</td>
<td>22.2</td>
<td>0.50</td>
<td>22.2</td>
</tr>
<tr>
<td>C</td>
<td>31.3</td>
<td>0.50</td>
<td>0.1</td>
<td>31.3</td>
<td>0.50</td>
<td>31.3</td>
</tr>
<tr>
<td>D</td>
<td>38.5</td>
<td>0.50</td>
<td>0.1</td>
<td>38.5</td>
<td>0.50</td>
<td>38.5</td>
</tr>
<tr>
<td>E</td>
<td>66.7</td>
<td>0.50</td>
<td>0.1</td>
<td>66.7</td>
<td>0.50</td>
<td>66.7</td>
</tr>
<tr>
<td>F</td>
<td>59.6</td>
<td>0.50</td>
<td>0.1</td>
<td>59.6</td>
<td>0.50</td>
<td>59.6</td>
</tr>
<tr>
<td>G</td>
<td>5.8</td>
<td>0.50</td>
<td>0.1</td>
<td>5.8</td>
<td>0.50</td>
<td>5.8</td>
</tr>
<tr>
<td>H</td>
<td>5.8</td>
<td>0.50</td>
<td>0.1</td>
<td>5.8</td>
<td>0.50</td>
<td>5.8</td>
</tr>
<tr>
<td>I</td>
<td>18.2</td>
<td>0.50</td>
<td>0.1</td>
<td>18.2</td>
<td>0.50</td>
<td>18.2</td>
</tr>
<tr>
<td>J</td>
<td>15.9</td>
<td>0.50</td>
<td>0.1</td>
<td>15.9</td>
<td>0.50</td>
<td>15.9</td>
</tr>
</tbody>
</table>

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cases considered, care is taken to ensure that the spectrum of
the zonal component of energy has equilibrated. While there
are important differences between some of the cases in
Table 1, we postpone a detailed discussion of these differ-
ences to a later article, and go on to examine a representative
case—case C presently. An examination of the instantaneous,
zonally-averaged, zonal-velocity and relative-vorticity
fields plotted as a function of latitude in Figure 1 suggests
altering zonal jets of a characteristic width. To further
verify this, we examine a few other familiar diagnostics. First,
Figure 2 shows the time-mean two-dimensional zonal
velocity field after the flow has reached statistically
stationarity, and the alternating zonal jets are evident in this
figure. Note that a) even though the forcing is homogeneous,
the jets are more pronounced to the west, b) unlike with
observations, time-mean jet signatures are obtained and
analysed, and c) the geometry of the jets are not significantly
different when the time-varying components are analysed
(not shown). Finally, while the meridional gradient of time-
averaged potential-vorticity is dominated by $\beta$ (stable), the
instantaneous flow quite frequently violates the barotropic
stability criterion $u_{\nu} < \beta$.

[14] That these alternating zonal jets are related to aniso-
tropization of the inverse cascade of energy of two dimen-
sional turbulence by Rossby wave dispersion is verified by
the dumbbell shape near the origin, characteristic of the
process [e.g., Vallis and Maltrud, 1993], in the contour plot
of the two dimensional spectral density of energy in Figure 3.

[15] It is not our intent to verify various universal scalings
of spectra in this problem, but to use it as a diagnostic
to further confirm the nature of the dynamics. To this end,
we show in Figure 4 the range of spectra that we obtain
in the parameter range considered. These figures show
the 1D energy spectra averaged over an angle of $\pi/6$ around
$\phi = 0$ (residual flow) and $\phi = \pi/2$ (zonal flows) [Chekhlov et
al., 1996]. Both the residual and zonal spectra have
further been compensated for the $k^{-5/3}$ scaling. (A 222
compensation for $k^{2/3}$—following Kolmogorov scaling
$E(k) = C_k k^{2/3} k^{-5/3}$ is avoided since while that would be
appropriate for the residual component, it would not be
appropriate for a possibly different scaling of the zonal
component such as $E_z(k) = C_k^{*} k^{-5/3}$). However, $k^{2/3}$ com-
penlation has been applied to the residual spectra to
establish the value of the Kolmogorov constant $C_k$. A 231
common and important feature of all the cases is that in
232 the inverse-cascade regime, while at the high-wavenumber
end, the zonal and residual spectra scale similarly, at lower
234 wavenumbers the zonal spectra lie above the residual 235
spectra and show steepening before they peak. This behav-
237 ior is as expected and verified by various investigators in the
238 periodic case relevant to the atmosphere. Furthermore, like
239 in the computations given by Danilov and Gurarie [2001],
our spectra display significant non-universal behavior. For
240 example, while in cases A and F, the residual spectra clearly
241 verify the classic Kolmogorov scaling $C_k k^{2/3} k^{-5/3}$ with a
242 Kolmogorov constant $C_k$ of about 6, that is not the case for
cases C and J. We note parenthetically that (a) the distribu-
tion of spectral energy flux as a function of wavenumber
245 (not shown) bears remarkable resemblance to that derived
246 using Aviso, TOPEX/Poseidon, and ERS-1/2 data [Scott
247 and Wang, 2005], and (b) on using the spectral flux of 248
energy as a function of wavenumber, as opposed to a 249
constant value, $C_k$ remains close to 6 in the high wave-
250 number range of the inverse-cascade, but then begins to rise
251 at the lower wavenumbers.

