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# The Contour Method: A New Approach in Experimental Mechanics

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## ABSTRACT

The recently developed contour method can measure complex residual-stress maps in situations where other measurement methods cannot. This talk first describes the principle of the contour method. A part is cut in two using a precise and low-stress cutting technique such as electric discharge machining. The contour of the resulting new surface, which will not be flat if residual stresses are relaxed by the cutting, is then measured. Finally, a conceptually simple finite element analysis determines the original residual stresses from the measured contour. The contour method determines a 2-D stress map with a direct calculation whereas other methods require an inverse calculation and only give a 1-D stress profile. This paper reviews the theory and application of the contour method.

Finally, this talk discusses why the contour method is significant departure from conventional experimental mechanics. Other relaxation method, for example hole-drilling, can only measure a one-dimensional (1-D) profile of residual stresses, and yet they require a complicated inverse calculation to determine the stresses from the strain data. The contour method gives a two-dimensional (2-D) stress map over a full cross-section, yet a direct calculation is all that is needed to reduce the data. The reason for these advantages lies in a subtle but fundamental departure from conventional experimental mechanics. The new approach yields many advantages but also requires new assumptions and introduces new errors.

## INTRODUCTION

Residual stresses play a significant role in many material failure processes like fatigue, fracture, stress corrosion cracking, buckling and distortion [1]. Residual stresses are the stresses present in a part free from any external load, and they are generated by virtually any manufacturing process. Because of their important contribution to failure and their almost universal presence, knowledge of residual stress is crucial for prediction of the life of any engineering structure. However, the prediction of residual stresses is a very complex problem. In fact, the development of residual stress generally involves nonlinear material behavior, phase transformation, coupled mechanical and thermal problems and also heterogeneous mechanical properties. Those issues make predicting residual stresses very difficult. Therefore, the ability to accurately quantify residual stresses through measurement is an important engineering tool.

Recently, a new method for measuring residual stress, the contour method [2, 3], has been introduced. In the contour method, a part is carefully cut in two along a flat plane causing the residual stress normal to the cut plane to relax. The contour of each of the opposing surfaces created by the cut is then measured. The deviation of the surface contours from planarity is assumed to be caused by elastic relaxation of residual stresses and is therefore used to calculate the original residual stresses. One of the unique strengths of this method is that it provides a full cross-sectional map of the residual stress component normal to the cross section. The only common methods that can measure similar 2-D stress maps have significant limitations. The neutron diffraction method is nondestructive but sensitive to micro-structural changes, time consuming, and limited in maximum specimen size, about 50 mm, and minimum spatial resolution, about 1mm. Sectioning methods [4] are experimentally cumbersome, analytically complex, error prone, and have limited spatial resolution, about 1 cm. A limitation of the original contour method is that only one residual stress component is determined. Recent developments have extended the contour method to the measurement of multiple stress components [5, 6].

The contour method determines a 2-D stress map with a direct calculation whereas other methods require an inverse calculation and only give a 1-D stress profile. After reviewing the theory and application of the contour

method, this paper examines the fundamental departure from conventional experimental mechanics that allows the contour method to achieve this advantage.

## THEORY

This section reviews the theory for the contour method in order to allow for later discussion of the fundamental differences between the contour method and traditional experimental mechanics.

The contour method [2] shown in Figure 1 is based on a variation of Bueckner's superposition principle [7].

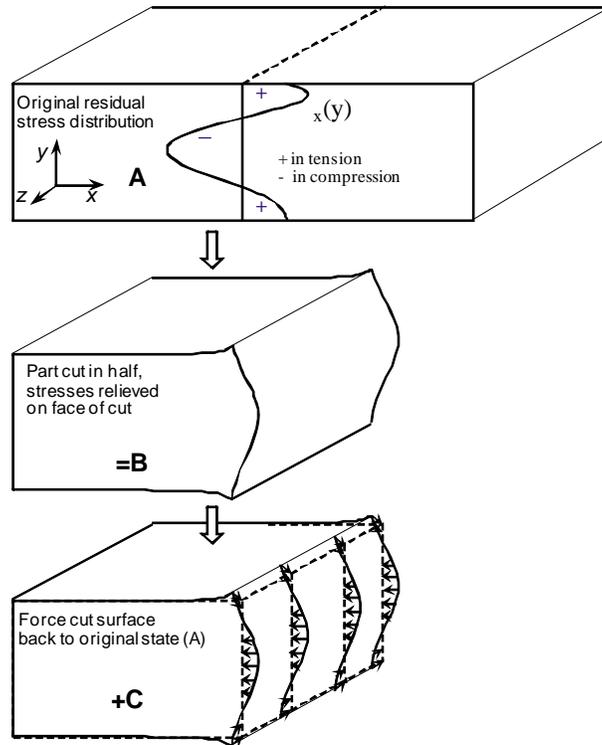


Figure 1. Superposition principle to calculate residual stresses from surface contour measured after cutting the part in two [6].

