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Transmission Network Expansion Planning with Complex Power Flow Models

Russell Bent, G. Loren Toole, and Alan Berscheid

***Abstract*—In recent years the transmission network expansion planning (TNEP) problem has become increasingly complex. As this problem is a non-linear and non-convex optimization problem, researchers have traditionally focused on approximate models of power flows to solve the TNEP problem. Until recently, these approximations have produced results that are straight-forward to adapt to the more complex problem. However, the power grid is evolving towards a state where the adaptations are no longer as easy (e.g., large amounts of limited control, renewable generation) necessitating new approaches. In this paper, we propose a Discrepancy-Bounded Local Search (DBLS) that encapsulates the complexity of power flow modeling in a black box that may be queried for information about the quality of a proposed expansion. This allows the development of an optimization algorithm that is *decoupled* from the details of the underlying power model. Case studies are presented to demonstrate cost differences in plans developed under different power flow models.**

***Index Terms*—TNEP, Transmission Network Expansion Planning, Simulation Optimization, Non-linear Optimization, Local Search.**

I. Introduction

RECENT years have brought an increased awareness of one of the major challenges of the 21st century: the problem of how to provide clean, sustainable, and cheap energy to the world's rising population [1], [2]. To address this challenge, the United States Department of Energy released a report in 2008 that stated the goal of having 20% of the U.S.'s energy come from wind by 2030 [3]. An important point is raised in the report on how to best upgrade and expand the electric power transmission grid to meet increased demand for energy and to incorporate sustainable, renewable energy sources that are often located in transmission deficient areas. This optimization problem has been well studied under the name of Transmission Network Expansion Planning (TNEP) [4], [5], [6], [7]; however, the requirements for the future grid raise a number of open challenges.

The challenges center on how power flows are modeled in the TNEP and the degree to which expansions and grid operations are constrained [8]. The literature has traditionally (with the exception of [9]) focused on linearized models of real (DC) power flows as these types of models account for most network utilization, generation is controllable, the networks are small, and planning horizons are short [7]. Under these assumptions, it is generally considered easy for a planner to modify a plan to accommodate the non-linear nature of power flows (as well as reactive (AC) power) [10]. However, recent studies by [3] that consider transmission planning for large-scale systems (the western United States), long planning horizons (30+ years), and large amounts of renewables (i.e., solar and wind, with limited control capabilities, thus, constraining operations) relax many of the assumptions prior approaches have relied upon and make the problem more difficult. Indeed, our preliminary work [11], [12] has shown that failing to take into account AC power flows can yield arbitrarily poor solutions. Even the seminal work of [13] indicated AC modeling on a small six-node benchmark was required as it contained a voltage violation.

To address these challenges, this paper presents a novel approach, Discrepancy-Bounded Local Search (DBLS), for embedding ideas from simulation optimization [14] in a local search procedure that generalizes constructive heuristics [15], [16], constraint-based local search [17], and is related to global search techniques, such as limited discrepancy search [18], [19]. The key idea of the approach is the encapsulation of the power model within a simulation black box. The DBLS is allowed to query the black box for power flow information about proposed expansion plans. In this paper, DBLS is used on the cost minimization variation of the TNEP problem and is constrained to only consider line and shunt compensation expansions. However, DBLS can be generalized to other expansion criterion such as carbon emissions and reliability and to other expansion options such as voltage upgrades and generation.

In short, the key contributions of this paper include:

- A TNEP algorithm that can use AC power flow modeling
- An algorithm that generalizes existing constructive TNEP heuristics to allow for more complete search and improves their performance
- Case studies on constrained problems that demonstrate the difference in cost of DC-based plans and AC-based plans can be large

Literature Review The literature on TNEP is extensive and references [4], [5], [6], [7], [10], [20] provide surveys of the field. In general, existing approaches have focused on modeling power flows with transportation models or the linearized DC model to reduce computational overhead [10]. Until recently, it has been easy to adapt plans derived from these models to more realistic conditions (see [21], [22]). TNEP algorithms tend to fall into three categories: complete search based on mixed-integer program (MIP) formulations [21], [23], locally optimal search such as constructive heuristics [15], [16], and meta-heuristics [8], [24], [25], [26].

One of the most relevant papers to the work presented here is that of [27], which presents an expansion planning scenario where generation is fixed (also studied in [16]) in order to model the challenges of market-based economic

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dispatching. In this paper, the primary motivation for fixing generation is the modeling of renewable energy sources. This is a pessimistic view of how power systems operate, but it is useful for modeling a worst case for dispatching and expansion planning. These models also provide an opportunity to understand the benefits and drawbacks of using approximate models of power flow in expansion planning when generation cannot be dispatched. Reference [28] also shares a number of interesting similarities with this paper. It presents a tree-based local search procedure that contains a truncation criterion not unlike the discrepancy parameter of the algorithm discussed later in this paper. Their approach utilizes combinations of the transportation model and linearized DC model for modeling power flows; they do state that the approach can be generalized to more complex models of power flow, but this was not tested. They also incorporate multiple scenarios and a reliability criterion into the planning environment. It is important to note that their search procedure is primarily guided by cost, whereas our approach is guided by both feasibility and cost.

DBLS also shares a number of similarities with branch and bound approaches. For example, reference [29] discusses a branch and bound approach for solving the TNEP using Bender's cuts. This approach has some interesting parallels with this paper where the cutting procedure could theoretically use more complex power models (like AC), not unlike those discussed in reference [30]. Reference [31] has also developed a branch and bound approach for the TNEP, with a focus on generalizing branch and bound to accommodate multiple competing objective functions.

Reference [9] is one of the few papers that uses AC power flows in the TNEP. They present a constructive heuristic that incrementally selects the expansion that most improves the load shed. Their approach was tested on problems that allowed for the dispatching of generation. DBLS can be viewed as a generalization of this approach when this constructive heuristic can be embedded as the branching heuristic in DBLS (discussed later). Reference [31] has also taken steps to include AC models in the TNEP. They evaluate a set of candidate plans constructed from a DC-based branch-and-bound algorithm with an N-1 reliability metric based on the AC power flow model.

Also important is the work of [32], [33], which is the basis of many of the results contained in [3]. These papers provide the fundamental motivations for the work of this paper. They studied how to best integrate large amounts of wind power into large geographic areas based upon transportation models of power transmission. We seek to address the question of how to incorporate non-linear models of power flow into such planning scenarios. Considerable effort was required to adapt solutions derived from the transportation models used in these planning scenarios [11].

Finally, it is important to note that the preliminary conference version of this paper appeared in [12] (referred to as limited discrepancy local search). However, this paper did not generalize existing constructive heuristics, compare with existing algorithms, or demonstrate the effectiveness of the algorithm on DC power-flow models.

