Dispersive Eddy Parameterization in a Barotropic Model

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Abstract

Recently a new class of coarse-grained equations, known as α-models, have been proposed for the mean motion of an ideal incompressible fluid. In this note, we consider the use of one such model to represent the time-mean component of a turbulent β-plane circulation that is characterized by potential-vorticity mixing. In particular, we focus on the wind-driven circulation in a shallow ocean basin, a problem that has been well studied as a prototype of more realistic ocean dynamics. We demonstrate the ability of an α-model to reproduce qualitatively the structure of a four-gyre circulation that forms (in the time mean) when the barotropic vorticity equation is driven by a symmetric, double gyre wind forcing, and when the dissipation is weak. We offer this as a first step in assessing the utility of the α-model approach to simulating more complex geophysical flows.

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I. INTRODUCTION

In a variety of turbulent geophysical flows, the tendency of potential vorticity to become homogenized inside a closed streamline has been recognized (Rhines & Young (1982)) and this homogenization has served as a basis for eddy parameterizations. In most such parameterizations, the divergent part of the eddy potential vorticity flux is modelled as a down-gradient transfer, resulting in a purely dissipative closure. In this article, we consider an alternate route to homogenization of potential vorticity, and present preliminary results from our investigation of a class of fluid descriptions to model geophysical fluid turbulence.

The evolution of the time mean flow in the simple case of the barotropic vorticity equation is given by

$$\frac{\partial \overline{q}}{\partial t} + \nabla.(\overline{\mathbf{u}}\,\overline{q}) + \nabla.\overline{\mathbf{u}'q'} = \overline{F} + \overline{D}, \tag{1}$$

where $q = \nabla^2\psi + \beta y$ is the potential vorticity, ψ is the velocity streamfunction, F is the forcing, D is the dissipation, an overbar denotes the time mean and a prime denotes the difference from the mean (see below for other notation). In (1) it is clearly only the divergent component of the eddy flux of potential voriticity $\overline{\mathbf{u}'q'}$ that can drive the mean flow and the above-mentioned down-gradient closure for (1) takes the form

$$(\overline{\mathbf{u}'q'})_{div} = -\kappa\nabla\overline{q}, \tag{2}$$

where $(\overline{\mathbf{u}'q'})_{div}$ denotes the divergent component of $\overline{\mathbf{u}'q'}$. This closure is purely dissipative.

Holm et al. (1998) proposed a model for the mean motion of ideal incompressible fluids wherein a momentum density $\mathbf{v}$, which is related to the usual velocity $\mathbf{u}$ by

$$\mathbf{v} = (1 - \alpha^2\nabla^2)\mathbf{u},$$

is transported by $\mathbf{u}$ in lieu of the usual self-advection of $\mathbf{u}$ resulting in

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{u}\cdot\nabla\mathbf{v} - \alpha^2\left(\nabla\mathbf{u}\right)^T\cdot\nabla^2\mathbf{u} = -\nabla p, \quad \nabla\cdot\mathbf{u} = 0. \tag{3}$$

(Note that when α is set to zero, the usual Euler equations result.) In a complementary fashion, the transport velocity $\mathbf{u}$ for such a flow may be viewed as Helmholtz-smoothed relative to the advected quantity $\mathbf{v}$. Such a model is obtained by introducing a length scale α—the smallest length-scale of interest—in the Euler-Poincare formalism and conserving an energy

$$\frac{1}{2}\int\left(|\mathbf{u}|^2+\alpha^2|\nabla\mathbf{u}|^2\right)d\mathbf{x}. \tag{4}$$

which also accounts for the spatial fluctuations of the velocity field in place of the usual kinetic energy

$$\frac{1}{2}\int|\mathbf{u}|^2 d\mathbf{x}. \tag{5}$$

Conservation of the latter kinetic energy results in the usual Euler equations. For details of the derivation of (3) and its other interesting properties, the reader is referred to Holm et al. (1998) and Shkoller (1998).

In the large eddy simulation of a turbulent flow, one attempts to model the dynamics of the smaller resolved scales in such a way as to (1) reproduce the same energy transfer characteristics (between the large interesting scales and the small scales) as would occur if the turbulent flow were fully resolved and to (2) dissipate the correct amount of energy. Such a modeling attempt can be viewed as specifying the stress-strain relationship at the numerically resolved scales in a manner that is specific to the flow under consideration. Thus, although the constitutive nature of the *fluid* is known at the molecular scale, the closure of the equations governing the large scales is equivalent to specifying a new *flow*-based constitutive law at the (larger than molecular) resolved scales. In this context, it is interesting to note that the α-model—a model for the mean motion of an ideal incompressible fluid—is isomorphic to the equations governing the behavior of second grade incompressible fluids (e.g., see Rivlin & Ericksen (1955)and Dunn & Fosdick (1974)).

