Quantum ripples in chaos

Andreas Albrecht

The differences between quantum and classical chaos show up on the smallest of scales. Although tiny, these differences have implications for our understanding of quantum mechanics.

A mere 100 years ago — a recent event in the history of human endeavour — mankind discovered that underlying all the familiar laws of physics is the strange world of quantum mechanics. Superficially, at least, the quantum world appears to contradict many things we intuitively 'know' to be true about nature. For example, in the quantum world, position and momentum cannot both be precisely specified. Solid particles have a wave-like nature that allows them to produce interference patterns like ripples on a pond. It is even possible for familiar objects to be assigned strange 'coherent superposition states' (for example, to be simultaneously dead and alive in the case of Schrödinger's famous cat).

Such is the contrast between 'classical' physics and the quantum world that even Albert Einstein was unwilling to fully embrace quantum mechanics. Yet those who have carefully studied quantum theory have learned many of its tricks. We have discovered how, under the right conditions, both position and momentum can be well specified. And we know that the classical world is rich with natural decoherence processes that rapidly destabilize most Schrödinger cat states (see ref. 1 for a review). Moreover, Schrödinger cat states have been created in the laboratory using real systems, such as the current flowing in a superconductor[2,3]. Not quite a cat, but not bad. By now, many aspects of quantum mechanics (both mundane and exotic) have been put to the test, and quantum mechanics has passed with flying colours. Still, there are exciting new phenomena to be explored, especially whenever the quantum and classical worlds overlap. On page 712 of this issue[4] Wojciech Zurek tackles one of these — the difference between classical and quantum chaos — with some surprising results.

Chaotic behaviour is well understood from a classical perspective, and is typically discussed in the context of a mathematical 'phase space', in which there are dimensions for both position, x, and momentum, p. A particle at a given instant can be specified as a point in classical phase space, and the time development of the particle describes a curve or trajectory in phase space. In chaotic systems, particles that start out in virtually identical states (that is, at very close points in phase space) rapidly evolve into completely different states (that is, distant parts of phase space).

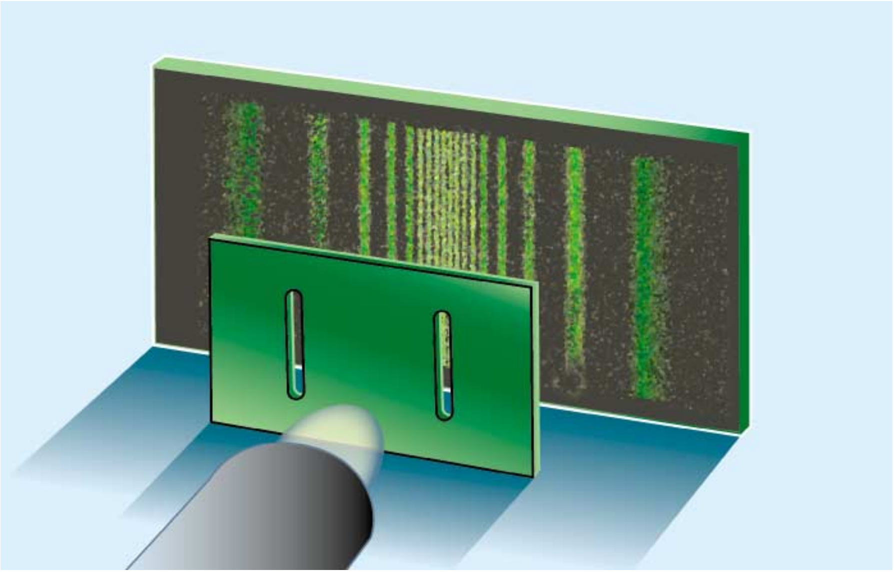

Figure 1 **The double-slit experiment. In this, slits are used to create two coherent beams of particles that interfere to produce a pattern on the screen. The coherent beams are a special case of the double-peaked 'Schrodinger cat states' discussed by Zurek[4]. The interference pattern has a characteristic size set by a combination of the distance to the screen, the separation of the slits and the momentum of the particles in the beam. The relevant momentum uncertainty is set by the width of the slits. These quantities can be arranged so that the scale of the interference fringes is much smaller than the minimum position uncertainty given by the uncertainty principle. But, as Zurek shows for the general case, there is no contradiction with quantum theory because the uncertainty in position used in the uncertainty principle refers to the width of the entire pattern, not the size of a single fringe. These sub-Planck-scale fringes have physical significance and are related to the sensitivity of quantum states to quantum decoherence.**