The remainder of this paper is organized as follows. Section II formally defines the TNEP. Section III describes the algorithm used to generate expansion plans. Section IV discusses heuristics used to guide the algorithm to reduce physical violations and cost. Section V discusses the experimental results and Section VI concludes this paper.

II. Problem Definition

Buses The problem is described in terms of a set of buses, $\mathcal{B}$, that represent geographically located nodes in a power network e.g., generators, loads, and substations. Each bus, i, is defined by parameters g_i, l_i, ι_i^-, ι_i^+, which represent generation, load (demand for power), minimum voltage (per unit), and maximum voltage (per unit). $P(g_i)$ and $Q(g_i)$ are used to denote the real and reactive components of generation. Similarly, $P(l_i)$ and $Q(l_i)$ are used to denote real and reactive components of load. For simplicity, $P_i = P(g_i) - P(l_i)$ and $Q_i = Q(g_i) - Q(l_i)$ are used to denote the real and reactive power injected at bus i. The decision variable c_i defines the number of control components at i (in this paper, shunt capacitors (compensation) for regulating AC power). c_i has discrete domain $\{c_i^-, c_i^- + 1, \dots,\ c_i^+ - 1,\ c_i^+\}$. c_i^- defines the number of control elements i starts with, ensuring that existing controls are included.

Transmission Corridors The TNEP is also described by a set of edges, $\mathcal{E}$, called transmission corridors, that connect pairs of buses. A transmission corridor i,j between buses i and j has a decision variable $c_{i,j}$ that defines the number of circuits (power lines) in the corridor. The variable has discrete domain $\{c_{i,j}^-, c_{i,j}^- + 1, \dots, c_{i,j}^+ - 1, c_{i,j}^+\}$ where $c_{i,j}^-$ is defined as the number of circuits the corridor starts with. $c_{i,j}^+ = c_{i,j}^-$ when no circuits may be added to a corridor. A circuit is also defined by parameter $\psi_{i,j}$, which denotes the capacity of a single circuit in the corridor. Similarly, $r_{i,j}$, $x_{i,j}$, and $b_{i,j}$ denote the resistance, reactance, and line charging of a single circuit in the corridor.

TNEP Solution A transmission network solution, σ, is defined as a set of variable assignments $\bigcup_{i\in\mathcal{B}}[c_i \leftarrow d_i] \cup \bigcup_{i,j\in\mathcal{E}}[c_{i,j} \leftarrow d_{i,j}]$, where d_i is drawn from the domain of c_i and $d_{i,j}$ is drawn from the domain of $c_{i,j}$[1]. By convention, unassigned variables are assumed to be c_i^- and $c_{i,j}^-$. $\sigma(c_i)$ and $\sigma(c_{i,j})$ are used to denote the variable assignments for σ. For convenience, we use the notation y to denote a variable of type c_i or $c_{i,j}$ when they may be used interchangeably.

Simulation TNEP algorithms have at their disposal a simulator $\mathcal{S}$ for determining the flow of power for σ. $\mathcal{S}(\sigma)$ returns true if it computes the flows without divergence. $\mathcal{S}_{F_{i,j}}(\sigma)$ denotes the flow in corridor i,j and $\mathcal{S}_{v_i}(\sigma)$ the voltage at bus i. For simplicity, this notation is shortened to $F_{i,j}$ and v_i when $\mathcal{S}(\sigma)$ is understood from context. $\mathcal{F}(i,j)$ and $\mathcal{T}(i,j)$ are used to denote the flow from and flow to bus of corridor i,j, respectively.

A TNEP solution σ is feasible when the following constraints are satisfied, i.e.,

$$\begin{cases} c_{i,j}^- \leq c_{i,j} \leq c_{i,j}^+ & (i,j \in \mathcal{E}) \quad (1) \\ c_i^- \leq c_i \leq c_i^+ & (i \in \mathcal{B}) \quad (2) \\ \mathcal{S}(\sigma) = \text{true} & \quad (3) \end{cases}$$

[1]This formulation can be generalized for multiple types of control components and circuits.

Physical constraints are relaxed and incorporated into the objective function in order to keep the search space connected (similar to Lagrangian Relaxation). The overload of σ is calculated as the sum of flow that exceeds the capacity of the circuits (overloads), i.e., $\eta(\sigma) = \sum_{i,j\in\mathcal{E}} \max(0, F_{i,j} - \psi_{i,j}c_{i,j})$. The voltage violation of σ is calculated as the sum of voltages that fall below ι_i^- or above ι_i^+, i.e., $\nu(\sigma) = \sum_{i\in\mathcal{B}} \max(0, \iota_i^- - v_i, v_i - \iota_i^+)$. Finally, the cost of σ is defined by $\kappa(\sigma) = \sum_{i,j\in\mathcal{E}} c_{i,j}\kappa_{i,j} + \sum_{i\in\mathcal{B}} c_i\kappa_i$, where κ_i is the cost of putting a control at bus i and $\kappa_{i,j}$ is the cost of putting a circuit in corridor i,j. The objective function, $f(\sigma)$, is then a lexicographic multi-objective function[2] of the form $\min f(\sigma) = \langle \eta(\sigma), \nu(\sigma), \kappa(\sigma)\rangle$.

III. Simulation Optimization DBLS Algorithm

As discussed earlier, branch and bound techniques have been successful on small-scale problems with DC models of flow [7], [29], [31]. On larger problems, heuristics and meta-heuristics work well under specific models for calculating power flows. Driven by the desire for TNEP algorithms that apply to a wide variety of behavior (flow) models, including non-linear models, we present a novel algorithm for addressing this need. This algorithm builds on simulation optimization ideas by encapsulating the behavior of the network into a "black box" that may be queried by the algorithm for information about how a TNEP solution behaves (i.e., $\mathcal{S}(\sigma)$) and embedding it in a DBLS that limits the full exploration of a branch and bound search tree. The intuition behind DBLS is to generalize constructive heuristics that make good decisions on how to build solutions, but make a few bad decisions from time to time. DBLS embeds the heuristic in a search tree as the branching heuristic and explores those solutions that are within δ violations (discrepancies) of the heuristic, where δ is a user-specified parameter. DBLS provides a natural way to incorporate constructive heuristics from the TNEP literature, e.g., [15], [16], into a more general framework and is related to the approach of [28]. The formal algorithm of DBLS for TNEP is presented in Figure 1.