In either scenario, the inviscid evolution of the (relative) vorticity in two dimensions can be written as

$$\frac{\partial\zeta}{\partial t}+\left(1-\alpha^2\nabla^2\right)^{-1}J\left[\psi,\left(1-\alpha^2\nabla^2\right)\zeta\right]=0, \tag{6}$$

where a vorticity-streamfunction formulation has been used and $\zeta=\nabla^2\psi$ is the (relative) vorticity. While in the mean-motion scenario, as mentioned before, α is a fluctuation length scale, in the second-grade fluid theory, α is a constitutive property of the fluid. We recall that the usual incompressible two dimensional Euler equation in this notation is

$$\frac{\partial \zeta}{\partial t} + J[\psi, \zeta] = 0. \tag{7}$$

By writing (6) in its Fourier representation,

$$\frac{d\hat{\zeta}_{\mathbf{k}}}{dt} = \frac{1}{1+\alpha^2|\mathbf{k}|^2} \sum_{\mathbf{p}+\mathbf{q}=\mathbf{k}} \frac{\mathbf{p}\times\mathbf{q}}{|\mathbf{p}|^2\,(1+\alpha^2|\mathbf{q}|^2)} \hat{\zeta}_{\mathbf{p}}\hat{\zeta}_{\mathbf{q}}, \tag{8}$$

phenomenologically it can be seen that the modification of the advective nonlinearity in (6) is such as to supress the nonlinear interactions between length scales smaller than α.

Until recently, most work on equations of the kind of (6) seems to have been of a mathematical nature. It is only recently with the rederivation by Holm et al. (1998), that their potential as models of coarse-grained hydrodynamics has been realized. Chen et al. (1998) used a steady and parallel flow solution of a viscous α-model to fit turbulent channel data and Chen et al. (1999) considered a viscous α-model to study homogeneous and isotropic turbulent scalings in three dimensions. Nadiga (2000) considered the statistical equilibria of (6) and showed that the scaling of the equilibrium states of the model for conserved energy and enstrophy reproduced the corresponding scaling for the Euler equations on the large scales, while at the same time greatly deemphasizing the importance of small scales.

In this article, we examine the utility of an extension of (6) to the β-plane barotropic vorticity equation in modelling a turbulent flow that can be directly realized by eddy-resolving simulations of the barotropic vorticity equation. The outline of the rest of the paper is as follows: In the next section, we present the α-model corresponding to the barotropic vorticity equation. In section 3, we briefly recapitulate the phenomenon of formation of a time-mean four-gyre circulation in a strongly turbulent β-plane barotropic vorticity equation flow driven by the classic double-gyre wind forcing. In section 4, we show that with a judicious choice of boundary conditions, the α-barotropic vorticity equation can model the important, eddy-driven, components of the time-mean four-gyre circulation, and we summarize our results in section 5.

II. THE BAROTROPIC VORTICITY EQUATION AND ITS α-MODEL

The unforced and inviscid barotropic vorticity equation may be written as

$$\frac{\partial q}{\partial t} + J[\psi, q] = 0 \tag{9}$$

where $q = Ro\zeta + y$ is the potential vorticity, $\zeta = \nabla^2\psi$ is the relative vorticity, ψ is the velocity streamfunction, $J[\ ,\ ]$ is the Jacobian operator given by $J[\psi, q] = -\frac{\partial\psi}{\partial y}\frac{\partial q}{\partial x} + \frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial x}$, and Ro is the Rossby number. We consider a rectangular, closed basin on a beta plane with the y-coordinate $(-L \leq y \leq L)$ increasing northwards and the x-coordinate $(0 \leq x \leq L)$ eastwards. For this case, we define the Rossby number by

$$Ro = \left(\frac{\delta_i}{L}\right)^2 = \frac{U}{\beta L^2},$$

where δ_i is the inertial boundary layer thickness, L, the width of the domain, is the reference length, U is a reference velocity, and β is the (North-South) gradient of rotation. For future reference, we mention here that in the forced-dissipative cases to be considered later, we choose the reference velocity to be the Sverdrup velocity corresponding to the applied wind-stress forcing (with no loss of generality). With such a choice, the magnitude of the wind stress curl forcing will be unity.

