Fully-Implicit Techniques for Ocean Circulation Problems

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(LA-UR: 07-2056)

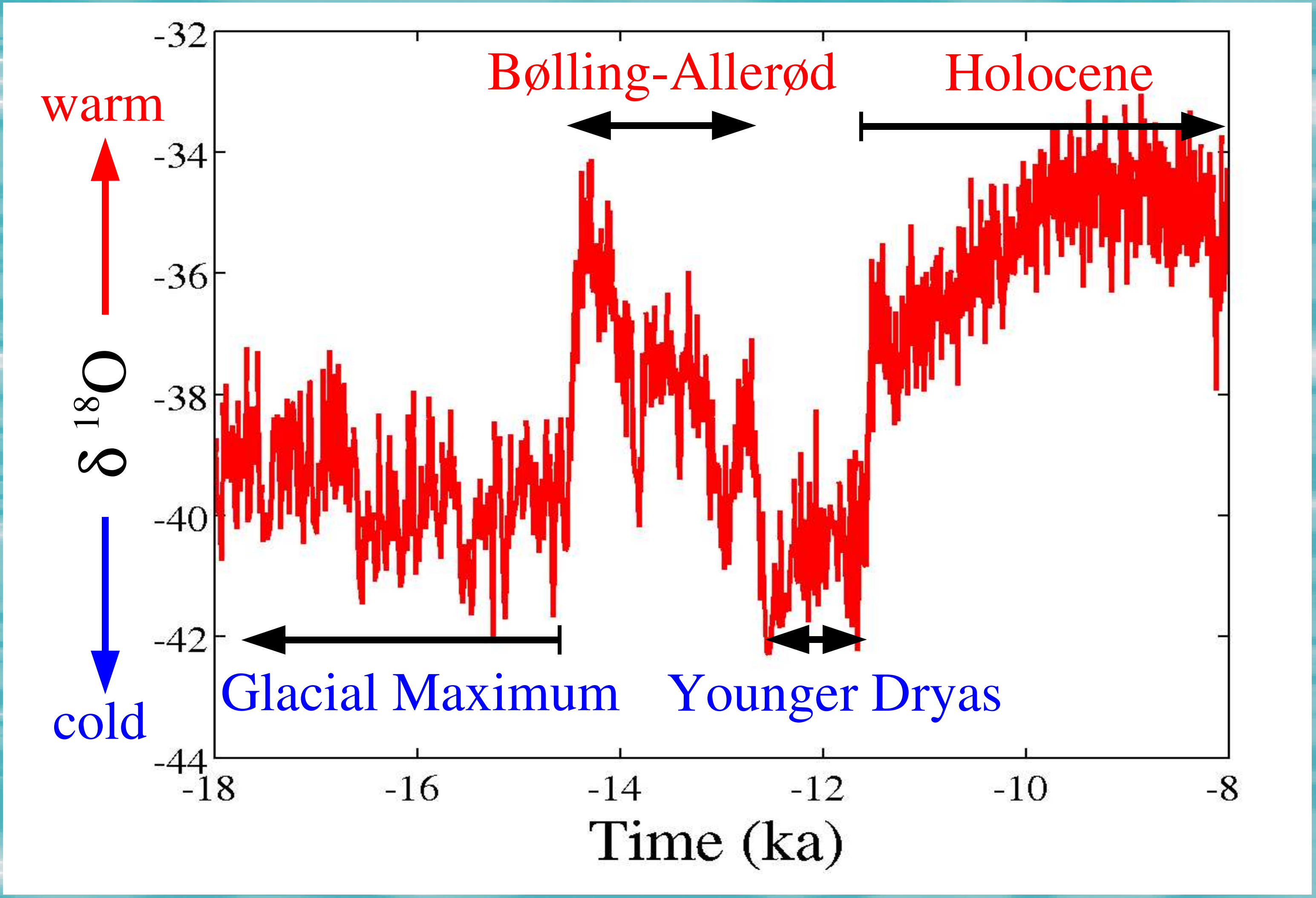
Overview

Studies of the Meridional Overturning Circulation

- Multiple Equilibria
  - Parameter continuation
- Oscillatory Modes
  - Linear stability analysis
- Optimal Perturbations
  - Generalized Stability Analysis

Abrupt Climate Change

Younger Dryas in Geenland ice core



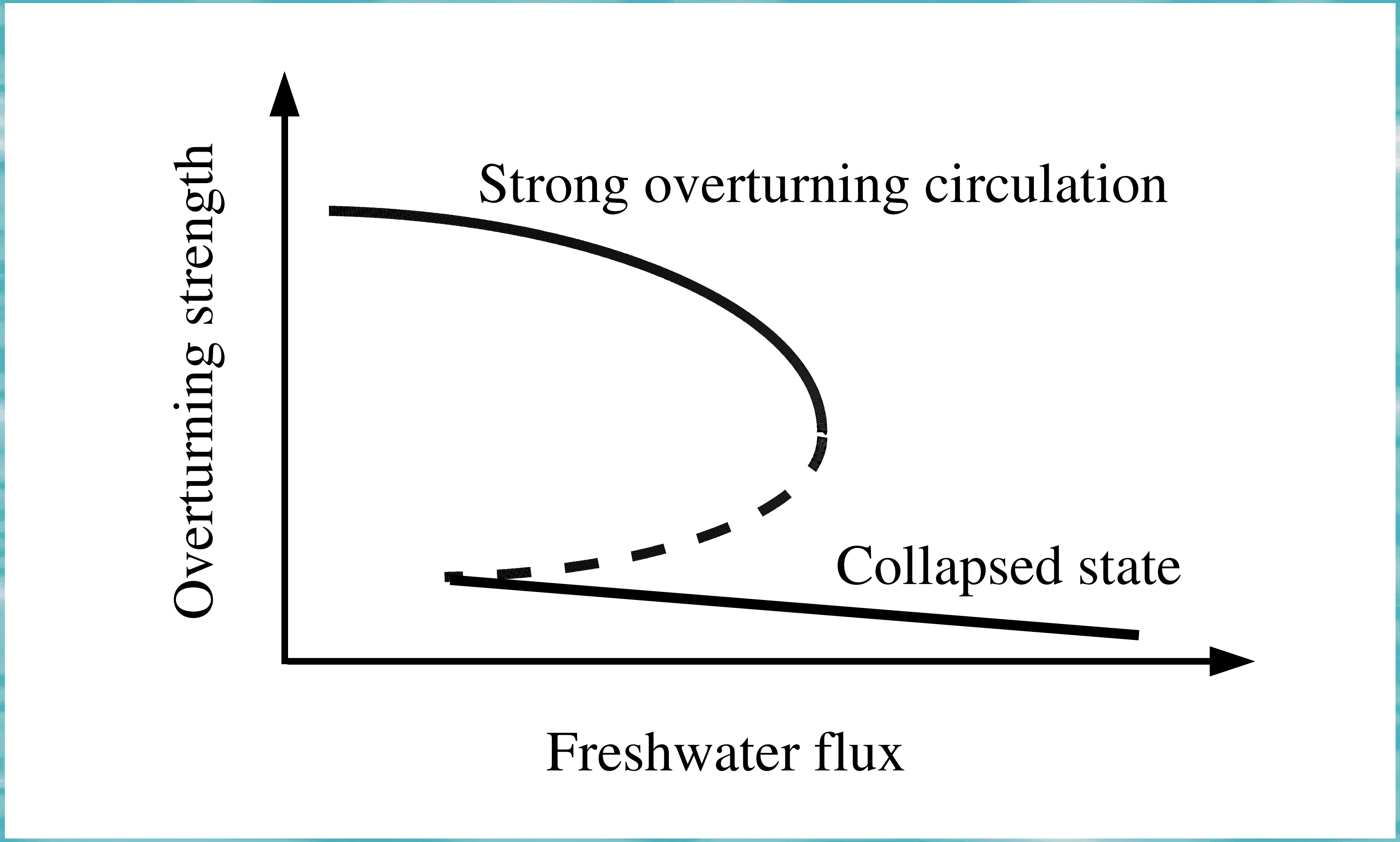
Abrupt Climate Change

Regime switches of the overturning circulation

- Leading hypothesis:
  - Transition between *multiple equilibria* of the overturning circulation