DBLS($\sigma, \mathcal{X}, \delta$)
1 **if** $\delta = 0$
2   **then return** σ;
3 $\sigma^* \leftarrow \sigma$;
4 $y \leftarrow$ chooseVariable($\mathcal{X}, \sigma$);
5 $\langle d_1, d_2, \ldots, d_k\rangle \leftarrow$ orderDomain(y);
6 $\sigma \leftarrow \sigma \setminus [y \leftarrow \sigma(y)]$;
7 **for** $i \leftarrow 1 \ldots k$
8 **do** $\sigma_i \leftarrow \sigma \cup [y \leftarrow d_i]$;
9   **if** $f(\sigma_i) \leq f(\sigma^*)$ **and** $\mathcal{S}(\sigma_i)$
10   **then** $\sigma^* \leftarrow \sigma_i$;
11   DBLS($\sigma_i, \mathcal{X} \setminus y, \delta - i$);
12 **return** σ^*;

Fig. 1. Discrepancy-Bounded Local Search

DBLS takes as arguments a solution σ, (often the current state of the network, i.e., $\bigcup_{i\in\mathcal{B}}[c_i \leftarrow c_i^-] \cup \bigcup_{i,j\in\mathcal{E}}[c_{i,j} \leftarrow c_{i,j}^-]$); a set of variables, $\mathcal{X}$, drawn from $\bigcup_{i\in\mathcal{B}} c_i \cup \bigcup_{i,j\in\mathcal{E}} c_{i,j}$; and a discrepancy parameter, δ. The first two lines of Figure 1 check if the number of discrepancies has dropped to 0. Line 3 initializes the best solution discovered with the current solution. Line 4 chooses a variable y to explore. Line 5 executes the heuristic for ordering the domain of y. When $\eta(\sigma) > 0$ or $\nu(\sigma) > 0$ the domain is ordered by component additions, no change ($\sigma(y)$), and component removals, i.e.,

$$\left\langle \sigma(y)+1, \ldots, y^+, \sigma(y), \sigma(y)-1, \ldots, y^- \right\rangle$$

otherwise it is ordered in reverse, i.e.,

$$\left\langle \sigma(y)-1, \ldots, y^-, \sigma(y), \sigma(y)+1, \ldots, y^+ \right\rangle$$

Line 6 unassigns the current variable assignment of y (if any) and lines 7–11 iterate over the ordered domain of the variable. δ is decremented by violations in the ordering heuristic. It is worth noting that line 9 implicitly updates attributes associated with the new σ executes $\mathcal{S}$. Line 12 returns the best solution discovered. From a search tree perspective, Figure 2 provides an illustration of DBLS's search on a binary tree for $\delta = 0, 1, 2$ and 3. As can be seen in the figure, the running time of DBLS is exponential in δ and $|\mathcal{X}|$ (the number of plans considered is $\sum_{i=1\ldots\delta}\binom{|\mathcal{X}|\times k}{i}$, where k is the maximum size of a variable's domain) [34].

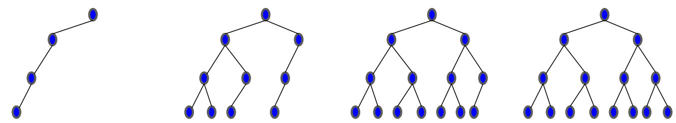


Fig. 2. An example of the DBLS algorithm on a simplified binary search tree. On the left is the portion of the search tree explored when $\delta = 0$, i.e., only expansion decisions suggested by the constructive heuristic are considered. The subsequent pictures show the portions of the search tree explored when $\delta = 1, 2$ and 3.

In studying the performance of DBLS, three key generalizations boost the quality of the results [12]. First, $f(\sigma)$ is non-monotonic. Adding components can cause $\eta(\sigma)$ and $\nu(\sigma)$ to rise or fall (sometimes referred to as Braess's paradox [35]). To control this behavior, we introduce a parameter, α, to limit the number of times in a row that $f(\sigma)$ may worsen. A similar parameter is used in [28]. Second, it is possible for $\mathcal{S}(\sigma)$ to fail for a given σ. A parameter β is introduced to limit the number of times in a row that $\mathcal{S}(\sigma)$ may fail. Finally, the performance of DBLS on TNEP was highly dependent on the quality of early decisions as it can take a considerable amount of time to return to these decisions. Thus, it was productive to iteratively restart DBLS with improving starting solutions (generally keeping δ small).

IV. Branching Heuristics

We next discuss five implementations of chooseVariable. The first four heuristics are motivated by the difficulty in minimizing $\eta(\sigma)$ and are invoked when $\eta(\sigma) > 0$ or $\nu(\sigma) > 0$. The fifth heuristic for minimizing cost is invoked when $\eta(\sigma) = 0$ and $\nu(\sigma) = 0$. The first three heuristics focus on circuit expansions. If no circuit expansions exist, the functions choose the bus with the lowest voltage to add or subtract shunt compensation

In the nomenclature of [16], these implementations represent least-effort heuristics that use electrical system performance to create a sensitivity index (ranking function) for each variable. The appendix provides formal definitions of each implementation in pseudo-code.

[2]Lexicographic objective functions define objective functions in order of primacy. The first objective is used to compare two solutions. In the case of ties, the second objective is used, and so forth.

Maximum Utilization The first implementation of CHOOSEVARIABLE is referred to as *maximum utilization* (MU). This heuristic chooses the corridor with the most utilized capacity. Interestingly, this choice can have a negative effect on overloaded corridors; adding capacity increases the conductance of the corridor and can increase the flow in the corridor. This observation provides the intuition for adding capacity to corridors that are not overloaded. Corridors that are near capacity are clearly attractive routes for power, so adding capacity (conductance) may divert power from areas that are overloaded.

Flow Diversion The second implementation of CHOOSEVARIABLE is referred to as *flow diversion* (FD). This heuristic looks for a circuit (i, j) that is overloaded and considers portions of the network that are upstream and downstream from the overloaded circuit. It selects circuit expansions that divert power away from upstream locations or accept power downstream. Intuitively, this selects circuits that may bypass the overloaded circuit more conductive.

Alternate Path Around The third implementation of CHOOSEVARIABLE is referred to as *alternate path around* (APA). This heuristic looks for a circuit (i, j) that is overloaded and selects expansions that are on power flows paths that bypass (i, j) and bring power from upstream generators to downstream loads.

Most Improving The fourth implementation of CHOOSEVARIABLE is referred to as *most improving* (MI) and it is based on constructive heuristics that choose an expansion variable whose modification most improves the current plan. Variations of this heuristic are presented in [9], [15], [16], among others. MI increases the number of simulations performed at each node in the search tree by $O(\mathcal{E})$, so it can be computationally burdensome. However, it can more accurately assess the impact to f when adding a component.

Minimize Cost This fifth heuristic is based on the cost reduction stages of constructive heuristics [15], [16]. It chooses the most expensive variable that has been expanded.