For simplicity, to obtain the α-model for the above barotropic vorticity equation, we exploit the relationship between the barotropic vorticity equation (9) and the two-dimensional Euler equation (7): In particular, the barotropic vorticity equation (9) can be obtained from the two-dimensional Euler equation (7) by replacing the relative vorticity ζ in the latter by potential vorticity q (given as above by $q = Ro\zeta + y$). So also, the α-model corresponding to the barotropic vorticity equation can be obtained from the Euler-α equation (6) by replacing the relative vorticity ζ with the potential vorticity $q = Ro\zeta + y$, leading to

$$\frac{\partial q}{\partial t} + (1 - \alpha^2\nabla^2)^{-1}J[\psi, (1 - \alpha^2\nabla^2)q] = 0. \tag{10}$$

We call this the α-barotropic vorticity equation. Alternately, one can start start from the full three-dimensional Euler-Boussinesq equations, construct the α-model for that set of equations and then go through the hydrostatic and quasi-geostrophic approximations to arrive at (10). For a derivation of the α-Euler-Boussinesq model, the reader is referred to Holm et al. (1998).

In (10), α is a nondimensional turbulence correlation length, which we consider to be the ratio of a length scale appropriate to the eddy activity to be modelled and the reference length L. We remark that (10) may also be written in the form

$$\frac{\partial q^\alpha}{\partial t} + J[\psi, q^\alpha] = 0, \tag{11}$$

where

$$q^\alpha = \left(1 - \nabla \cdot \alpha^2 \nabla\right) q, \tag{12}$$

This form makes transparent the conservative and advective nature of the model equation.

In (12), we have considered the turbulent correlation length scale α to be spatially variable. For example, this is the case when α is dependent on the local value of eddy kinetic energy. However, in this preliminary study, we consider α to be spatially uniform and defer considerations of variable α to later analysis. This simplifies (12) to

$$q^\alpha = \left(1 - \alpha^2 \nabla^2\right) q. \tag{13}$$

Even though the net number of derivatives in (10) is the same as in (9), in the sense that the Helmholtz operator acting on q is reversed by an inverse Helmholtz operation after the Jacobian is evaluated in (10), it remains that a new boundary condition has to be specified for q. This is again evident by considering the evolution of q^α as given, e.g., by (11) and (13).

Here, we rely on previous experience: In Greatbatch & Nadiga (2000) (henceforth referred to as GN), the qualitative aspects of the four-gyre circulation were shown to be reproducible in a parameterized model in which the divergent part of the eddy potential vorticity-flux was represented as a down-gradient transfer of potential vorticity. There also, a boundary condition had to be specified in conjunction with the divergence of the modelled dissipative flux ($\nabla \cdot \kappa \nabla \overline{q}$) and the appropriate boundary condition was that of no normal flux of potential vorticity along the model boundaries. Although the present parameterization has a very different physical basis to it, mathematically, the new singular term is again $\nabla^2 q$ and to that extent we choose the same boundary conditions

$$\frac{\partial q}{\partial n} = 0 \tag{14}$$

along the model boundaries to invert the Helmholtz operator in (13).

III. FOUR-GYRE CIRCULATION WITH DOUBLE-GYRE WIND FORCING

In this section, we briefly recapitulate the phenomenon of formation of a time-mean four-gyre circulation when the weakly dissipative barotropic vorticity equation is subjected to a symmetric double-gyre wind stress in a closed rectangular β-plane basin as described in GN. Consider the barotropic vorticity equation (9) now with forcing

$$F = \sin(\pi y) \tag{15}$$

and weak dissipation D. Figure 1 shows the time-mean streamfunction and potential vorticity contours for a case with $Ro = 0.01$. In this simulation, a Laplacian mixing of momentum with an associated Munk scale $(\frac{\delta_2}{L})$ of 0.04 has been used in conjunction with superslip boundary conditions:

$$D = \left(\frac{\delta_2}{L}\right)^3 \nabla^2 \zeta \tag{16}$$

with

$$\frac{\partial \zeta}{\partial n} = 0 \tag{17}$$

along boundaries. Here $(\frac{\delta_2}{L})^3 = \frac{A_2}{\beta L^3}$ and δ_2 is the scale for the width of the Munk boundary layer. The flow was resolved on a 51x101 grid giving a grid spacing $\Delta x/L$ of 0.02, enough to marginally resolve the Munk scale, and, therefore, no subgrid scale model was required.

Unless stated otherwise, all computations presented in this article were carried out using second-order finite difference spatial discretization with Arakawa Jacobian and a (nominally) fifth-order, adaptive time-step, embedded Runge-Kutta Cash-Karp algorithm time marching scheme. (For details, see GN.)

While the instantaneous flow in this simulation is highly turbulent, the time-mean circulation shown in the contour plots of Fig. 1 shows a four-gyre structure. The two inner gyres circulate in the same direction as conventional wind-driven gyres, while the two outer gyres at the northern and southern ends of the basin circulate in the opposite direction, and are driven by the eddy flux of potential vorticity. As noted in GN, the four-gyre structure is the result of weak dissipation.