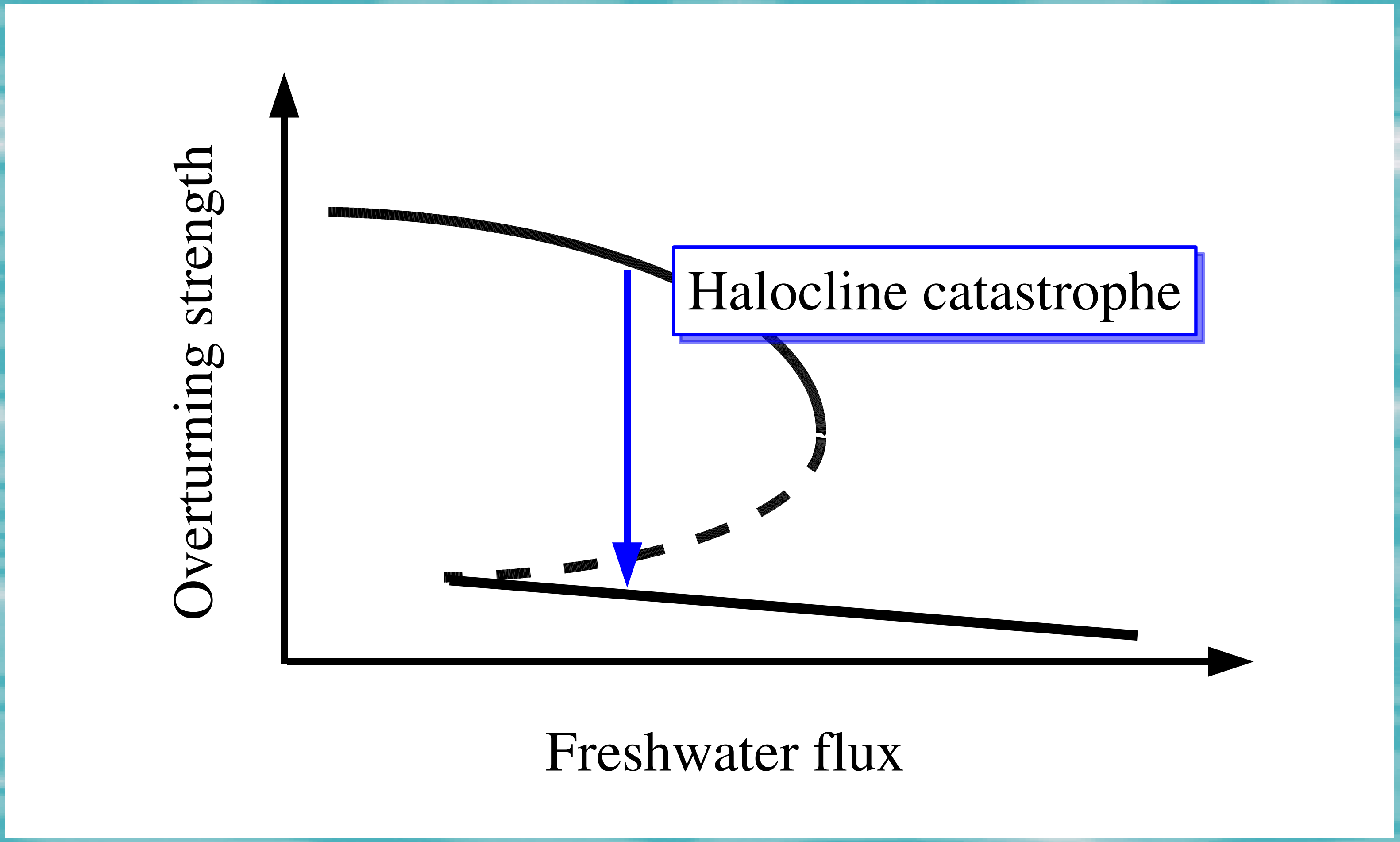
Abrupt Climate Change

Regime switches of the overturning circulation



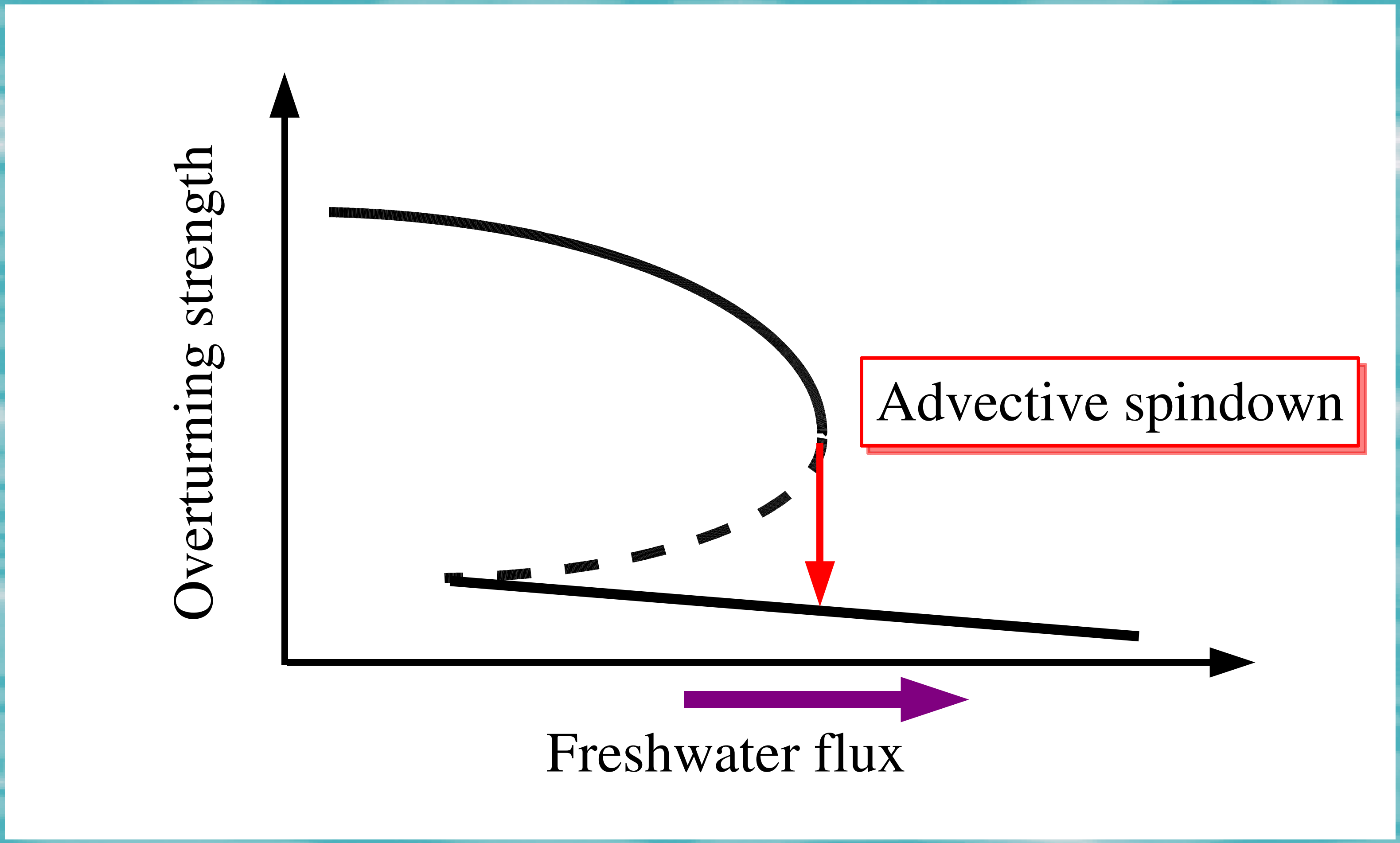
Abrupt Climate Change

Regime switches of the overturning circulation



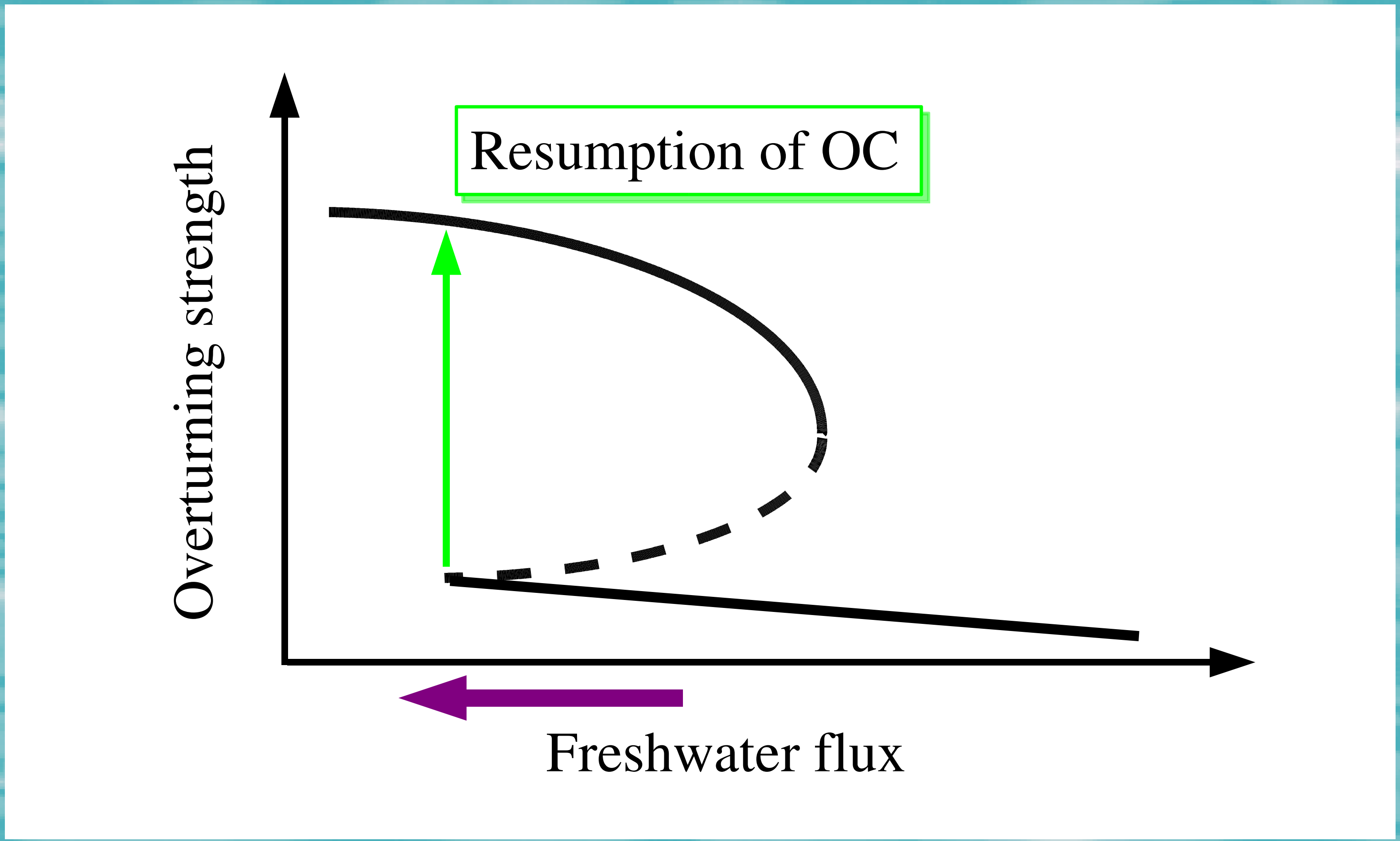
Abrupt Climate Change

Regime switches of the overturning circulation



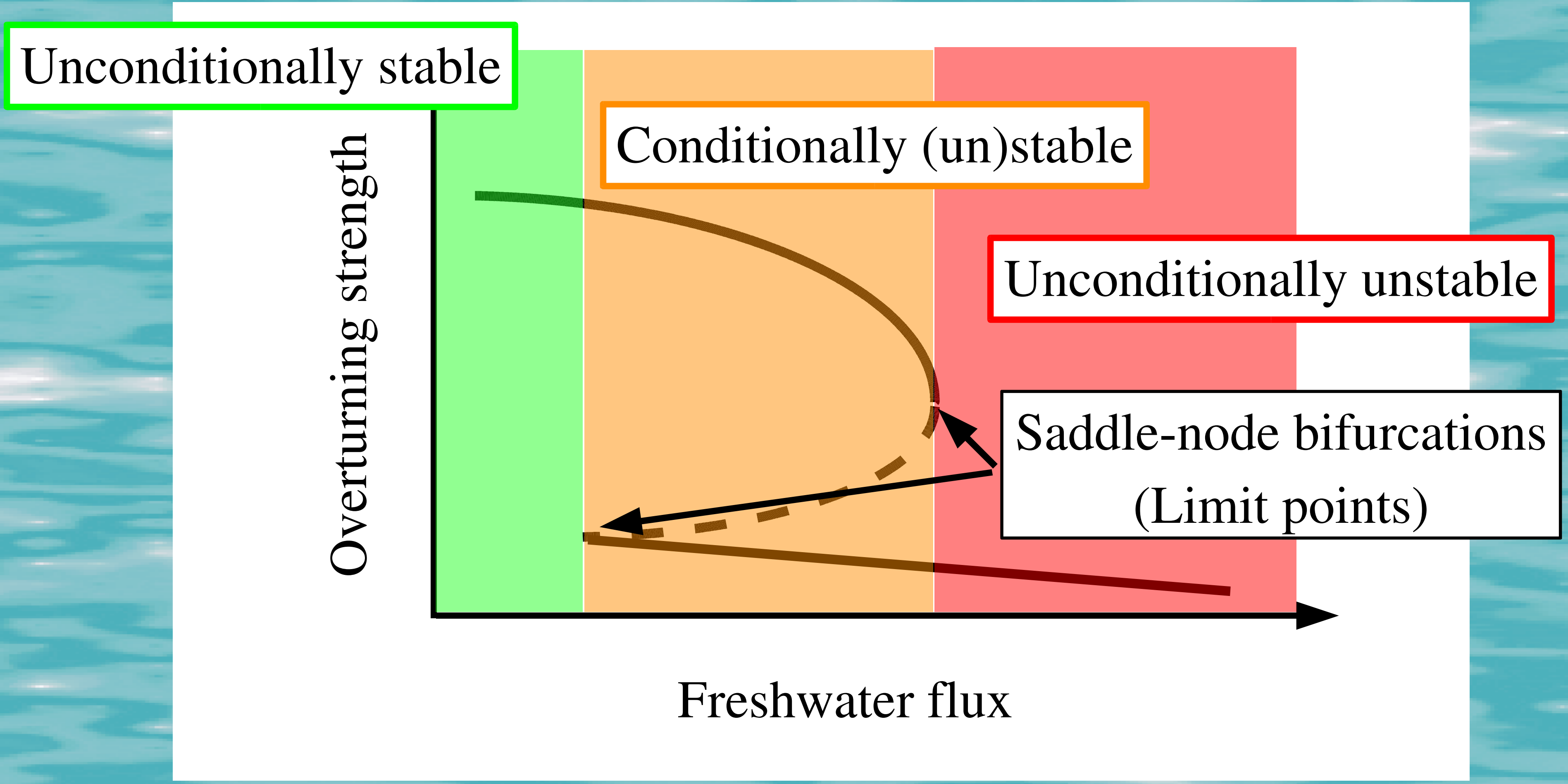