V. Experimental Results

In order to evaluate our approach, we considered four benchmarks from the TNEP literature [27] and an expansion scenario based on the electric power grid in New Mexico. The New Mexico scenario uses the load and wind generation growth projections of [3]. The commercial electric power simulation package T2000 [36] and the linearized DC flow model are used as implementations of $\mathcal{S}$. All results are obtained using an Intel Xeon 2.83 Ghz processor. The intent is to 1) demonstrate that DBLS can find high-quality solutions to the TNEP, 2) compare the performance of different constructive heuristics when embedded in DBLS, 3) demonstrate that DBLS can be used with the AC power flow model, and 4) show examples where solutions based on the DC power flow model need considerable reinforcement under the AC power flow model.

Reference [27] proposed 4 TNEP benchmarks (G1, G2, G3, and G4) based on the 24-bus RTS-79 problem of [37]. Reference [27] grew demand and generation of the RTS by 200–300%. The problems allow up to 3 additional circuits in the 34 existing corridors and in 7 new corridors (the domain of each variable has size 4). The problems pessimistically assume that generation cannot be dispatched. This provides worst case scenarios, e.g., all generation is wind-based.

TABLE I
AC Generation

Bus	G1 $Q(g)$	G2 $Q(g)$	G3 $Q(g)$	G4 $Q(g)$	$Q(g)_{max}$	$Q(g)_{min}$
1	94.43	76.24	94.43	85.25	240.0	-150.0
2	46.8	46.8	46.8	42.32	240.0	-150.0
7	193.5	155.23	193.5	174.58	540.0	0.0
13	758.8	609.43	623.55	684.32	720.0	0.0
14	41.1	41.1	41.1	41.1	200.0	-150.0
15	0.15	0.15	0.08	0.13	330.0	0.0
16	75.66	75.66	45.88	68.17	240.0	-150.0
18	412.2	412.2	207.13	246.63	600.0	-150.0
21	324.6	324.6	257.24	291.32	600.0	-150.0
22	-89.28	-89.28	-89.28	-89.28	288.0	-180.0
23	64.6	195.45	406.08	287.94	930.0	-375.0

TABLE II
AC Load

Bus	$Q(g)$	Bus	$Q(g)$	Bus	$Q(g)$	Bus	$Q(g)$	Bus	$Q(g)$
1	66	5	42	8	105	13	162	18	204
2	60	6	84	9	108	15	192	19	111
3	111	7	75	10	120	16	60	20	78
4	45								

The definition of the original RTS problems provide all the parameters for solving AC and DC power flows. However, as reference [27] used DC power flows, some information was not provided in the new problems, namely growth in AC generation and demand and line charging for circuits in new corridors. Thus, we scaled the AC load and generation by the same factors as [27]. We also modeled the generators as voltage controlled. The AC generation parameters for problems G1, G2, G3, and G4 are in Tables I, II, and III. Finally, bus 6 is allowed up to three additional inductive shunts (c_i), each of size 100 MVar and cost \$1000.

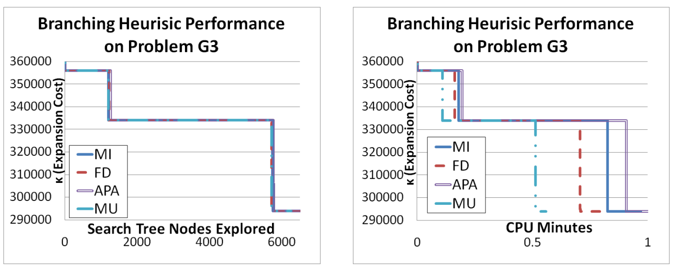
TABLE III
New Corridor Line Charging

Bus	Bus	b	Bus	Bus	b	Bus	Bus	b
1	8	0.043	13	14	0.088	19	23	0.122
2	8	0.034	14	23	0.14	16	23	0.179
6	7	0.052						

DC Power Flow Model The first set of results considers these benchmarks when $\mathcal{S}$ is implemented with the linearized DC power flow equations. This allows a comparison of DBLS with results in the literature. Table IV reports the performance of the different branching heuristics on these benchmarks. It describes the best result (and the parameter settings for achieving the result) for minimizing η and κ. The possible parameters are drawn from $\delta = \{1, 2, 3, 4, 5\}$ and $\alpha = \{1, 2, 3, 4, 5\}$ (divergence does not occur in the DC power flow model, so β does not impact the search). In the case of a tie, the result with the minimal number of search tree node explorations is reported. In these problems, each branching heuristic is able to achieve the same quality solution. From a search efficiency perspective, the results are less clear as no heuristic dominates in terms of number of search node explorations required to achieve their best solution. However, the difference between the best and worst approaches is never greater than double. This provides some evidence that, for these problems for the DC power flow model, all of the branching heuristics are reasonable choices. It is important to recall that all four feasibility-based branching heuristics use the same heuristic

TABLE IV
BEST SOLUTIONS OBTAINED FOR DIFFERENT IMPLEMENTATIONS OF CHOOSEVARIABLE UNDER THE DC MODEL

		η				κ			
	BH	δ	α	f	NC	δ	α	f	NC
G1	MI	1	1	15	0	3	1	390K	552
	FD	1	1	12	0	3	1	390K	669
	APA	1	1	12	0	3	1	390K	707
	MU	1	1	15	0	3	1	390K	539
G2	MI	1	1	27	0	3	1	424K	697
	FD	1	1	13	0	3	1	424K	841
	APA	1	1	13	0	3	1	424K	835
	MU	1	1	18	0	3	1	424K	714
G3	MI	1	1	7	0	5	3	294K	5757
	FD	1	1	7	0	5	3	294K	5735
	APA	1	1	7	0	5	3	294K	5780
	MU	1	1	7	0	5	3	294K	**5725**
G4	MI	1	1	8	0	4	2	354K	64
	FD	1	1	8	0	4	2	354K	**55**
	APA	1	1	8	0	4	2	354K	**55**
	MU	1	1	8	0	4	2	354K	**55**



(a) (b)

Fig. 3. The performance of the branching heuristics for $\delta = 5$ and $\alpha = 3$ on problem G3 for the DC power flow model in terms of number of search tree nodes (a) and CPU time (b).

for reducing cost and they are invoked when the search re-enters portions of the search tree where $\eta > 0$.

Figure 3 drills deeper into the results of Table IV to better understand the performance of the branching heuristics as they progress towards achieving the best result. This figure considers $\delta = 5$ and $\alpha = 3$ on problem G3. The figure plots the best solution seen during the search as a function of number of search tree nodes explored (a). In terms of nodes explored, all four branching algorithms behave in essentially the same way. This is a typical result for a wide range of parameters on all four problems. As all five heuristics require about the same number of search tree node explorations to achieve similar quality results, a useful metric of comparison is CPU time (b). MU is a slightly better performing branching heuristic (all results require 1 CPU minutes regardless), which is not surprising given the computational simplicity of its selection process. The other three algorithms require traversals of the network or multiple executions of $\mathcal{S}$ to choose a variable.