That the outer gyres are driven by the eddy flux of potential vorticity and that dissipation in this flow is weak can be seen in the mean vorticity balance for the four gyres shown in Table 1: For the inner gyres (labelled sub-tropical wind-driven and sub-polar wind driven in the table), the forcing and dissipation terms are of opposite sign while for the outer gyres (labelled southern outer and northern outer in the table), the forcing and dissipation terms have the same sign. For the outer gyres, the sign of the dissipation term is the opposite of the sign of the (divergence of the) eddy flux term. Furthermore, from Table 1 it is clear that the dominant balance is between the wind stress forcing and the (divergence of) eddy flux of potential vorticity, while the explicit dissipation plays a weak role. Since this balance is true for each of the four gyres, it is also true for each half of the basin separated by the Zero Wind stress Curl Line (ZWCL), and shows that practically all the wind stress curl that is input in one half of the basin is transported across the ZWCL to the other half of the basin by transient eddies.

When the importance of the eddy flux of potential vorticity is reduced by increasing explicit dissipation, the outer counter-rotating gyres disappear and only a conventional double gyre circulation appears in the time mean (see Fig. 8 of GN). The formation of such a time-mean four-gyre circulation given a double-gyre wind forcing is actually quite robust and does not depend sensitively on either the Rossby number, the form of dissipation or the boundary conditions, as long as dissipation is weak. (For details, see GN.)

The tendency for potential vorticity to homogenize in the northern and southern parts of the domain due to eddy mixing is clear in the above example. In GN it was shown that qualitative aspects of the time-mean flow were obtainable (in a coarser resolution run) by a down-gradient parameterization of the eddy flux of potential vorticity as in (2). In such a case, the eddy-parameterized model equations are

$$\frac{\partial q}{\partial t} + J[\psi, q] = F + D \tag{18}$$

with

$$D = \nabla.(\kappa \nabla q). \tag{19}$$

Here, the model variables are to be interpreted as time averages, and, as mentioned earlier,

$$\frac{\partial q}{\partial n} = 0 \tag{20}$$

along the boundary. Equation (19) parameterizes the divergent part of the eddy flux of potential vorticity as a down-gradient transfer. For an approriate value of κ in (19), the steady state solution of (18) is then expected to represent the time-mean circulation of the unsteady eddy-permitting simulation previously described.

Figure 2 shows the same four-gyre structure in the parameterized model as in the eddy resolving case, and the association of the outer gyres with a tendency for potential vorticity to homogenize. In this parameterized case, a spatially uniform κ corresponding to a Munk scale of about $0.17L$ was used. The Rossby number was 0.01, same as before, but the resolution was cut in half in each direction—the grid-spacing was $0.04L$. In this parameterized case, the balance is by design between the forcing and dissipation terms and this is verified in Table 2. We remark that the correspondance between the time-mean of the unsteady simulation (Fig 1) and the steady state of the down-gradient parameterized model (Fig 2) is only qualitative and we speculate that a better quantitative agreement would require a spatially variable κ.

IV. AN ALTERNATIVE PARAMETERIZATION

In this section, we consider steady states of the forced-dissipative version of the α-barotropic vorticity equation (10),

$$\frac{\partial q}{\partial t} + (1 - \alpha^2 \nabla^2)^{-1} J[\psi, (1 - \alpha^2 \nabla^2) q] = F + D, \tag{21}$$

as a model for the eddy-driven mean flow previously discussed. Recognizing that the problem is forced (with a double gyre wind stress curl (15)), and the left hand side of (21) is conservative, one is forced to introduce dissipation D in order to ensure that kinetic energy and potential enstrophy remain bounded. In this context, we briefly discussed in the previous section (1) how eddies in the simulation that permitted eddies tended to homogenize potential vorticity and (2) how such a homogenization was achievable in a parameterized run by using a down-gradient approximation for the eddy potential vorticity flux, resulting in a dissipation of the form indicated in (19). Now, in the context of the α-parameterization contained in the left hand side of (21), if we were to choose the dissipation D to be of the same form as in (19), our conclusion would not be compelling. That is to say, it would be difficult to separate out the effects of the α-parameterization and the dissipation term (related to the down-gradient approximation). To this end, we choose a form of dissipation that is *least effective* in modelling the down-gradient nature of the eddy flux of potential vorticity. We choose Rayleigh damping of relative vorticity ζ:

$$D = -\frac{\delta_s}{L}\zeta.$$

In Fig. 1, the return western boundary flow in the conventional gyres extend to about $0.45\,L$ in the east-west direction and the outer gyres extend to about $0.40\,L$ latitudinally. Thus the scale over which eddy activity affects the mean flow is about $0.45\,L$ and we choose this as the length scale α in (21). Fig. 3 shows the steady state circulation resulting from the parameterized model. In this parameterized setup, while the domain size is the same as before, the resolution has been cut in half in each direction compared to the eddying run—$\Delta x/L = .04$. The averaging length scale α is $0.45\,L$ and a Rayleigh friction corresponding to a Stommel boundary layer scale of $0.07L$ was used at the same value of Rossby number of 0.01. While we chose α as roughly the average of the lengths of the two nonlinear features as previously discussed, qualitatively similar results are obtained for quite a wide range of α ($0.3\,L \leq \alpha \leq 0.7\,L$). Again, with this alternative parameterization, the four-gyre structure and the homogenization of potential vorticity in the region of the outer gyres is evident.

As further proof that it is indeed the α-parameterization which is modeling the effects of eddies and thereby homogenizing potential vorticity, we repeat the previous run but with α set to zero. The steady state solution for this case is shown in Fig. 4, and corresponds to the classic double gyre picture of the Stommel problem. For this case, the Arakawa form of the Jacobian, which respects both energy and enstrophy constraints, was not used since it leads to grid scale oscillations in the potential vorticity field near the western boundary. Instead, following Veronis (1966), we chose a form of the Jacobian which respects only the energy constraint, but has the useful property that the boundary values of relative vorticity are never needed. It is for this reason, that potential vorticity is not defined on the boundary points in Fig. 4.

Indeed, what is interesting with the α-parameterized run is that there is no explicit down-gradient transfer term and the modelling of eddies results from a modification to the advective term. To the extent that the modelling of eddy effects in the α-model is adiabatic, the α-model is similar to the Gent-McWilliams parameterization of the effects of eddies in non-eddy resolving models (see, e.g., Gent et al. (1995)). In the latter parameterization, a bolus or eddy-transport velocity is added to the velocity which advects scalars.

The vorticity balance for the case with dispersive parameterization is shown in Table 3. If each of the gyres is considered individually, it is clear that equilibrium is achieved by a three term balance with the inviscid nonlinear dispersive term playing a significant role. This is unlike the forcing-dissipation balance which is achieved when the down-gradient parameterization is used. Further, if one considers the two halves of the basin separated by the ZWCL, the dominant balance in the case of α-parameterization turns out to be between the wind stress curl forcing and the inviscid model term, with the dissipation playing only a minor role in close analogy with the mean vorticity balance of the eddy-resolving case.

Finally, we note that it is only the qualitative aspects of the four-gyre structure that are reproduced in the new parameterization considered, in that neither the magnitude of the circulation nor the structure of the potential vorticity field is accurately reproduced. This is clearly indicative of the need for a spatially variable α in the dispersive parameterization. This conclusion is supported by noting that the down-gradient parameterization with spatially constant diffusivity also suffers from the same lack of detailed correspondance with the time-mean circulation of the eddy-resolving simulation, and that the two parameterized runs produce rather similar results.

V. SUMMARY AND DISCUSSION

These are preliminary results from a study of the applicability of the α-models to modeling geophysical fluid turbulence, and more work is necessary to understand the relevance of these models to describing the oceans and the atmosphere. That apart, we have shown that a time mean circulation of the barotropic vorticity equation with an important eddy-driven component can be reproduced using one such eddy parameterization. This parameterization arises from a specialization to the barotropic vorticity equation of a class of ideal incompressible fluid models proposed to model mean flow. These models are derived by considering the mean dispersive effects of rapid fluctuations over a length scale α.

From the point of view of geophysical fluid dynamics, this new parameterized model shows that the tendency of eddies to mix potential vorticity in certain situations can be successfully parameterized in a manner very different from the usual dissipative mixing term which arises from assuming a down-gradient form for the eddy flux of potential vorticity. As to whether the new model is indeed better than the usual down-gradient mixing parameterization is a question that requires further study.

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Gyre	$-\overline{J(\psi', q')}$	$\overline{F}$	$\overline{D}$
Southern Outer	0.148	−0.141	−0.008
Sub-Tropical Wind-Driven	0.476	−0.492	0.017
Sub-Polar Wind-Driven	−0.421	0.438	−0.016
Northern Outer	−0.203	0.195	0.007

TABLE I The mean vorticity balance for the eddy resolving run. The dominant balance is between forcing and the divergence of eddy flux of potential vorticity.

Gyre	F	D
Southern Outer	−0.267	0.268
Sub-Tropical Wind-Driven	−0.369	0.367
Sub-Polar Wind-Driven	0.369	−0.367
Northern Outer	0.266	−0.268

TABLE II The mean vorticity balance for the down-gradient-parameterization case. The balance is by design between the forcing and dissipation.