Because nothing is ever measured with absolute precision, one can never realistically talk about 'points' in phase space. Instead, every point (x,p) in phase space is typically assigned a probability, $P(x,p)$. For a well-specified particle this probability peaks sharply at a localized point in phase space. For an ordinary classical object, such as a single billiard ball, a phase-space probability distribution that starts out sharply peaked will remain peaked over time; a small uncertainty in the starting point results in a similarly small degree of ignorance at a later time.

Chaotic systems are dramatically different. A sharply peaked initial distribution gets torn apart by the chaotic evolution, as neighbouring phase-space trajectories rapidly head off in different directions (see Fig. 1d on page 713). A small amount of ignorance at the beginning rapidly translates into huge uncertainties later on, as the distribution becomes highly delocalized.

So how does a chaotic system behave when a full quantum calculation is made? Zurek[4] gives clear answers to this question, and in the process debunks some widely held beliefs on the subject. He uses the Wigner formulation of quantum mechanics[5], which gives the state of a particle using the Wigner function, $W(x,p)$. Under suitable conditions, $W(x,p)$ can be interpreted as the classical $P(x,p)$, but W is a more general object that incorporates the full range of quantum behaviour. (For example, W can be negative, whereas P is always positive.)

Heisenberg's famous 'uncertainty principle' of quantum mechanics says that $\hbar/2$ is the minimum value of the product of uncertainties in position and momentum (where $\hbar$ is Planck's constant). There is a widely held belief that the uncertainty principle requires W to have no features below the 'Planck scale' — that is, no features in phase space with an area smaller than $\hbar/2$ ($\approx 10^{-34}$ J s). Zurek shows this belief to be completely false. Along with a nice general analysis, he gives specific examples of how the Wigner function can have features on scales well below the Planck scale. Zurek shows that there is a smallest scale for structure in *W* that is set by interference effects, not the uncertainty principle. More importantly, he shows that these features are physically significant: they represent the susceptibility of quantum states to decohering processes.

The idea that quantum systems may have physical features below the Planck scale can be illustrated by the classic example of the double-slit experiment (Fig. 1). In this experiment, a pair of slits creates two coherent particle beams that interfere to produce striking interference fringes. The scale of the interference fringes can be arranged to be much smaller than the minimum uncertainty given by the uncertainty principle. But there is no contradiction with the laws of quantum mechanics because the uncertainty in position used in the uncertainty principle refers to the width of the entire pattern, not the size of a single fringe.

Zurek[4] shows how a generalized version of these fringes can appear in the Wigner function — typically as ripples on sub-Planck scales — when coherent parts of a quantum state interfere with one another. These fringes are present whenever there is quantum interference. Zurek shows that in the special case of isolated chaotic systems the fringes are essentially always present and emerge very rapidly. Even states that might initially look like localized classical states are quickly pulled apart by the chaotic evolution into pieces that interfere with one another (see Fig. 1a–c on page 713). Zurek then points out that there are physical effects that can destroy these fringes. This destruction is perpetrated not by the uncertainty principle, but by the coherence-destroying effects of interactions with other physical systems (that is, with the environment). This decoherence is a generalization of another well-known aspect of the double-slit experiment. If the particle beams interact with a measurement apparatus or some other part of the environment in a way that is different for each slit, the coherence is destroyed and the interference fringes disappear.