Finally, it is important to consider how varying α impacts the search. Figure 4 shows how the performance of MI changes when $\delta = 4$ and α is varied. It is clear from these results that allowing α to increase to 3 improves the quality of the results. After that point it is more productive to increase δ rather than α. Surveying the results, allowing α to be more than 3 rarely improves the quality or efficiency of the results. This provides empirical evidence that it is important to prune the search tree when worsening solutions are encountered.

Finally, Table V compares the quality of the results discovered by DBLS with the best solutions in the reviewed

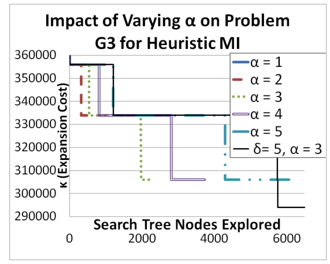


Fig. 4. The performance of MI for $\delta = 4$ and varying α on problem G3 for the DC power flow model.

TABLE V
BEST DC SOLUTIONS TO BENCHMARKS OF [27]

Problem	Best Known	Ref	Best Found
G1	438K	RRMS	390K
G2	451K	FH	424K
G3	218K	RRMS	294K
G4	376K	FH	354K

literature. In this table, RRMS refers to [16] and FH refers to [27]. In three cases, DBLS improved the best known solutions in the literature for the DC model. These solutions are shown in Table VI.

AC Power Flow Model The second set of results uses a non-linear AC power flow model implemented by the commercial T2000 software package [36] for $\mathcal{S}$. This model was tested on the RTS-79 benchmarks of the previous section. As the initial network topologies with modified generation and load do not generally exhibit convergent behavior in $\mathcal{S}$, the best solutions obtained under the DC power flow model are used as a starting point (Table VI). Note that the algorithm is allowed to remove edges proposed by the DC power model.

Table VII compares the solutions for RTS benchmarks when the AC power flow model is used. It describes the best result (and the parameter settings for achieving the result). The possible parameter settings are down from $\delta = \{1, 2, 3, 4, 5\}, \alpha = \{1, 2, 3, 4, 5\}$, and $\beta = \{1, 2, 3, 4, 5\}$. Once again, in the case of ties, the result with the minimal number of search tree explorations is reported. From these results, MI is the best heuristic. It removes all overloads in all four problems (every other heuristic fails to achieve this result at least once). MI also achieves the best result κ in all four cases (although at higher node counts). These results also suggest that $\alpha = \beta$ is a good parameter choice in general.

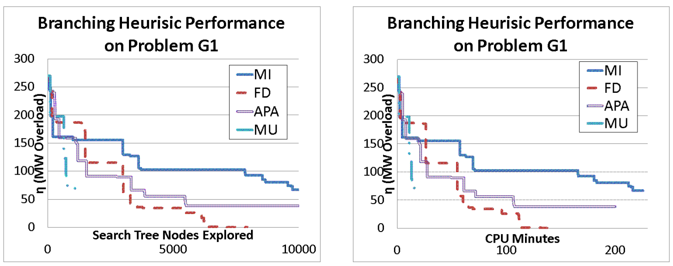
Figure 6 considers the results of Table VII more closely for $\delta = 5, \alpha = 3, \beta = 3$ on problem G1. In the figure, (a) plots the best solution of η seen during the search as a function of the number of search tree nodes explored. There are a couple of interesting points that are common across many parameter choices. First, MU and FD are very good at finding high-quality solutions quickly, however, they terminate more quickly as α and β are invoked more often and prune out large

TABLE VI
SOLUTIONS TO THE RTS-79 BENCHMARKS FOR DC POWER FLOW MODEL.

Line	G1	G2	G4	Line	G1	G2	G4
(1,5)	1	1	2	(14,16)	1	1	1
(3,24)	1	1	1	(15,24)	1	1	1
(6,10)	1	1	1	(16,17)	2	2	1
(7,8)	2	1	2	(16,19)	1	1	0
(10,12)	0	1	1	(17,18)	2	2	0

TABLE VII
BEST SOLUTIONS OBTAINED FOR DIFFERENT IMPLEMENTATIONS OF CHOOSEVARIABLE UNDER THE AC MODEL

	BH	η					κ				
		δ	α	β	f	NC	δ	α	β	f	NC
G1	MI	3	4	1	**0**	2184	4	4	1	**1303K**	8521
	FD	3	3	3	**0**	943	5	5	5	1425K	14928
	APA	2	2	1	**0**	**205**	5	5	5	1510K	50758
	MU	3	5	5	**0**	2843	5	5	5	1895K	37381
G2	MI	5	3	3	**0**	31464	5	3	3	**1681K**	31975
	FD	5	5	5	11	36444	X	X	X	X	X
	APA	3	5	5	5	4886	X	X	X	X	X
	MU	2	5	5	7	76347	X	X	X	X	X
G3	MI	5	5	5	**0**	36298	5	5	5	**1128K**	38537
	FD	4	1	1	9	871	X	X	X	X	X
	APA	5	3	3	8	5825	X	X	X	X	X
	MU	3	5	5	**0**	**3263**	5	3	3	1919K	5436
G4	MI	4	4	1	**0**	**6063**	4	4	1	**1802K**	6223
	FD	5	5	5	1	51714	X	X	X	X	X
	APA	5	3	3	12	13461	X	X	X	X	X
	MU	5	5	5	**0**	66042	5	5	5	2417K	X



(a) (b)

Fig. 5. The performance of the branching heuristics for $\delta = 5$, $\alpha = 3$, and $\beta = 3$ on problem G1 on the AC power flow model in terms of search nodes (a) and CPU time (b).

portions of the search tree space. Second, in the mid-range, APA is the best, however, MI is typically best in the long run. In short, MI does not direct the search into bad regions of the search space (where α and β are invoked) as often as the other heuristics and is able to perform a more complete and productive search for a given δ. In terms of CPU time, as seen in (b), the results are similar. MU is seen to be very efficient, but has a sparse search tree. These results suggest a mixed strategy, where MU is used to find a high quality solution quickly, followed by a switch to MI to refine the results.

Finally, it is important to consider the impacts of varying α and β on the search. Generally speaking, the smaller of α or β dominates the pruning, so this figure only considers results for $\alpha = \beta$. It is clear that allowing α and β to increase up to 3 provides benefits, however, beyond that point there is limited improvement and increasing δ is more productive. This provides additional evidence that pruning based upon α and β leads to better results empirically.