Abrupt Climate Change

Regime switches of the overturning circulation



Abrupt Climate Change

Regime switches of the overturning circulation



Abrupt Climate Change

Hosing Experiments

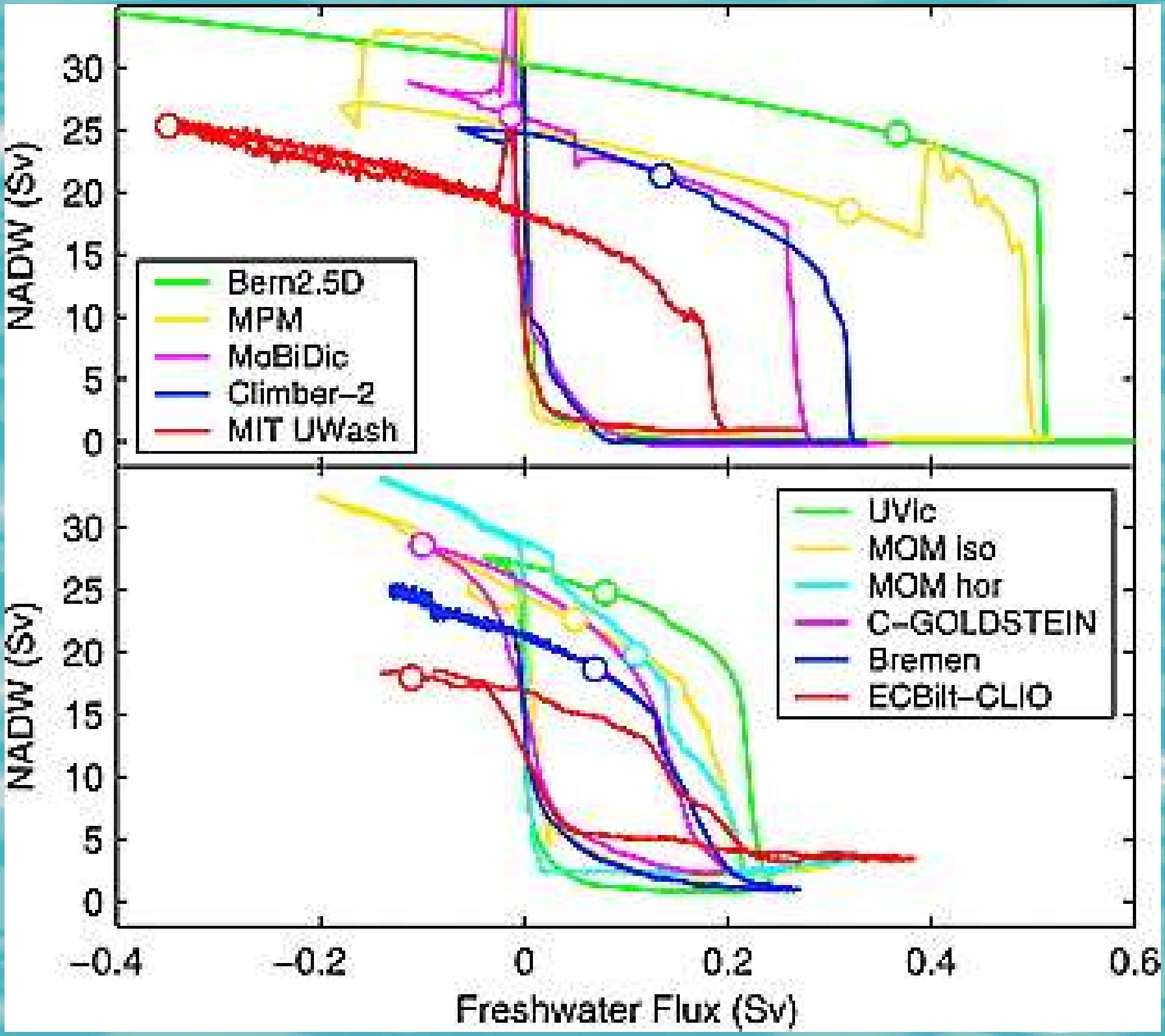
- Traditionally studied with *hosing experiments*:
  - Slowly increase freshwater input in North Atlantic
  - Follow quasi-steady ocean circulation