Gyre	$H^{-1}J\left[\psi, Hq\right]$	F	D
Southern Outer	0.388	−0.136	−0.252
Sub-Tropical Wind-Driven	0.264	−0.500	0.235
Sub-Polar Wind-Driven	−0.265	0.500	−0.235
Northern Outer	−0.387	0.135	0.252

TABLE III The mean vorticity balance for the dispersive-parameterization case. Note the importance of the inviscid model term in this three term balance as compared to the forcing-dissipation balance in the down-gradient parameterization case. H here stands for the Helmholtz operator $(1 - \alpha^2\nabla^2)$.

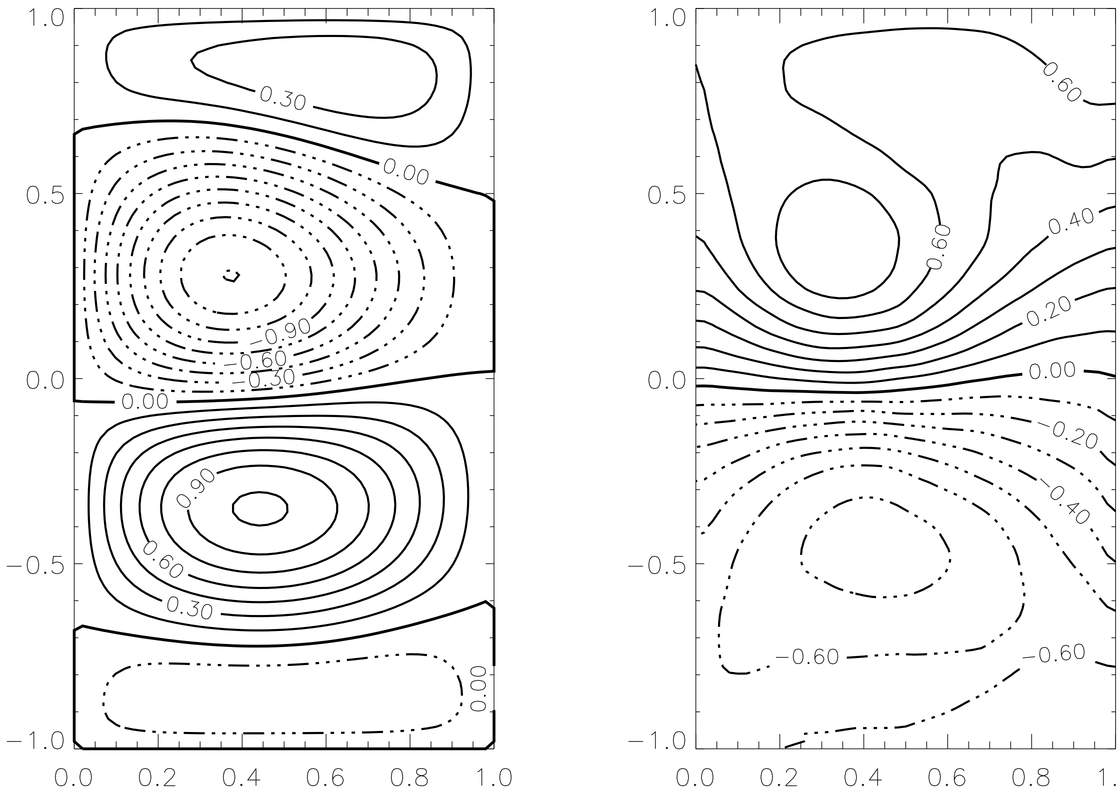

FIG. 1 Contour plots of the time-averaged streamfunction (left panel) and potential vorticity (right panel). The Rossby number for this double gyre wind-forced beta-plane circulation is 0.01, the Munk scale is 0.04 with superslip sidewall boundary conditions, and the grid-spacing is 0.02. The contour intervals are 0.15 for streamfunction and 0.10 for potential vorticity. The instantaneous flow is highly turbulent.