Table VIII compares the quality of the best results for κ discovered for the DC power flow model and the AC power flow model. Note that the cost of expansion under the AC power flow model can be as much as 5.1 times more expensive (solution G3) than the DC power flow model. This indicates that under constrained planning scenarios, the plans obtained under the DC approximation underestimate the required costs considerably. The solutions are shown in Table IX. For these solutions, no shunt compensation was added to bus 6.

It is important to note that the large additional expense is a product of the problems being over-constrained (no dispatching, the DC solution having nearly maximized reinforcements in many corridors, and limited options for adding shunt compensation). If the constraints are relaxed, the expenses drop considerably. For example, if compensation is allowed at all nodes up to 300 MVar in 100 MVar increments (inductive and capacitive), the solutions are considerably less expensive. Table X provides the results. The solution to problem G1 adds 200 MVar of compensation to buses 6 and 18. Solutions to problems G2, G3, and G4 add 200 MVar in compensation at bus 6 (used in capacitive mode). Under this scenario, the most expensive addition is now roughly a factor of 4 (G3).

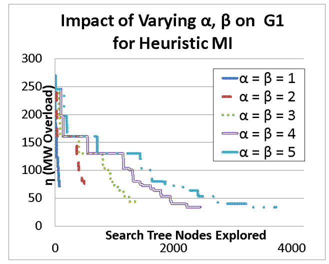


Fig. 6. The performance of MI for $\delta = 3$ and varying α and β on problem G3 for the AC power flow model.

TABLE VIII
COMPARISON OF THE BEST COST DC AND AC SOLUTIONS TO BENCHMARKS OF [27]. THE COLUMN LABELED η PROVIDES THE OVERLOAD FOR THE DC PLAN WHEN SOLVED USING THE AC MODEL. THE COLUMN LABELED ν PROVIDES THE CUMULATIVE VOLTAGE VIOLATIONS FOR THE DC PLAN WHEN SOLVED USING THE AC MODEL. IN BOTH COLUMNS THE NUMBER IN PARENTHESIS IS THE NUMBER OF CORRIDORS AND BUSES IN VIOLATION, RESPECTIVELY.

Problem	DC Best κ	η	ν	AC Best κ
G1	390K	269.60 (3)	.0267 (3)	1303K
G2	424K	197.28 (3)	.0787 (3)	1681K
G3	218K	267.08 (3)	.1585 (3)	1128K
G4	354K	158.23 (2)	.0414 (3)	1802K

We also tested DBLS on the RTS problem of reference [9], which is based on the G0 problem of [27]. This problem has excess generation capacity that can be dispatched and allows for unlimited compensation. We were able to achieve the same solution as reference for [9] for $\delta = 1$ and found no better solution for $\delta = 5$. Interestingly, the AC solution with dispatching and unlimited compensation is considerable less costly than the DC solutions for similar problems G1–G4. This also provides evidence that the loosening constraints reduces the cost impact of AC power flows.

New Mexico Test Case Finally, we tested the approach on a problem based on a real system. This problem comes from the

TABLE IX
SOLUTIONS TO THE RTS-79 BENCHMARKS FOR AC POWER FLOW MODEL

Line	G1	G2	G3	G4	Line	G1	G2	G3	G4
(1,2)	2	1	2	2	(9,12)	1	1	3	2
(1,3)	1	3	0	1	(11,13)	1	2	1	2
(1,5)	2	1	0	2	(12,13)	0	0	1	0
(2,4)	1	0	1	1	(14,16)	1	2	1	2
(2,6)	3	3	3	3	(15,16)	1	0	0	0
(3,9)	0	1	1	0	(15,21)	0	1	0	1
(3,24)	3	3	0	3	(15,24)	1	2	0	2
(5,10)	2	3	3	3	(16,17)	2	2	1	1
(6,7)	3	3	3	3	(16,19)	1	0	2	0
(6,10)	1	1	2	1	(17,18)	1	2	0	0
(7,8)	3	2	3	3	(18,21)	1	0	0	1
(8,9)	1	2	2	2	(19,23)	0	0	0	1
(8,10)	2	1	1	1	(20,23)	0	0	1	0
(9,11)	1	2	2	3	(21,22)	0	0	0	1

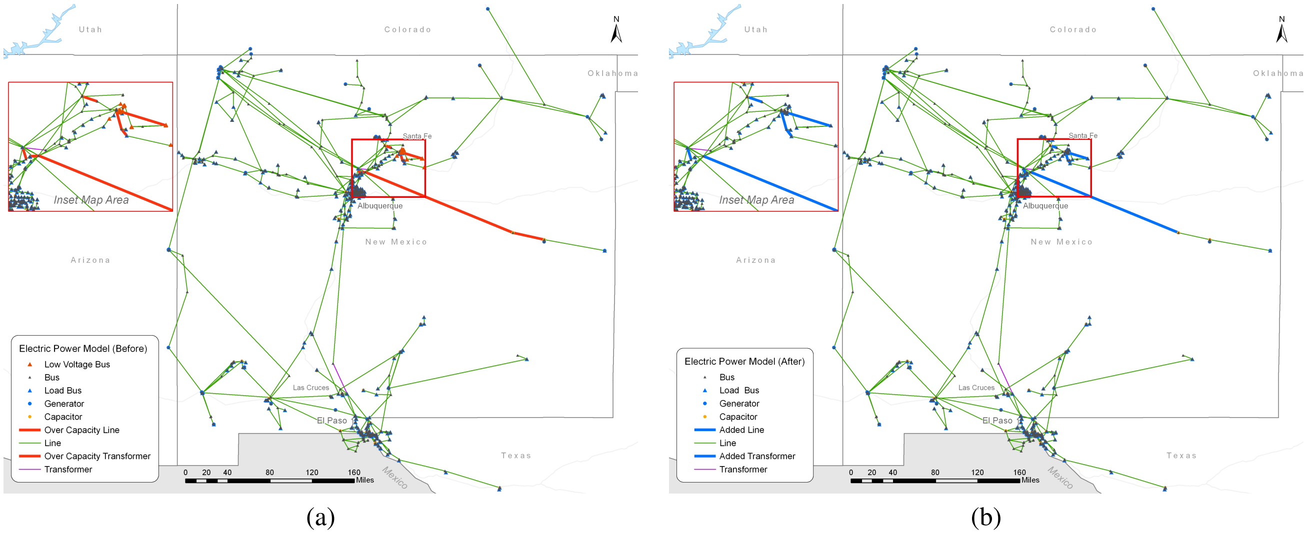


Fig. 7. New Mexico expansion example for 2020 renewable energy penetration. (a) shows the network state after 2020 load growth and renewable generation are added to the existing system. Lines colored red (dark gray) are over capacity. Triangles marked in red (dark gray) are buses with voltage problems. (b) shows the network after expansion. Lines colored in blue (dark gray) indicate where lines were added.