In **A**, the part is in the undisturbed state including the residual stresses that we desire to determine. In **B**, the part has been cut in two and has deformed as residual stresses were released by the cut. In **C**, the free surface created by the cut is forced back to its original flat shape. Assuming elasticity, superimposing the partially relaxed stress state in **B** with the change in stress from **C** would give the original residual stress throughout the part:

$$\sigma^{(A)} = \sigma^{(B)} + \sigma^{(C)} \quad (1)$$

where  $\sigma$  without subscripts refers to the entire stress tensor.

This superposition principle assumes elastic relaxation of the material and that the cutting process does not introduce stress that could affect the measured contour. With proper application of this principle it is possible to determine the residual stress over the plane of the cut. Experimentally, the contour of the free surface is measured after the cut and analytically the surface of a stress-free model is forced back to its original flat configuration by applying the opposite of the measured contour as boundary conditions. Because the stresses in **B** are unknown, one cannot obtain the original stress throughout the body. However, the normal and shear stresses on the free surface in **B** must be zero ( $\sigma_x$ ,  $\tau_{xy}$  and  $\tau_{xz}$ ). Therefore, **C** by itself will give the correct stresses along the plane of the cut:

$$\begin{aligned}
\sigma_x^{(A)} &= \sigma_x^{(C)} \\
\tau_{xy}^{(A)} &= \tau_{xy}^{(C)} \\
\tau_{xz}^{(A)} &= \tau_{xz}^{(C)}
\end{aligned}
\tag{2}$$

In practice, only the normal stress component  $\sigma_x$ , can be experimentally determined. The experimental measurement of the contour only provides information about the displacements in the normal (x) direction, not those in the transverse directions. Therefore, the surface is forced back to the original flat configuration (step C) in the x-direction only. The shear stresses ( $\tau_{xy}$  and  $\tau_{xz}$ ) are constrained to zero in the solution. This stress-free constraint is automatically enforced in most implicit, structural, finite-element analyses if the transverse displacements are left unconstrained. Even if residual shear stresses were present on the cut plane, averaging the contours measured on the two halves of part still leads to the correct determination of  $\sigma_x$  [2].

The measured surface contour has an arbitrary reference plane, resulting in three arbitrary rigid body motions in defining the surface. These three arbitrary motions are uniquely determined by the need for the stress distribution over the cross section to satisfy three global equilibrium conditions: force in the x-direction and moments about the y and z axes. It is not necessary to explicitly enforce these constraints. In a finite element calculation, appropriate boundary conditions are applied to the cut plane, including three extra constraints to prevent rigid body motions. The remainder of the body is unconstrained. In the static equilibrium step used to solve for stress, the free end of the body will automatically translate and rotate such that the equilibrium conditions are fulfilled.

A small convenience is taken in the data analysis. Modeling the deformed shape of the part for **C** in Figure 1 would be tedious. Instead, the surface is flat in the finite element model, and then the part is deformed into the shape opposite of the measured contour. Because the deformations are quite small, the same answer is obtained but with less effort.

## EXPERIMENTAL IMPLEMENTATION AND VALIDATION

The experimental implementation and validation of the contour method are briefly reviewed here.

Making the cut is the most important experimental aspect of the contour method. The ideal cut would be zero width, introduce no stresses, and allow no plasticity at the tip of the cut. So far, the only method that has been successfully applied for the contour method is wire Electric Discharge Machining (EDM). Wire EDM has been shown to introduce very small stresses when used correctly [8], so it has long been used for residual stress measurements.

Firmly restraining the part during cutting generally improves the accuracy of the contour results. First, the cut plane will move less as stresses are relaxed, which reduces errors. The restraint also minimizes any crack closing or opening load resulting from the stress release. Such loads result in concentrated stresses at the cut tip which can cause plasticity errors [9], and also strain the material ahead of the cut and change the effective cut width. Both of these effects cause errors. Figure 2 shows a special fixture to securely clamp a specimen during EDM cutting. Less complicated fixtures can also be used to obtain good results.