Abrupt Climate Change

Hosing Experiments

2D models

3D models



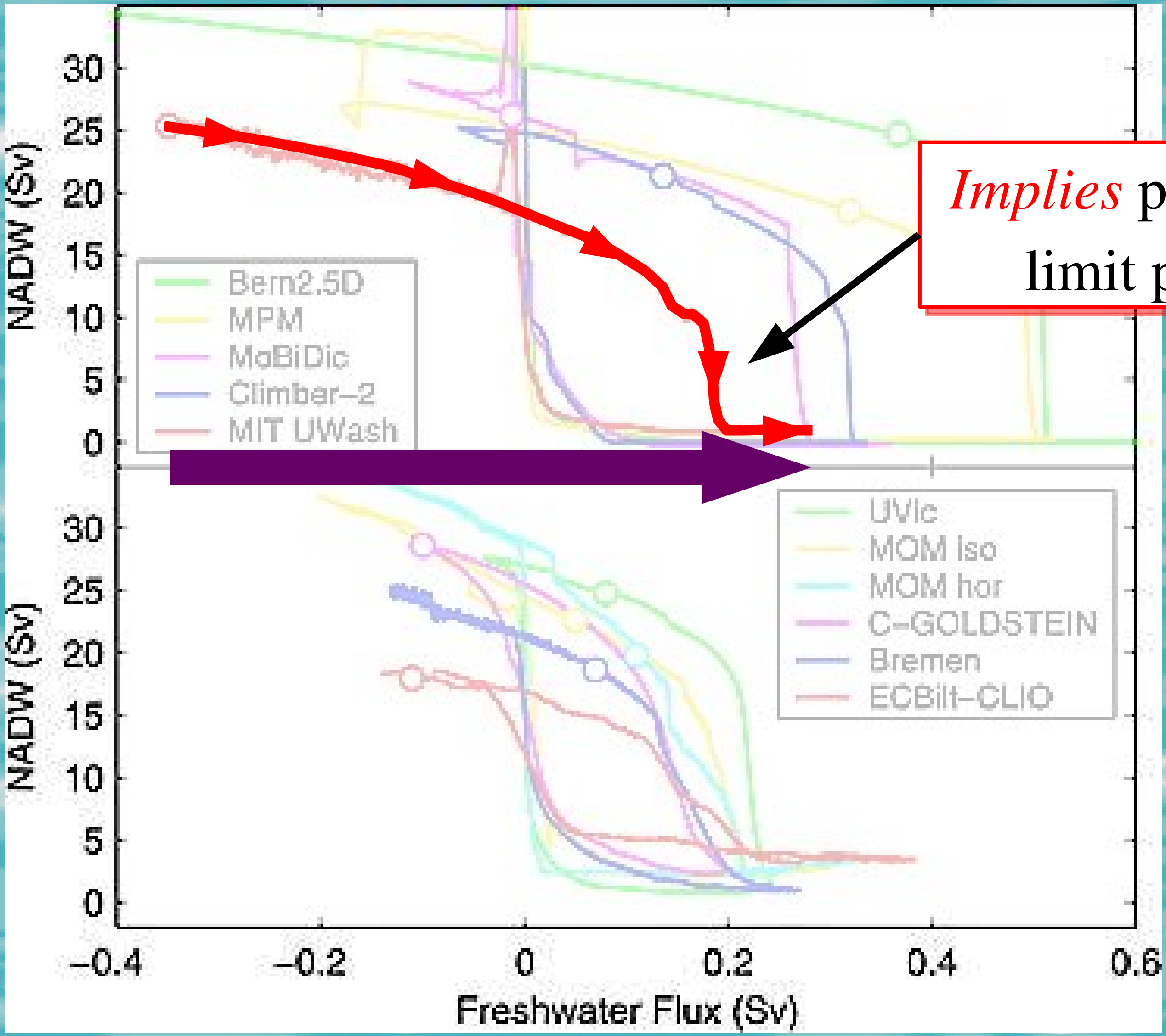
Abrupt Climate Change

Hosing Experiments

2D models

3D models

Implies position of limit point

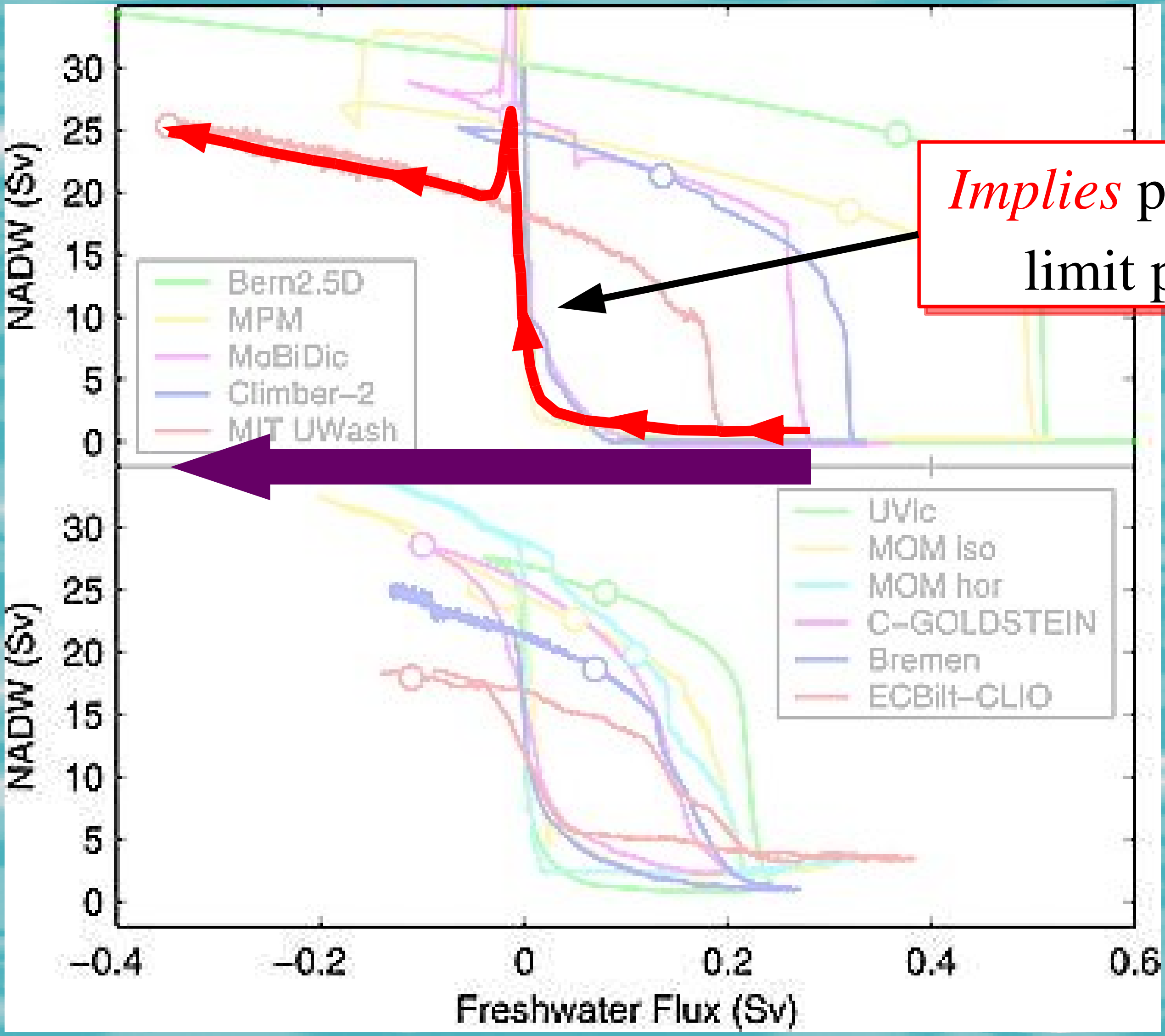


Abrupt Climate Change

Hosing Experiments

2D models

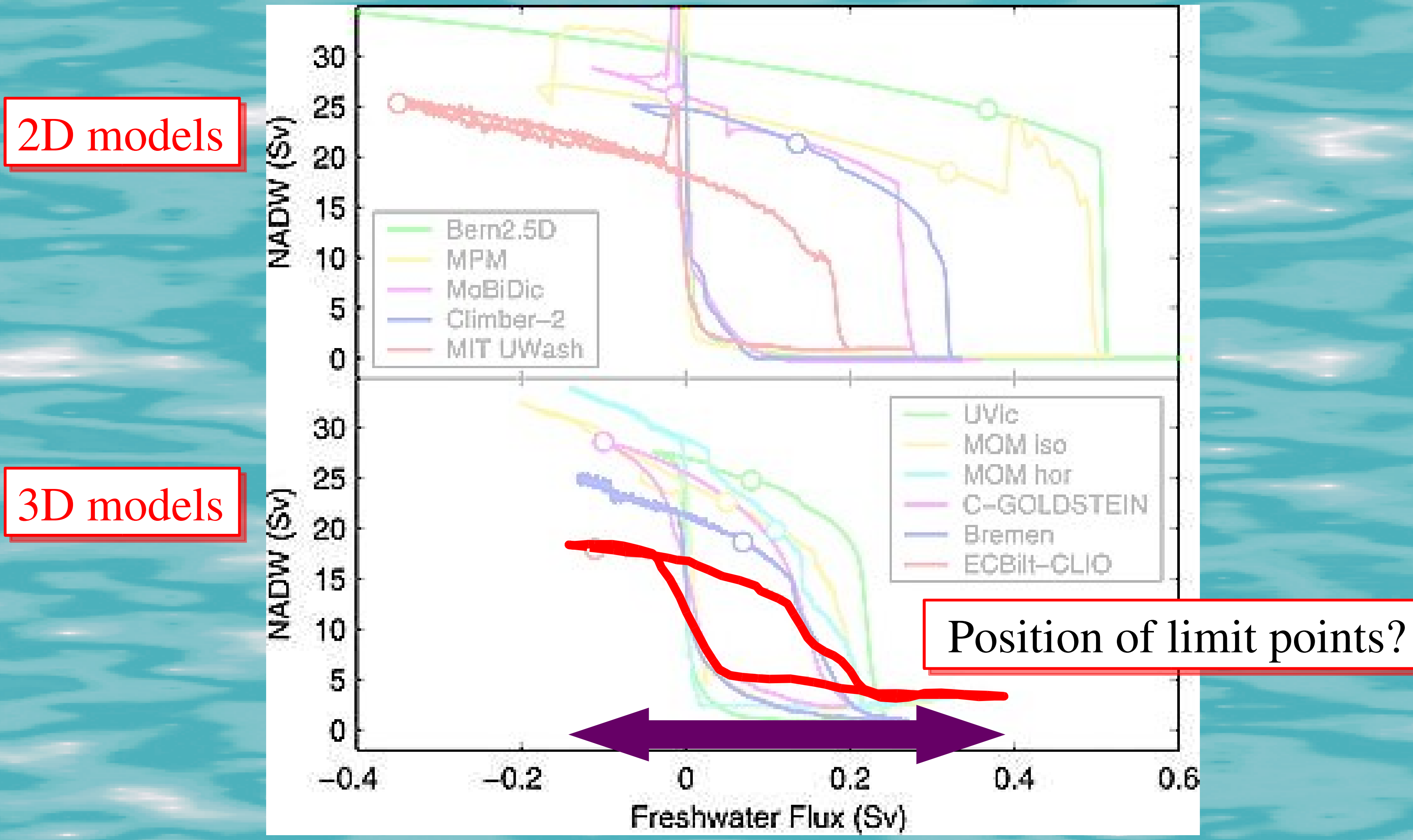
3D models



Implies position of limit point

Abrupt Climate Change

Hosing Experiments



Abrupt Climate Change

Hosing Experiments

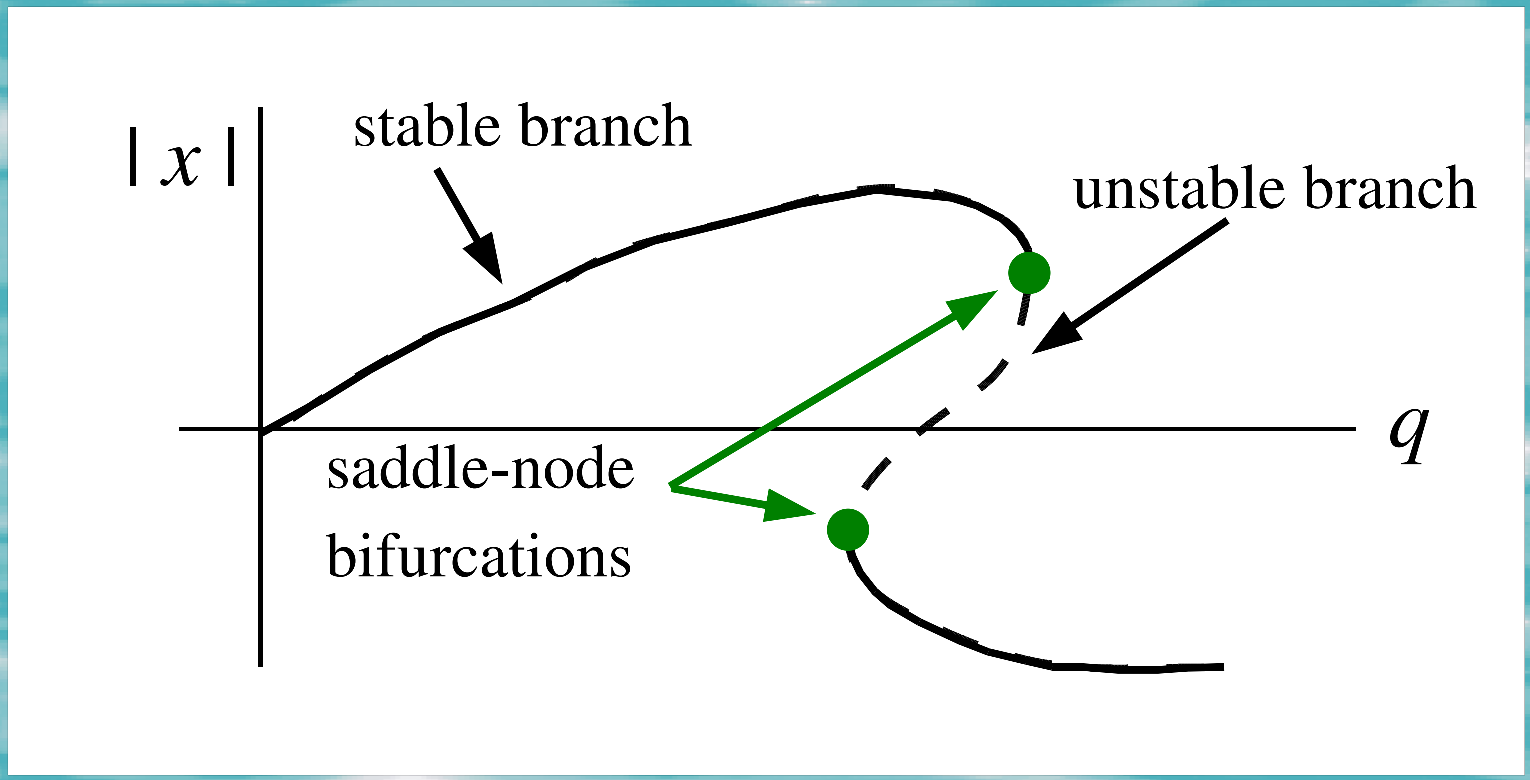
- Drawbacks:
  - Inaccurate due to transient response of MOC
  - Inefficient due to long time integrations