TABLE X
SOLUTIONS TO THE RTS-79 BENCHMARKS FOR AC POWER FLOW MODEL WITH ADDITIONAL SHUNT COMPENSATION OPTIONS.

Line	G1	G2	G3	G4	Line	G1	G2	G3	G4
(1,3)	0	0	2	0	(10,12)	1	1	2	0
(1,5)	2	1	0	2	(11,13)	0	2	1	1
(2,4)	1	0	1	0	(12,13)	1	0	0	0
(2,6)	0	1	0	0	(12,23)	0	0	1	0
(3,9)	0	1	0	0	(14,16)	1	2	0	2
(3,24)	3	2	0	2	(15,16)	1	0	0	0
(6,7)	0	0	1	0	(15,21)	1	1	0	0
(6,10)	2	2	3	2	(15,24)	1	1	0	1
(7,8)	3	2	2	2	(16,17)	1	2	1	1
(8,9)	0	0	2	0	(16,19)	1	0	2	0
(8,10)	0	0	2	0	(17,18)	1	2	0	0
(9,11)	0	2	2	0	(20,23)	0	0	1	0
(10,11)	0	0	0	2					
	733K	911K	877K	592K					

transmission system of the state of New Mexico. This problem is an order of magnitude larger than problems traditionally considered in the TNEP literature, with 900 buses and 1020 transmission corridors. The results on this scenario provide some evidence of the scalability of DBLS. The problem uses the demand and renewable generation penetration assumptions of [3]. Under the DC power flow model there are 840 MW in overloads spread across 27 transmission corridors. The cost to upgrade is roughly $60 million, using the expansion cost estimates of [38]. The upgrades include adding 25 lines in 20 different corridors.

Under the AC power flow model, there are 1450 MVA in overloads, spread across 29 transmission corridors. There are also 15 buses with voltages below .9 pu. The cost to upgrade is roughly $115 million, also using the expansion cost estimates of [38] (almost twice as much as the DC model). Figure 7 shows the network before expansions are applied and where expansions are added to remove the physical violations. This solution puts 78 additional lines in 28 corridors.

VI. CONCLUSION

The electric power system is undergoing a revolutionary transformation that requires new approaches for solving the TNEP. Increased desire and the need to incorporate sustainable power generation that is less controllable, such as wind and solar, creates a situation where nonlinear flows must be accounted for when evaluating plans. We have shown that DBLS is a powerful approach for solving problems with non-linear representations. It relies on encapsulating portions of the problem's model as a black box, similar to simulation optimization. The core algorithmic contribution of this paper is a general search procedure that achieves solutions to the TNEP using linear and non-linear flow equations.

Given, the success of the approach described in this paper, it will be interesting to explore how to further exploit $\mathcal{S}$, especially when $\mathcal{S}$ fails to converge. It will also be important to account for uncertainty in the planning process as described in [39], [40], [41], in particular as it relates to the intermittent output of renewable energy. It will also be interesting to generalize DBLS to handle non-lexicographic, multi-objective functions such as those developed in [31], [42]. Finally, it will be important to the study the effects on solution quality when dispatchable generation, load management, or other types of control are included in the models of power systems [43]. This may reduce the need for expansion.

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APPENDIX

In this section we provide formal descriptions of the implementations of CHOOSEVARIABLE discussed in Section IV. For ease of presentation, $\mathcal{E}(\mathcal{X})$ is used to denote those corridors that have circuit variables in $\mathcal{X}$, i.e., $\bigcup i,j \in \mathcal{E} \mid c_{i,j} \in \mathcal{X}$. We use $\mathcal{B}(\mathcal{X})$ to denote those buses that have shunt compensation variables in $\mathcal{X}$, i.e., $\bigcup i \in \mathcal{B} \mid c_i \in \mathcal{X}$.

MU is described formally in Figure 8. MU chooses the corridor with the most utilized capacity (line 2 of Figure 8). If no circuit variables exist, the function chooses the bus with the lowest voltage to add or subtract shunt compensation (lines 4–5).

CHOOSEVARIABLE-MU$(\mathcal{X}, \sigma)$
1 **if** $|\mathcal{E}(\mathcal{X})| > 0$
2   **then** $i,j \leftarrow \arg\max_{i,j \in \mathcal{E}(\mathcal{X})} |F_{i,j}| - \psi_{i,j}\sigma(c_{i,j})$;
3   **return** $c_{i,j}$;
4 $i \leftarrow \arg\min_{i \in \mathcal{B}(\mathcal{X})} v_i$;
5 **return** c_i;

Fig. 8. Maximum Utilization (MU) Branching Heuristic

FD is described in Figure 9. FD heuristic first looks for a circuit (i,j) that is overloaded (lines 1–3) and expands on the variable that is more overloaded (if it exists). Line 4 creates a temporary set of edges $(\hat{\mathcal{E}})$ to consider for flow diversion. It then iteratively considers overloaded circuits and chooses the expansion variable of a transmission corridor that could divert flow away from the overloaded circuits (lines 5–12). Line 6 determines the most overloaded transmission corridor remaining in $(\hat{\mathcal{E}})$. Line 7 collects the buses $(\hat{\mathcal{B}})$ that are within n corridors (hops) of the overloaded circuit (where n is a user parameter), using the function NEIGHBORS. Line 8 constructs the set of all transmission corridors formed by the cross product of $(\hat{\mathcal{B}})$, which is intersected with the set of available expansion variables. The expansion variable of the circuit with the highest flow diversion value, F, is chosen (line 10), where F is calculated by

$$F(i,j) = \begin{cases} i = \mathcal{F}(i,j) \textbf{ or } j = \mathcal{T}(i,j) & |F_{i,j}| \quad (1) \\ j = \mathcal{F}(i,j) \textbf{ or } i = \mathcal{T}(i,j) & -|F_{i,j}| \quad (2) \\ i \in \mathcal{B}_{\mathcal{F}} \textbf{ and } j \in \mathcal{B}_{\mathcal{F}} & -|F_{i,j}| \quad (3) \\ i \in \mathcal{B}_{\mathcal{T}} \textbf{ and } j \in \mathcal{B}_{\mathcal{T}}, & -|F_{i,j}| \quad (4) \\ i \in \mathcal{B}_{\mathcal{F}} \textbf{ or } j \in \mathcal{B}_{\mathcal{T}} & |F_{i,j}| \quad (5) \\ j \in \mathcal{B}_{\mathcal{F}} \textbf{ or } i \in \mathcal{B}_{\mathcal{T}} & -|F_{i,j}| \quad (6) \end{cases}$$

This function favors corridors that conduct lots of power away from the topological neighborhood of the overloaded corridor. For example, equation (5) favors corridors that move power out of the neighborhood of $\mathcal{T}(i,j)$. Intuitively, if outgoing corridors other than i,j become more conductive, then some of the power entering the neighborhood may exit the neighborhood on a corridor other than i,j. If no corridor exists (i,j) is removed from $(\hat{\mathcal{E}})$ (line 12) and the process repeats. If no circuit variables exist, the function chooses the bus with the lowest voltage to add or subtract shunt compensation (lines 14–15).