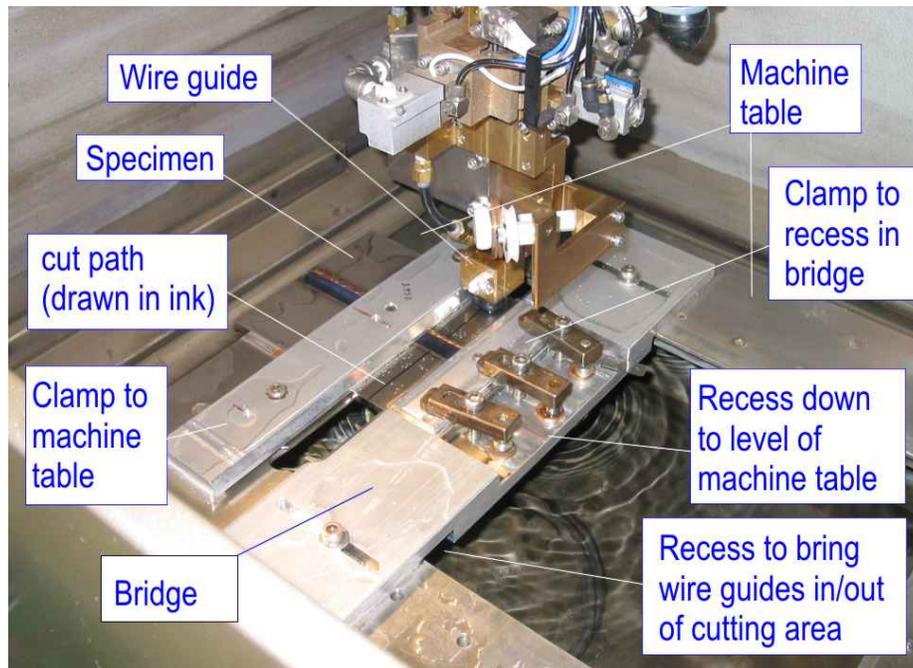


Figure 2. A special fixture to securely clamp a specimen during EDM cutting

After removing the part from the fixtures, the surface contour must be measured. It is relatively straightforward to measure the surface contour with all necessary precision, see Figure 3. Machining (EDM) surface roughness affects the contour and is representative of noise as compared to “signal” caused by stress relaxation. Hence, refining the surface measurement beyond some point only increases your resolution of the noise. Coordinate measuring machines (CMM) are widely available and very effective for measuring the surface contour [2]. Laser scanners can also be used to provide a faster measurement and to not contact the part, and they give similar accuracy for the stresses [3].