The struggle to adapt our intuition and insight to the quantum world has been quite an adventure, leading to such creations as transistors, Bose–Einstein condensates and the idea of quantum computation. It is clear that this adventure is far from over, and I expect many more remarkable developments to come. Zurek's article clears away some old misunderstandings and helps us develop a better quantum intuition, which we will need in this exciting future. ■

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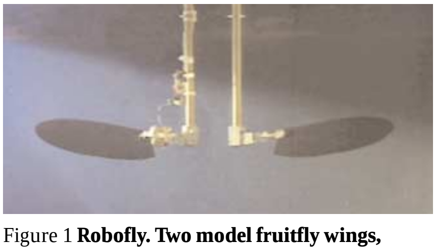
Aerodynamics

Flight of the robofly

George V. Lauder

Qualitative studies of airflow over insect wings have long been possible, thanks to the use of smoke trails. With a new robotic fly, flow and force can be analysed quantitatively, so theories of insect flight can be tested.

The problem of studying how air moves around flying animals has attracted attention from zoologists, aeronautical engineers and computational fluid dynamicists, but has remained generally unresolved. It is terribly difficult to measure patterns of airflow accurately in three dimensions, especially around insect wings, which are typically small and move rapidly in a complex manner. Yet quantifying such patterns is essential for understanding the aerodynamic mechanisms of insect flight and for testing theories about wing function. On page 729 of this issue, Birch and Dickinson[1] describe how they used a dynamically scaled robotic insect to obtain new data on how insect wings function during hovering. The importance of their work goes beyond the specific hypothesis that they test, and shows the power of a laboratory model that combines quantitative analyses of airflow with direct measurements of the forces produced by wings.



J. M. BIRCH & M. H. DICKINSON

Figure 1 **Robofly. Two model fruitfly wings, which can be controlled precisely in three dimensions, are attached to force sensors and immersed in a vat of mineral oil.**

Our understanding of the aerodynamics of insect flight has been helped greatly by observations of tethered insects flying in a wind tunnel[2,3]. The introduction of smoke or dust streams into the tunnel allows researchers to observe how wing movements deflect oncoming air, and offered a first look at the vortices produced in the insects' wake. These data, combined with detailed analyses of wing kinematics in freely flying insects[4], provided a basis for evaluating theories about the aerodynamics of insect flight[5]. But it is extremely difficult to obtain repeatable data using live insects, and their small size complicates any effort to quantify airflow.

Against this background, five years ago Ellington *et al.*[6] published an influential paper showing that the insect wing supports a particular type of vortex, the leading-edge vortex. This is a region of rapidly circulating air, found near the front (leading) edge of the wing, with a low-pressure core. This vortex is stable during the wing's downstroke and might enhance lift, perhaps in part explaining how insects can generate surprisingly large lift forces. The authors were able to describe this phenomenon in detail because they used a mechanical model of a hawkmoth (the 'flapper') with a wingspan of over a metre, which allowed repeatable observations of airflow at a large scale. By injecting smoke directly along the wing's leading edge, the authors revealed that the leading-edge vortex had a helical structure.

Birch and Dickinson[1] have taken this approach considerably further. First, their dynamically scaled model fruitfly (robofly; Fig. 1) has two 19-centimetre-long clear plastic wings whose motion can be precisely controlled. The model is immersed in a large vat of mineral oil, making it much easier to quantify fluid flow over the wing using the technique of digital particle image velocimetry (DPIV) — an increasingly popular tool for studying the mechanics of animal locomotion in fluids[7–9]. By seeding the mineral oil with small air bubbles and illuminating a two-dimensional slice with a pulsed sheet of laser light, the movement of fluid above and below the wing and in its wake can be quantified with precision. DPIV obviates the need for creative interpretation of smoke trails. Furthermore, the light sheet can be repositioned along the length of the wing to construct a complete three-dimensional picture of flow.

Second, small force sensors at the base of one of the wings (where it joins the fly's body) make it possible to measure the forces perpendicular and parallel to the wing as it flaps, at the same time that DPIV data are acquired. Third, the wing can be manipulated (by adding fences across it to disrupt fluid flow from base to tip), as can the nearby environment (by building a wall curving around the wing tip).

Birch and Dickinson programmed robofly to move its wing in a hovering motion, and the result is the most detailed picture ever obtained of flow over an insect