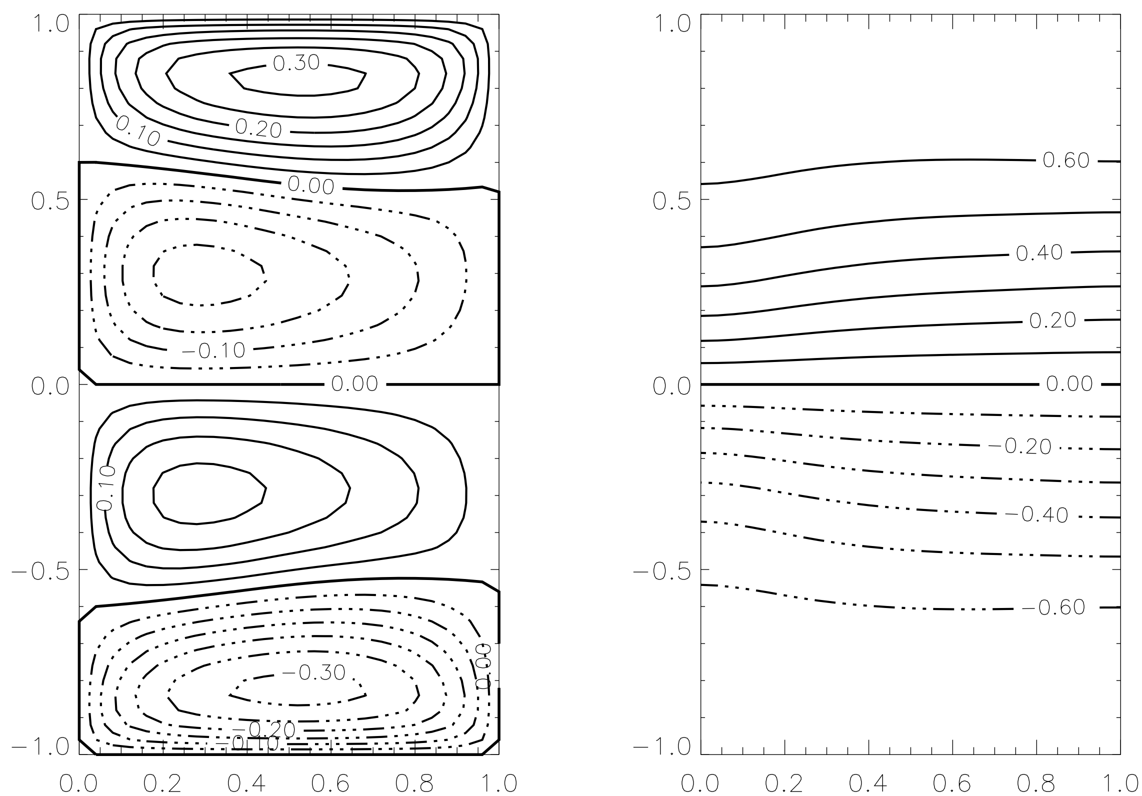

FIG. 2 The steady state fields using a down-gradient parameterization. Rossby number is 0.01 and the dissipative term, a Laplacian mixing of potential vorticity, arises from a parameterization of the divergent component of the eddy pv-flux as a down the gradient of mean pv. The associated Munk scale is 0.17 and a no-normal-flux-of-pv boundary condition is used. The grid spacing is 0.04. The contour intervals are 0.05 for streamfunction and 0.10 for potential vorticity.