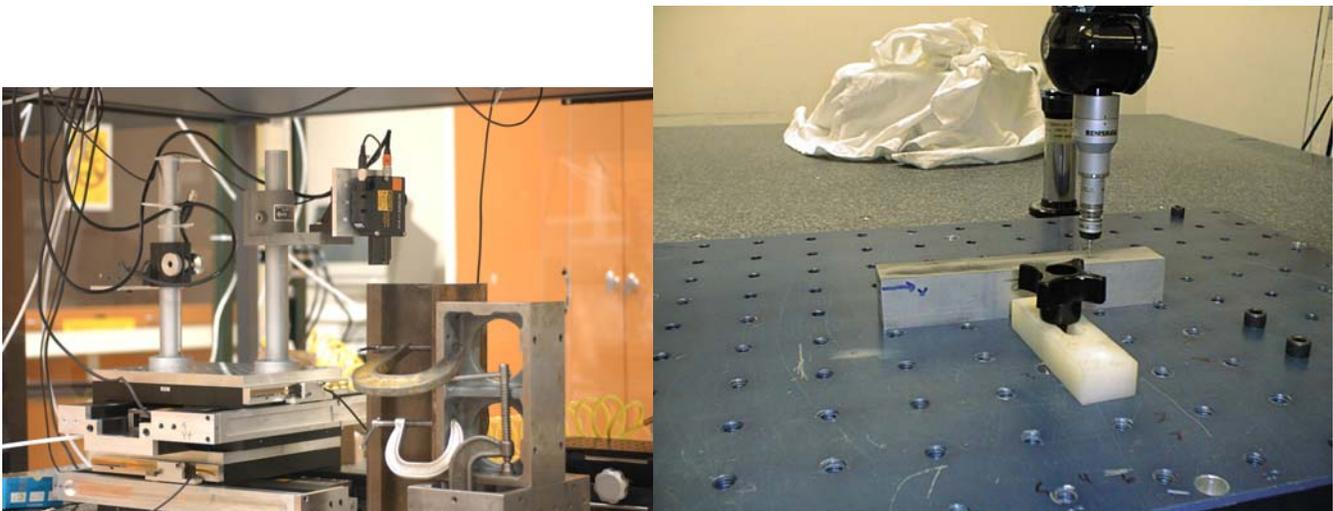


Figure 3. Measuring the surface contour. Left: on a railroad rail with a laser scanner [10].  
Right: on a friction stir weld with a CMM

The data is numerically processed to align the data from the two halves of the part, average the two halves, smooth the data, and evaluate the smoothed surface at the location of the nodes in the finite element mesh [3, 11]. An elastic, static-finite-element simulation that displaces the cut surface into the opposite of the measured contour is used to calculate the stresses. Figure 4 shows a finite element model that has been deformed into the opposite shape of the measured contour and the resulting stress.

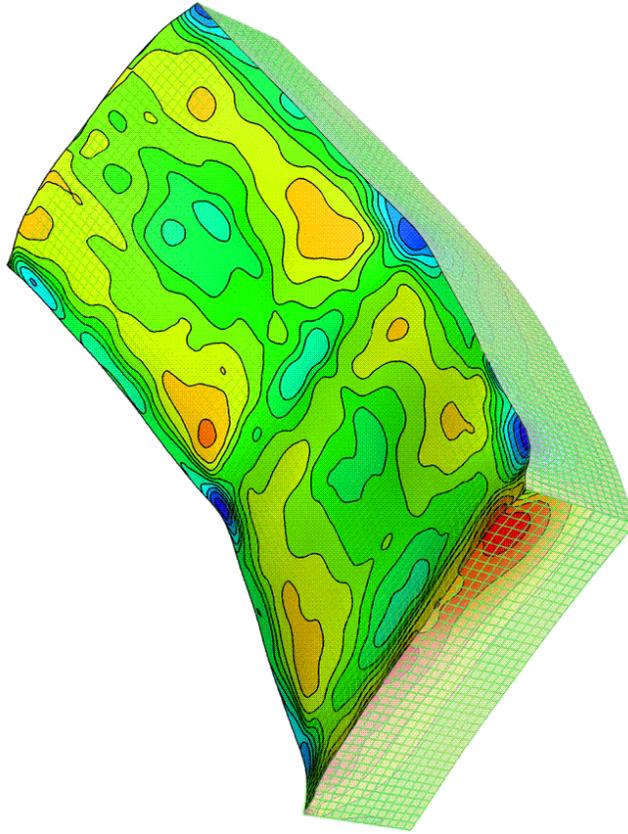


Figure 4. A finite-element model deformed into the opposite shape of the contour measured on the cut surface. Example for an aluminum forging [12]. Displacements are exaggerated by a factor of 200.

The contour method has been extensively validated by comparing with known stresses [2], with stresses measured by neutron diffraction [3, 11, 13-18], synchrotron diffraction [11, 18, 19], and sectioning [20].

## EXPERIMENTAL MECHANICS

**Comparison with traditional methods.** At least in principle, the contour method measures more than other methods for less effort. The “more” refers to measuring a 2-D stress map as compared to a 1-D stress profile. The “less” refers to using a direct calculation to reduce the data to stress as compared to solving an elastic inverse problem. The comparison here is limited to relaxation methods for measuring residual stress.

The majority of conventional relaxation methods follow the same model for stress measurement. Figure 5 illustrates the common examples of layer removal, incremental hole drilling, and incremental slitting (crack compliance). In these relaxation methods, material is incrementally removed, which causes stress relaxation by the creation of free surface. The deformation resulting from the relaxation is measured on some convenient free surface. Assuming elastic stress relaxation, the deformation from a known stress profile could be written as

$$\epsilon(a) = \int_0^a \sigma(x) A(a, x) dx \quad (3)$$

Where  $a$  is the depth of material removal in the  $x$  direction and  $A$  is a function of geometry and material properties. In the general case,  $A$  is not analytic and is determined from finite element calculations. Therefore, the equation cannot be inverted, and the stress profile  $\sigma(x)$  can only be determined using inverse calculations. To simplify the concept: an inverse calculation requires making some guess about the stress profile and then

adjusting the guess to best approximate the data. The inverse calculations, such as the integral and series expansion methods [21], can be unstable and error prone.