  - No info about unstable steady state connecting stable branches

Abrupt Climate Change

Parameter Continuation

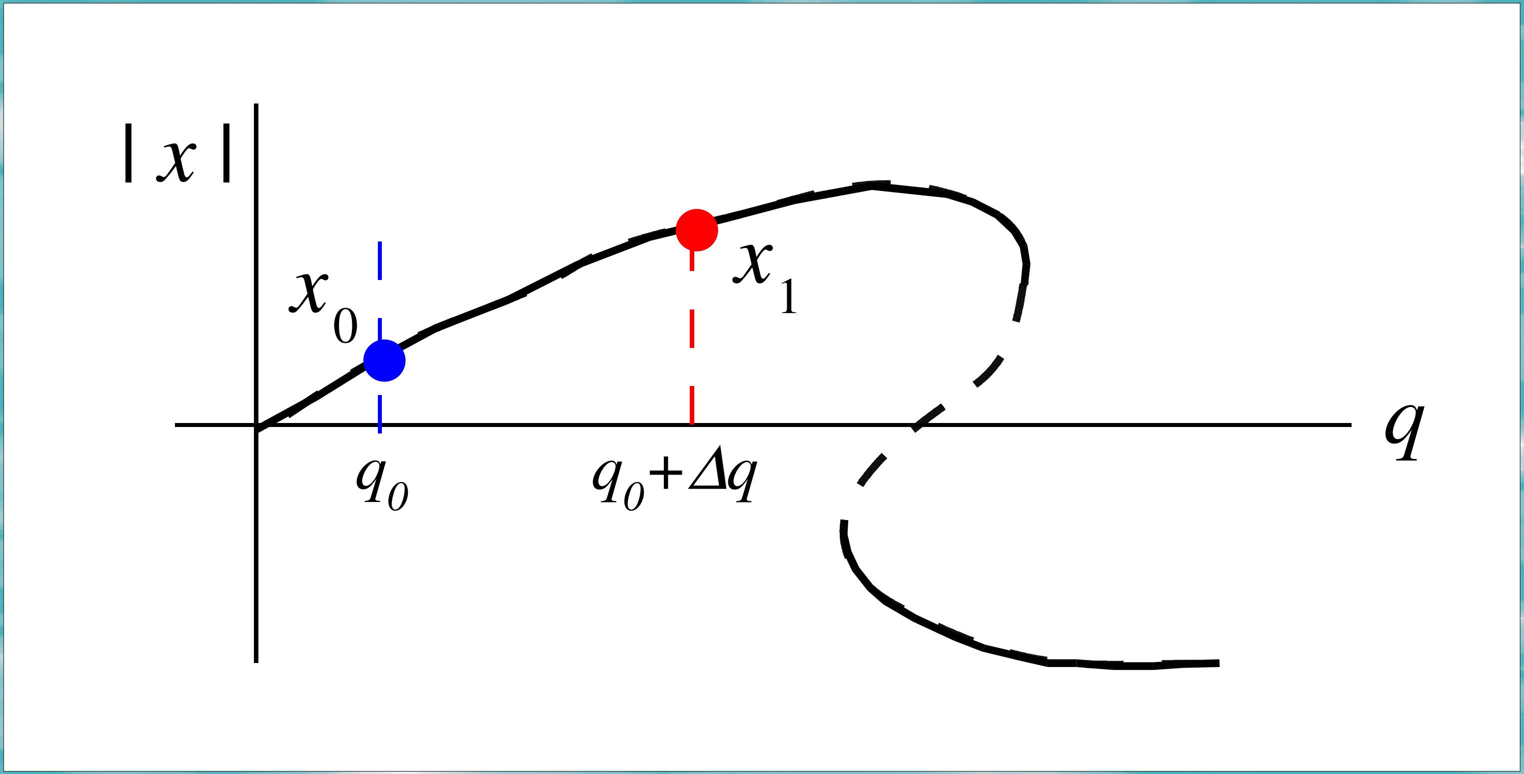
- Drawbacks:
  - Inaccurate due to transient response of MOC
  - Inefficient due to long time integrations
  - No info about unstable steady state connecting stable branches
- Alternative: *parameter continuation*
  - Calculate branches of steady states as function of parameter

Parameter Continuation



$$\frac{dx}{dt} = \Phi(x, q) = 0$$

Parameter Continuation



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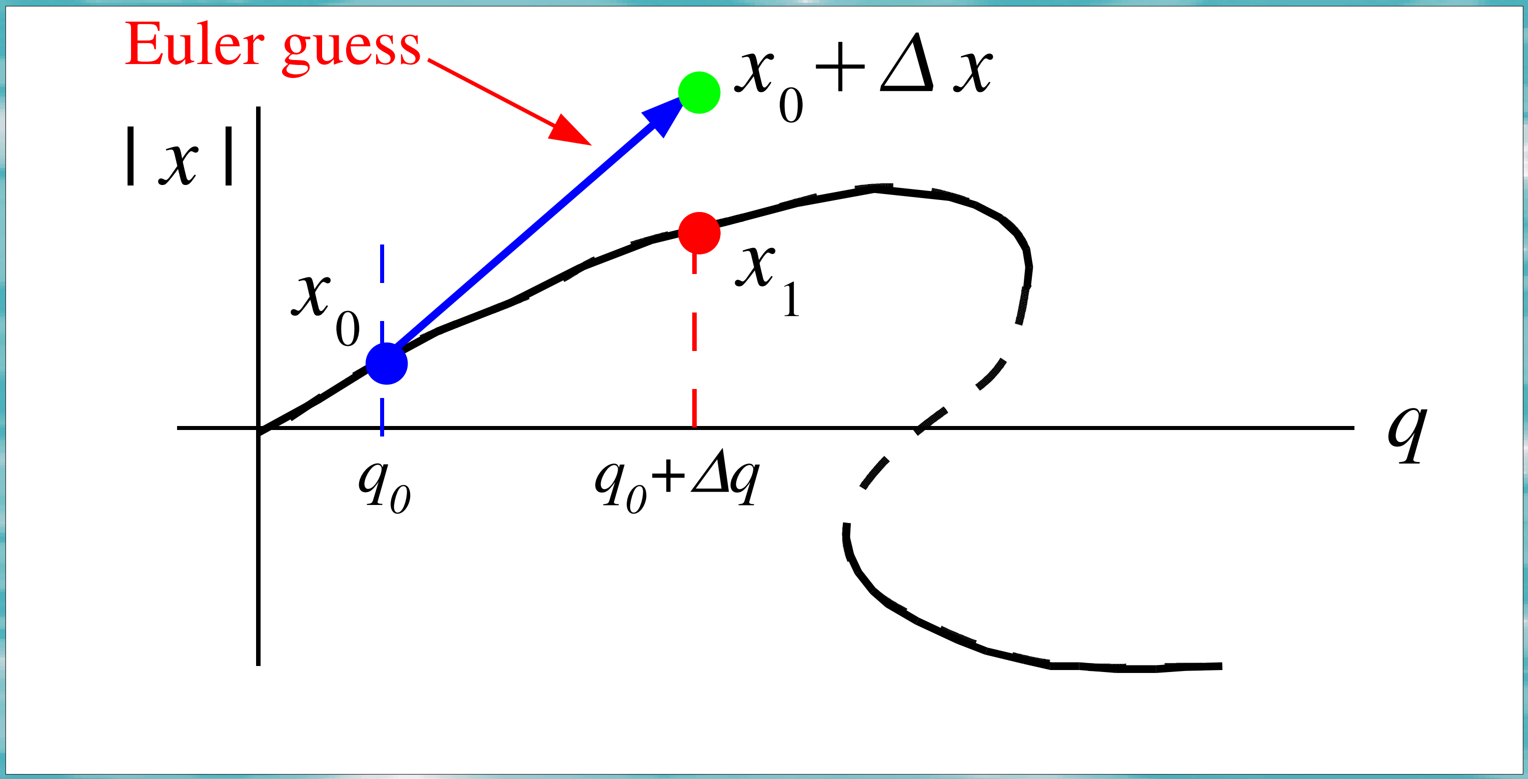
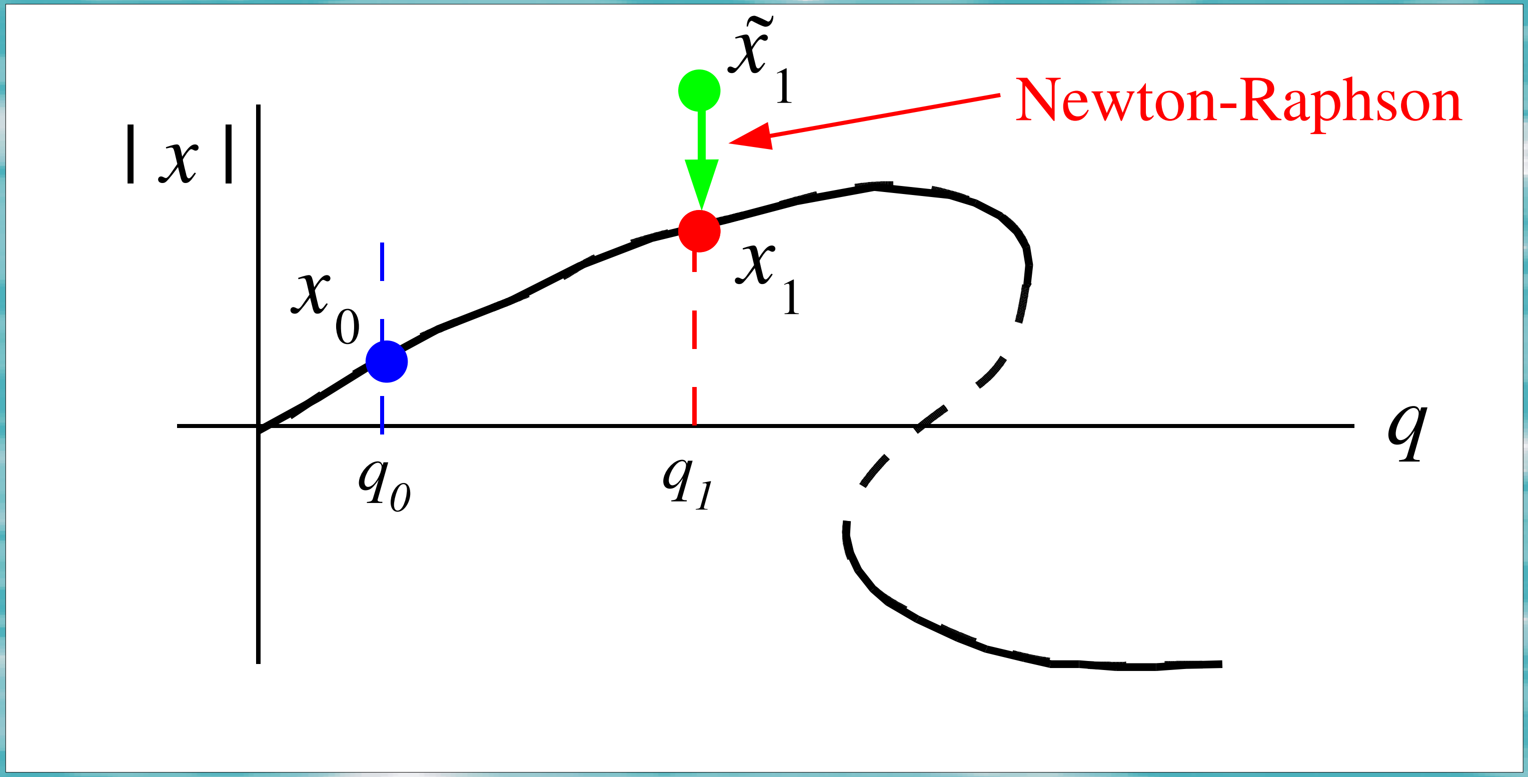
Parameter Continuation