APA is described in Figure 10. This heuristic uses a function EXISTSFLOW(a,b,c,d) that determines if there exists a path of flow from a bus in set a to a bus in set b using corridor c but not corridor d. The wild card $(*,*)$ is used to denote any corridor. It first looks for an overloaded circuit variable (lines 1–3) and selects the one that is most overloaded (if it exists).

CHOOSEVARIABLE-FD$(\mathcal{X}, \sigma)$
1 $i,j \leftarrow \arg\max_{i,j \in \mathcal{E}(\mathcal{X})} |F_{i,j}| - \psi_{i,j}\sigma(c_{i,j})$;
2 **if** $|F_{i,j}| - \psi_{i,j}\sigma(c_{i,j}) > 0$
3   **then return** $c_{i,j}$;
4 $\hat{\mathcal{E}} \leftarrow \mathcal{E}$;
5 **while** $|\hat{\mathcal{E}}| > 0$
6 **do** $i,j \leftarrow \arg\max_{i,j \in \hat{\mathcal{E}}} |F_{i,j}| - \psi_{i,j}\sigma(c_{i,j})$;
7    $\hat{\mathcal{B}} \leftarrow$ NEIGHBORS$(\mathcal{F}(i,j), n) \cup$ NEIGHBORS$(\mathcal{T}(i,j), n)$;
8    $\hat{\mathcal{E}}_{i,j} \leftarrow (\hat{\mathcal{B}} \times \hat{\mathcal{B}}) \cap \mathcal{E}(\mathcal{X})$;
9   **if** $|\hat{\mathcal{E}}_{i,j}| > 0$
10   **then** $\hat{i,j} \leftarrow \arg\max_{\hat{i,j} \in \hat{\mathcal{E}}_{i,j}} F(\hat{i,j})$;
11   **return** $c_{\hat{i,j}}$;
12    $\hat{\mathcal{E}} \leftarrow \hat{\mathcal{E}} \setminus i,j$;
13 $i \leftarrow \arg\min_{i \in \mathcal{B}(\mathcal{X})} v_i$;
14 **return** c_i;

Fig. 9. Flow Diversion (FD) Branching Heuristic

Line 4 creates a temporary set of edges $(\hat{\mathcal{E}})$ to consider for flow diversion.

APA then iteratively considers the overloaded circuits and chooses a circuit (i,j) on an alternate path bringing power from generators to loads downstream from (i,j) (lines 5–13). Line 6 determines the most overloaded transmission corridor remaining in $(\hat{\mathcal{E}})$. Line 7 calculates the loads $(L_{\mathcal{T}(i,j)})$ that are downstream from (i,j). Line 8 collects all the possible generators $(G_{\mathcal{T}(i,j)})$. Line 9 collects all the corridors that have expansion variables and lie on a flow path from $G_{\mathcal{T}(i,j)}$ to $L_{\mathcal{T}(i,j)}$ that does not include (i,j). The expansion variable of the circuit with the greatest flow value is chosen. If no corridor exists, (i,j) is removed from $(\hat{\mathcal{E}})$ (line 12) and the process repeats. If no circuit variables exist, the function chooses the bus with the lowest voltage to add or subtract shunt compensation (lines 14–15).

CHOOSEVARIABLE-APA$(\mathcal{X}, \sigma)$
1 $i,j \leftarrow \arg\max_{i,j \in \mathcal{E}(\mathcal{X})} |F_{i,j}| - \psi_{i,j}\sigma(c_{i,j})$;
2 **if** $|F_{i,j}| - \psi_{i,j}\sigma(c_{i,j}) > 0$
3   **then return** $c_{i,j}$;
4 $\hat{\mathcal{E}} \leftarrow \mathcal{E}$;
5 **while** $|\hat{\mathcal{E}}| > 0$
6 **do** $i,j \leftarrow \arg\max_{i,j \in \hat{\mathcal{E}}} |F_{i,j}| - \psi_{i,j}\sigma(c_{i,j})$;
7    $L_{\mathcal{T}(i,j)} \leftarrow \bigcup_{b \in \mathcal{B} \mid l_b > 0}$ **and** EXISTSFLOW$(\{\mathcal{T}(i,j)\}, \{b\}, (*,*), (i,j))$;
8    $G_{\mathcal{T}(i,j)} \leftarrow \bigcup_{b \in \mathcal{B} \mid g_b > 0}$;
9    $\hat{\mathcal{E}}_{i,j} \leftarrow \bigcup_{\hat{i,j} \in \mathcal{E}(\mathcal{X}) \mid \text{EXISTSFLOW}(G_{\mathcal{T}(i,j)}, L_{\mathcal{T}(i,j)}, (\hat{i,j}), (i,j))}$;
10   **if** $|\hat{\mathcal{E}}_{i,j}| > 0$
11   **then** $\hat{i,j} \leftarrow \arg\max_{\hat{i,j} \in \hat{\mathcal{E}}_{i,j}} |F_{\hat{i,j}}|$;
12   **return** $c_{\hat{i,j}}$;
13    $\hat{\mathcal{E}} \leftarrow \hat{\mathcal{E}} \setminus i,j$;
14 $i \leftarrow \arg\min_{i \in \mathcal{B}(\mathcal{X})} v_i$;
15 **return** c_i;

Fig. 10. Alternate Path Around (APA) Branching Heuristic

MI is described in Figure 11. MI chooses the expansion that improves the current plan the most. In Figure 11, line 1 makes the first assignment $(d_1(y)))$ of each variable in $\mathcal{X}$ and chooses the one that creates an expansion plan with the best objective value.

CHOOSEVARIABLE-MI$(\mathcal{X}, \sigma)$
1 **return** $\arg\min_{y \in \mathcal{X}} f(\sigma \cup [\sigma(y) \leftarrow d_1(y)])$;

Fig. 11. Most Improving (MI) Branching Heuristic

The Minimize Cost heuristic is formally presented in Figure 12, where line 1 chooses the most expensive variable (y) that has been expanded.

CHOOSEVARIABLE-COST$(\mathcal{X}, \sigma)$
1 **return** $y \leftarrow \arg\max_{y \in \mathcal{X} \mid \sigma(c_y) > c_y^-} \kappa_y$;

Fig. 12. Cost Reduction Branching Heuristic