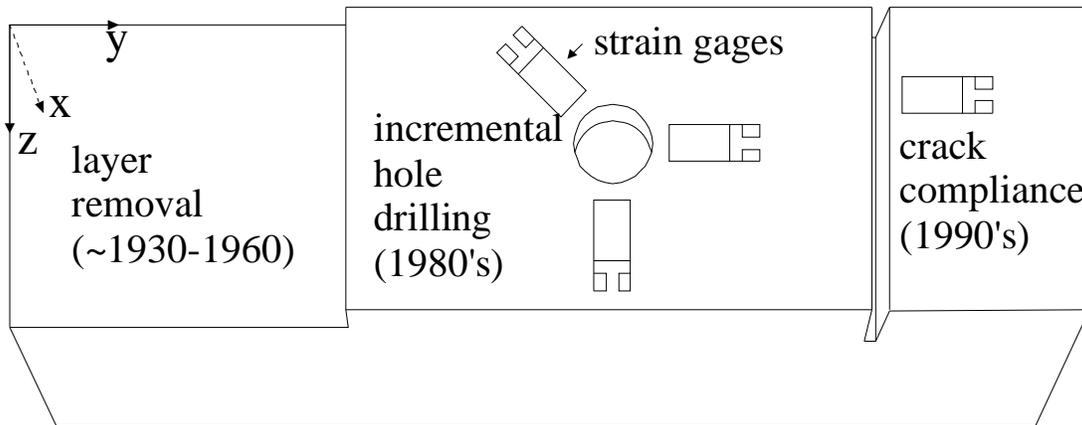


Figure 5. Conventional relaxation methods measure residual stress through incremental material removal.

The contour method uses a direct, although non-analytic, solution. Figure 1 shows that the calculation involves displacing the surface by a known amount. There is no guesswork involved. Because an analytic solution is generally not possible, the elastic boundary value problem is solved using straightforward application of the finite element method.

**Departure from Traditional Methods.** One might guess that the advanced capabilities of the contour method result from the extension from measuring a continuous data map as compared to discrete data. Figure 6 illustrates that traditional methods such as slitting or hole drilling use deformation data measured at a finite number of discrete locations, such as with strain gauges. Because the measurement of surface contour is not practically limited in the number of points taken, the contour method effectively uses a continuous map of data. Although the map of data is necessary in order to get a map of stresses, it is not able to explain all of the advances of the contour method. Two-dimensional data maps have been used with the hole-drilling method to increase accuracy [22] or for experimental simplicity [23] but still only resulting in a 1-D depth profile of stresses. With more extensive assumptions and analysis such data can give stress variation around the hole circumference as well [24], but the inverse solution is more complicated and less stable and accurate.