$$\Delta x = -(\Phi_x)^{-1} \Phi_q \Delta q$$

Parameter Continuation



$$\Delta x = -(\Phi_x)^{-1}\Phi(\tilde{x}_1, q_1)$$

Parameter Continuation

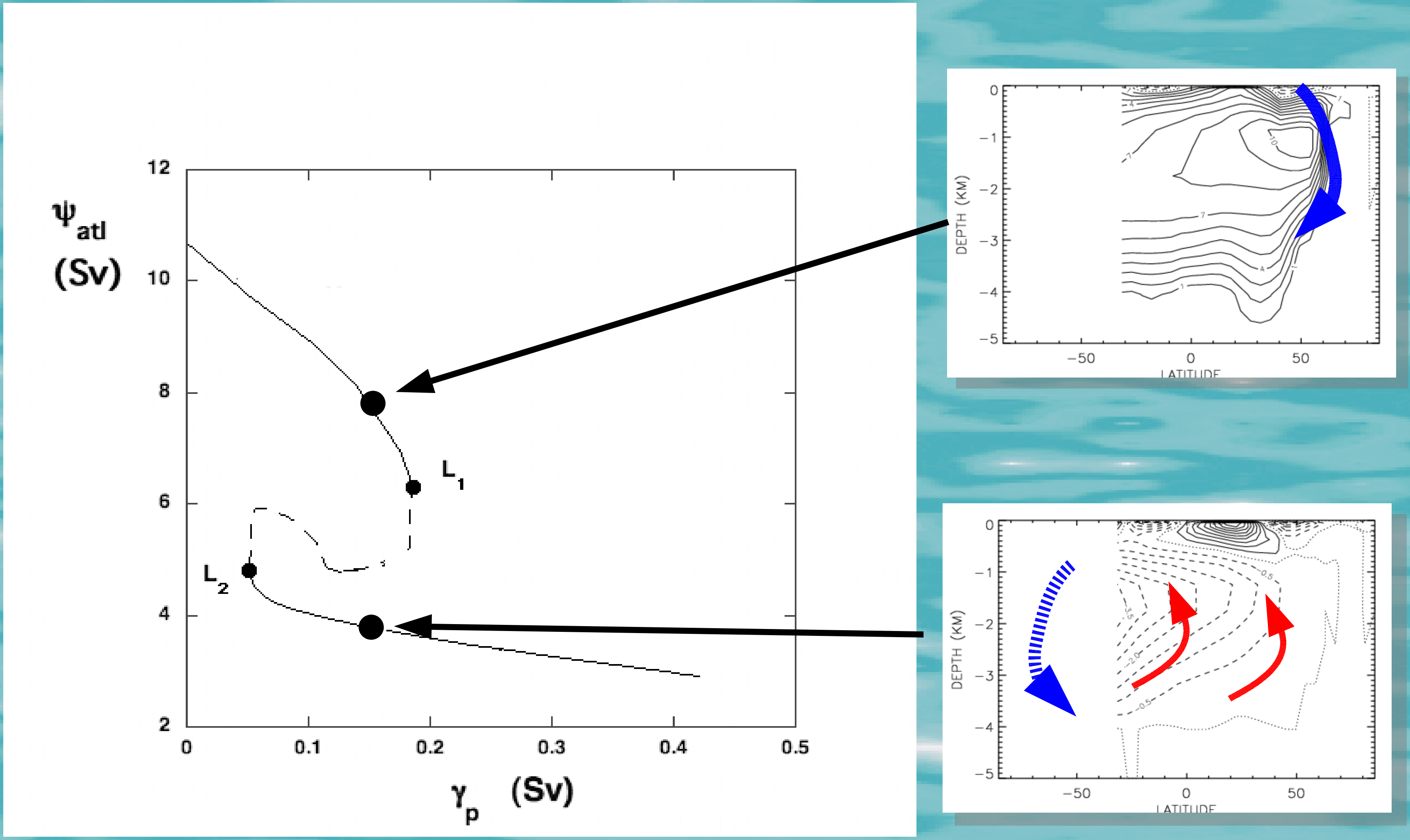
- Fully implicit ocean model: THCM *(Weijer et al., 2003)*
  - Jacobian explicitly constructed

- Algebraic system solvers:
  - GMRES: iterative linear systems solver *(Saad & Schultz, 1986)*
  - MRILU: preconditioner *(Botta & Wubs, 1999)*

Parameter Continuation

- Global configuration
  - Realistic bathymetry
  - 4^{o} resolution, 12 levels
- Variational Parameter
  - γ_p: North Atlantic freshwater flux perturbation

Parameter Continuation



Climate Variability

Linear Stability Analysis

- What are the “most dangerous” perturbations?

- Traditional approach: Linear Stability Analysis
  - Determine least stable normal modes
  - Stationary/Oscillatory
  - Spatial patterns, growth/decay rate, frequency

Eigenvalue Analysis

Steady state:

$$\boxed{M\frac{d\,\bar{x}}{dt}=\Phi(\bar{x})=0}$$

Eigenvalue Analysis

Steady state:

$$M\frac{d\bar{x}}{dt}=\Phi(\bar{x})=0$$

$$M\frac{dx'}{dt}=J(\bar{x})\,x'$$

$$x=\bar{x}+x'$$

Eigenvalue Analysis

Steady state:

$$M\frac{d\bar{x}}{dt}=\Phi(\bar{x})=0$$

$$x=\bar{x}+x'$$

$$M\frac{dx'}{dt}=J(\bar{x})\,x'$$

$$x'=x'_0\exp(\sigma t)$$

$$\sigma M x'_0=J(\bar{x})\,x'_0$$

Eigenvalue Analysis

Steady state:

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$$M\frac{dx'}{dt}=J(\bar{x})\,x'$$

$$x'=x'_0\exp(\sigma t)$$

$$\sigma M x'_0=J(\bar{x})\,x'_0$$

$$\sigma A x=B x$$

Generalized eigenproblem

Eigenvalue Analysis

- Eigenvalue solvers:
  - Jacobi-Davidson QZ-method: modes closest to “target”