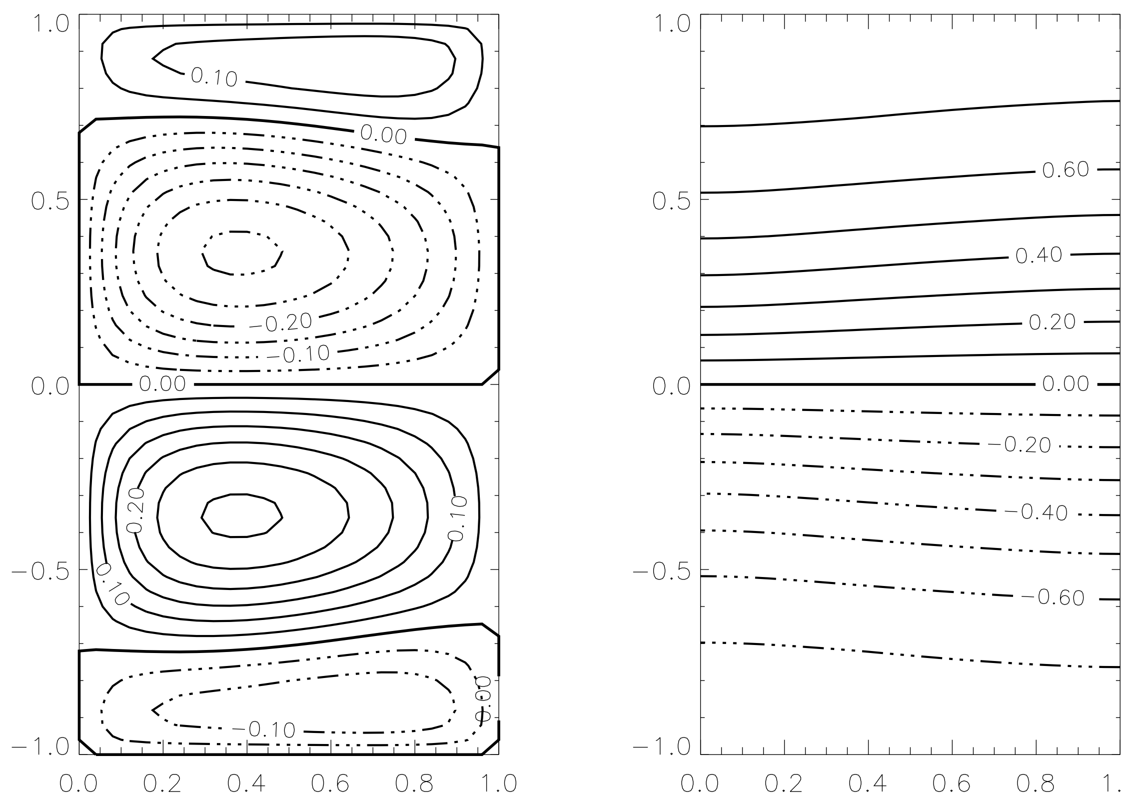

FIG. 3 The steady state fields from the α-parameterized model. Rossby number is 0.01 and the dissipative term is chosen to be Rayleigh damping of relative vorticity to ensure that this term does not represent down-gradient transfer. The Stommel scale associated with this dissipation operator is $0.07L$. The grid spacing is 0.04. The contour intervals are 0.05 for streamfunction and 0.10 for potential vorticity.

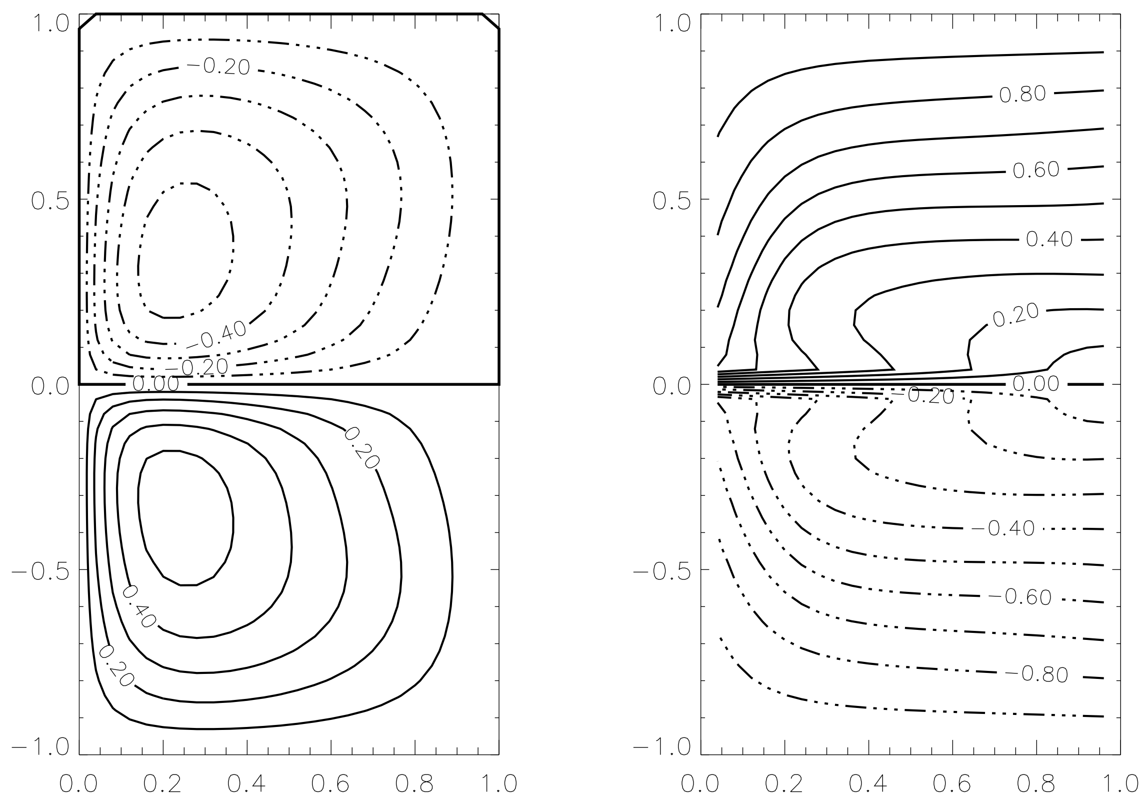

FIG. 4 The steady state fields from the previous run with α-parameterization turned off. The contour intervals are 0.10 for streamfunction and 0.10 for potential vorticity.