[16] Next, we identify the jet width with the wavenumber
at which the (uncompensated) zonal-spectrum peaks, $k_p$, 254
and verify it by referring to physical-space pictures like in 255

Figure 2. The alternating zonal jets are evident in the
time-averaged, two-dimensional zonal-velocity field. While
forcing is homogeneous, jets are more prominent in western
regions.

Figure 3. The time-averaged two-dimensional energy
spectrum displays the familiar anisotropic ‘dumbbell’ shape.

Figure 4. Time-averaged one-dimensional zonal (dot-
dashed line) and residual (solid line) energy spectra. Both
have a $k^{-5/3}$ compensation. See text for details.
Figures 1 and 2. This number is recorded for each of the cases in Table 1 and line to the linear least squares fit.

Figures 1 and 2. This number is recorded for each of the cases in the last column of Table 1. We note, that the wavenumber at which the residual spectrum peaks is close to this wavenumber as well. In Figure 5, we plot the above measure of jet width ($k_p$) against the Rhines’ scale ($k_p^R$) and find excellent agreement, like in the periodic (atmospheric) case [e.g., Vallis and Maltrud, 1993; Danilov and Gurarie, 2001].

4. Discussion

[17] The simulations and analyses presented in the previous section show clearly that there are parameter regimes wherein the dynamics of the alternating zonal jets in a midlatitude ocean basin are not controlled in a fundamental manner by meridional boundaries. That is to say, in these parameter regimes, the dynamics of the jets are governed largely by spatially local interactions, and the arrest of the inverse-cascade is mediated by free Rossby waves as opposed to Rossby basin modes. However, we still need to check if such a parameter regime is of relevance to the oceans in order to establish the importance of this mechanism in the oceans.

[18] The pronounced signature of the observed jets in western regions of ocean basins [Maximenko et al., 2005, Figure 1] would generally be attributed to the elevated levels of TKE in the separated western boundary current (WBC) regions. However, our simulations use homogeneous forcing but still display similar enhanced jet signatures in the west. This leads us to suspect that the enhanced signature of the jets in the west is more due to its internal dynamics, rather than due to the elevated levels of TKE in the WBC regions, and that the jets are controlled more by the ambient (lower) levels of internal TKE. An approximate range of 25 to 100 cm$^2$/s$^2$ is obtained for the latter by examining an altimetry-derived TKE map for the North Atlantic (R. B. Scott, personal communication, 2006). Keeping this in mind, first consider case C discussed extensively above: the peak wavenumber $k_p$ is 26 (Table 1); using the observed [Maximenko et al., 2005] dominant wavelength of 280 km. leads to $L_{ref}$ of 1160 km. Then, using a typical midlatitude value of $\bar{\beta}_{ref}$ of $2 \times 10^{-11}$ m$^{-1}$s$^{-1}$ and an r.m.s. value of 7.5 cm/s (mid-range) for the domain-averaged $\bar{\beta}$, $\beta_{ref}$($\bar{\beta}_{ref}$, $U_{ref}$) of 360, corresponding well with 320 used for case C. As for the ranges of parameters considered, $5 \leq k_p \leq 55$ (Table 1), using a range of wavelengths of 500 to 250 km and domain-averaged $U_{rms}$ level corresponding to $5 \leq U_{rms} \leq 10$ cm/s, gives $\beta_{ref}$ in the range 30–1900 (see range of 80–1280 in Table 1). In light of this, we suggest that the local mechanism wherein the arrest of the inverse cascade is mediated by free Rossby waves may be important in explaining the formation of alternating zonal jets in the world oceans. Obviously, further work is necessary to definitively establish the relevance of this mechanism to the oceans.

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References


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