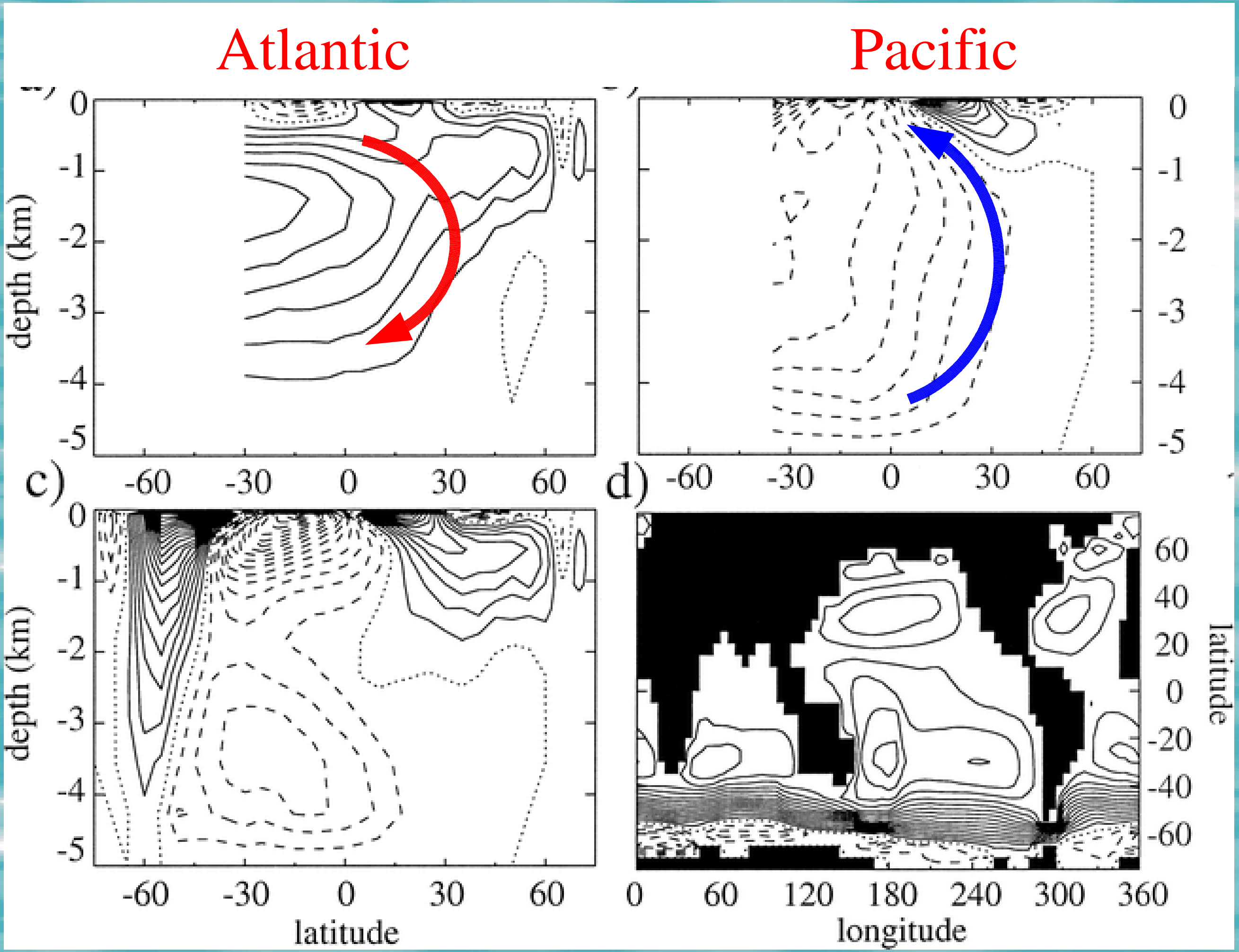
    (Sleijpen & Van der Vorst, 1996)
  - Simultaneous Iteration Technique: most “dangerous” modes

    (Steward & Jennings, 1981)

Climate Variability

Linear Stability Analysis

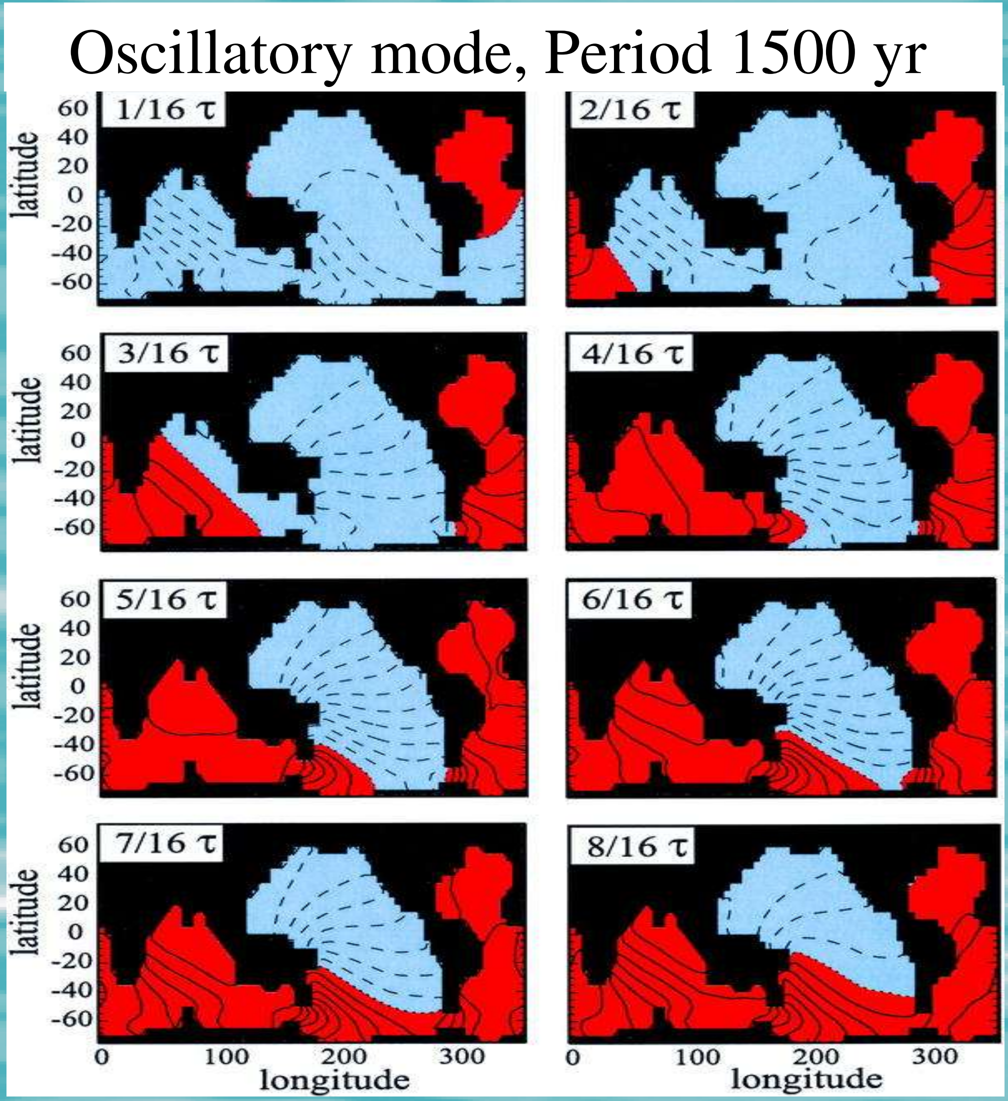
Basic State



(Weijer and Dijkstra, 2003)

Climate Variability

Linear Stability Analysis



T-anomaly at 300 m depth

(Weijer and Dijkstra, 2003)

Climate Variability

Generalized Stability Analysis

- What are the “most dangerous” perturbations?

- Traditional approach: Linear Stability Analysis
  - Determine least stable normal modes
  - Stationary/Oscillatory
  - Spatial patterns, growth/decay rate, frequency

- Alternative approach: Generalized Stability Analysis
  - Determine perturbations that lead to optimal (initial) growth
  - Uses adjoint of tangent linear model

Generalized Stability Analysis

Steady state:

$$M\frac{d\bar{x}}{dt}=\Phi(\bar{x})=0$$

$$x=\bar{x}+x'$$

$$M\frac{dx'}{dt}=Ax'$$

Generalized Stability Analysis

Steady state:

$$M\frac{d\bar{x}}{dt}=\Phi(\bar{x})=0$$

$$x=\bar{x}+x'$$

$$M\frac{dx'}{dt}=Ax'$$

$$x'(t_{n+1})=x'(t_n)P(t_{n+1};t_n)$$

$$P(t_{n+1};t_n)=\exp(M^{-1}A\Delta t)$$

Generalized Stability Analysis

Steady state:

$$M\frac{d\bar{x}}{dt}=\Phi(\bar{x})=0$$

$$x=\bar{x}+x'$$

$$M\frac{dx'}{dt}=Ax'$$

$$x'(t_{n+1})=x'(t_n)P(t_{n+1};t_n)$$

$$P(t_{n+1};t_n)=[M-\theta\Delta t A]^{-1}[M+(1-\theta)\Delta t A]$$

Generalized Stability Analysis

Norm

$$P(t_n;0)^T X P(t_n;0)x_e = \lambda X x_e$$

Adjoint of TLM

Tangent Linear Model (TLM)

Generalized Stability Analysis

$$P(t_n;0)^T X P(t_n;0) x_e = \lambda X x_e$$

$$A x = \sigma B x$$

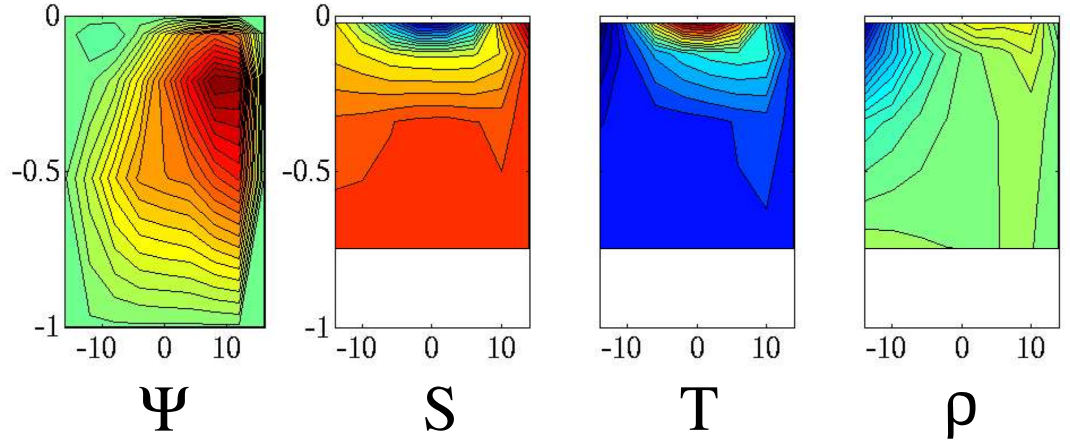
Generalized eigenproblem

Generalized Stability Analysis

Proof of concept

- Model THCM
  - Domain: 4x8x4

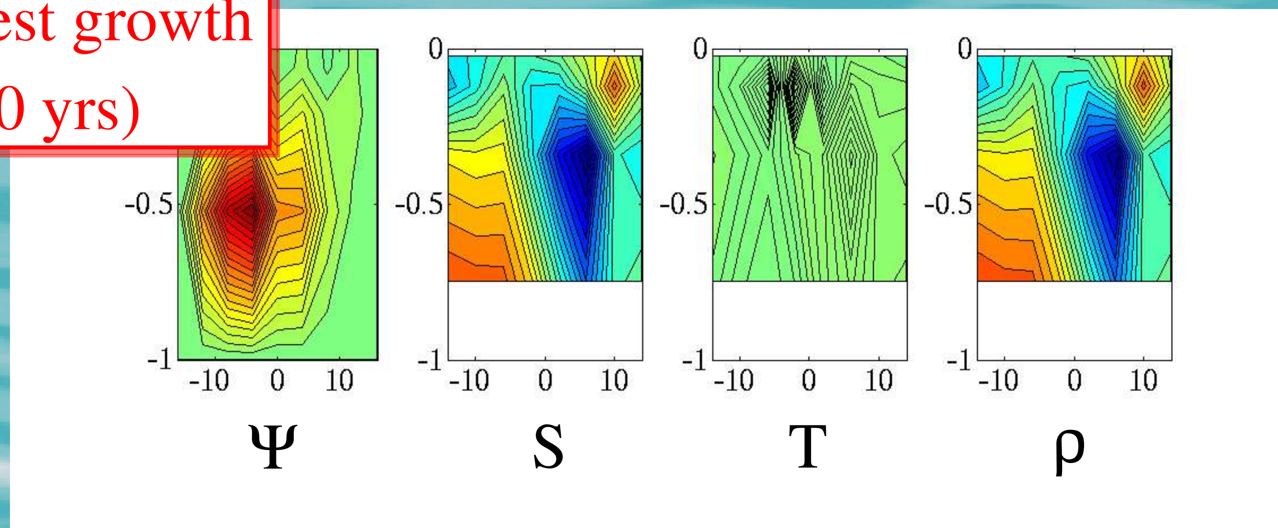
Basic State



Generalized Stability Analysis

Proof of concept

Initial perturbation with strongest growth (after 10 yrs)

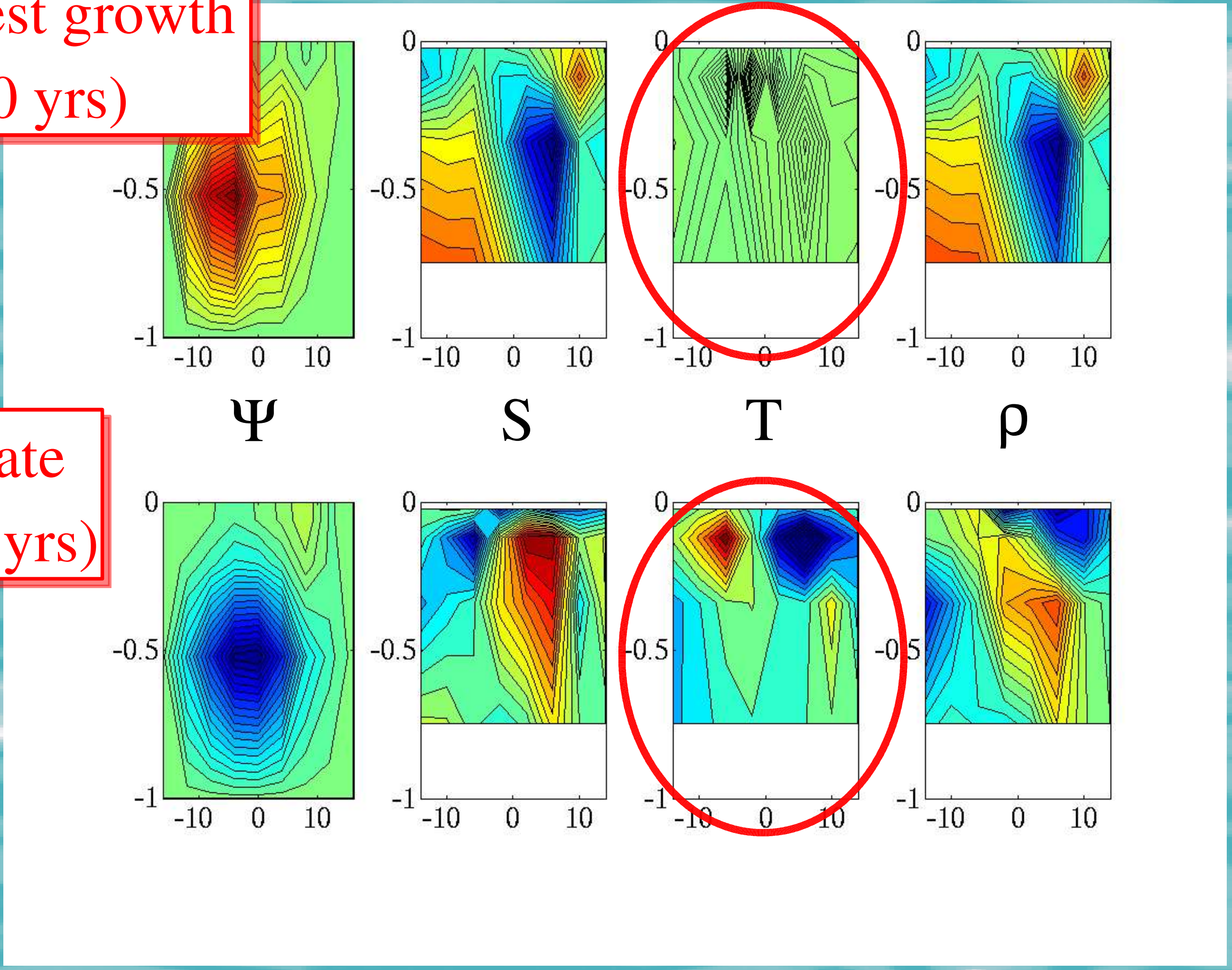


Generalized Stability Analysis

Proof of concept

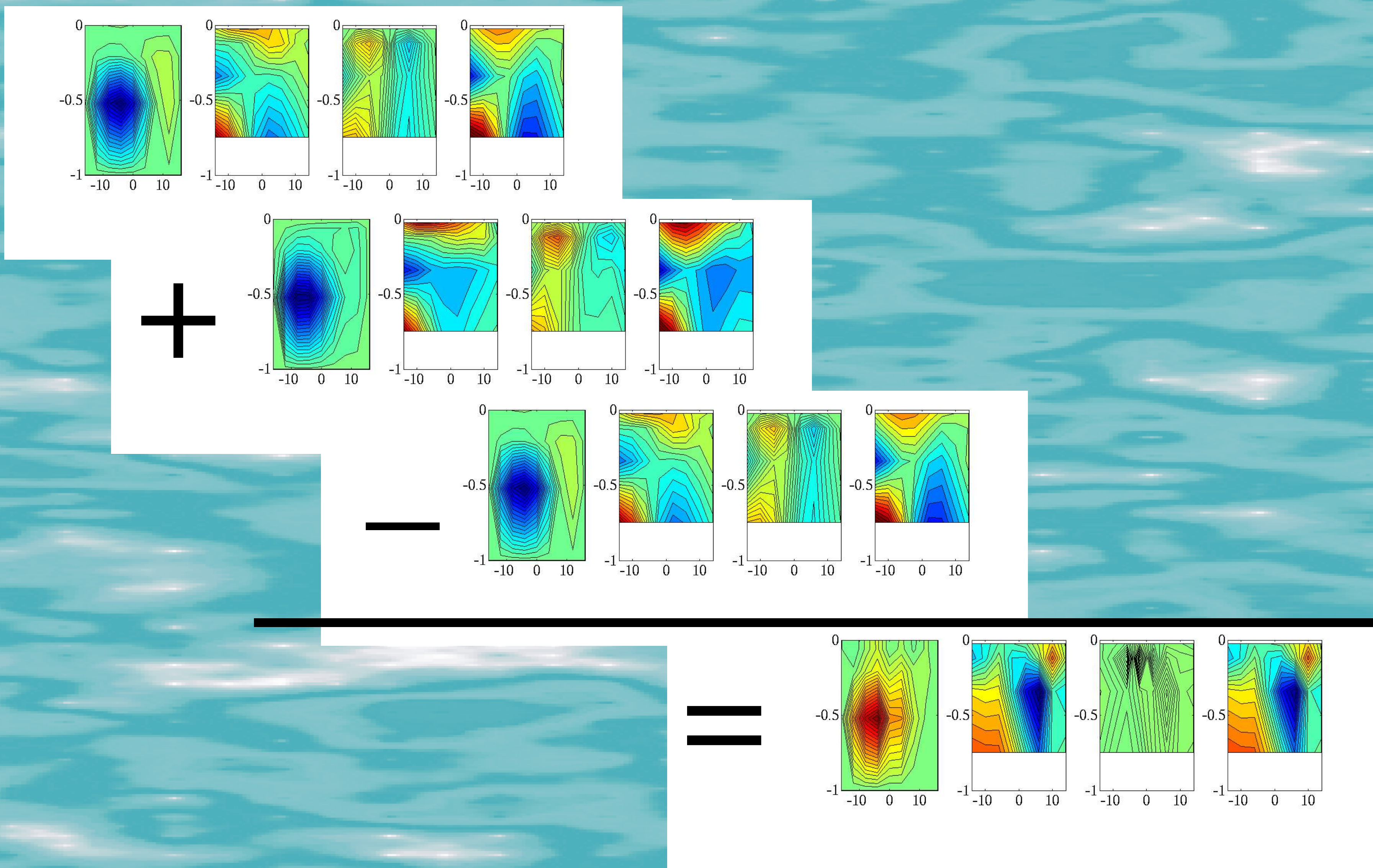
Initial perturbation with strongest growth (after 10 yrs)

Final state (after 10 yrs)



Generalized Stability Analysis

Proof of concept



Conclusions

- Implicit methods powerful tools to study ocean dynamics
  - Efficient time-stepping
  - Parameter continuation
  - Stability analysis: Linear & Generalized
- Current focus:
  - More efficient solvers
  - Improved ocean physics