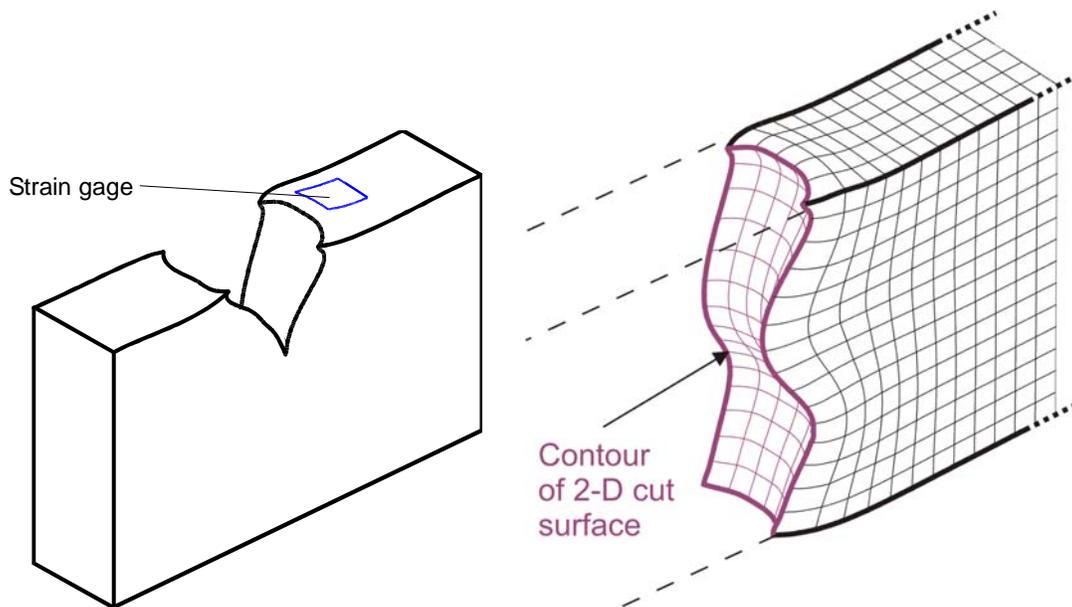


Figure 6. Traditional methods (left) use discrete deformation data and the contour method uses a continuous map.

In summary, the measurement of a 2-D map of data is necessary but not sufficient to get a 2-D map of stresses using a direct calculation.

The first part of the fundamental difference of the contour method is the measurement location, as shown in Figure 7. Traditional methods measure deformation at *pre-existing* free surface. By definition, such a location is *remote* from the location of full stress relief which is the new free surface created by the material removal. The amount of stress relaxation and, therefore, deformation at the remote location is a complicated function of geometry. Conceptually, the partial stress relief is embodied by  $A$  in Equation 2. Where the contour measures data (at the new free surface of the cut) the stress relaxation is guaranteed to be 100% because of the free surface condition. It is analogous to having  $A = 1$  in Equation 1 and making the equation invertible.

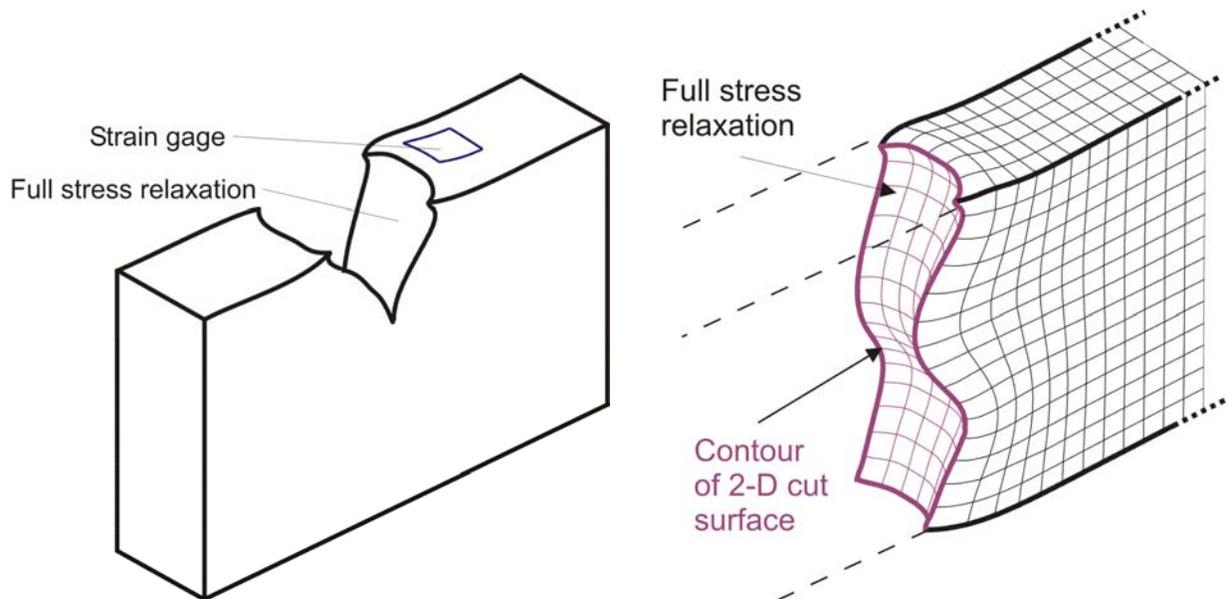


Figure 7. Conventional methods measured deformation remote from the location of stress relief. The contour method measures it at the location of full stress relief.

The second part of the fundamental difference of the contour method allows the first difference and marks the significant departure from traditional experimental mechanics. The difference is the quantity that is measured. The contour method uses a shape measurement as compared to deformation. Shape requires no reference state. Traditional deformation measurements require a pre-existing free surface in order to get the reference state for the deformation. The reference state on the cut plane is not accessible prior to the cut.

The new measurement approach requires significant new assumptions. In order to infer deformation from a shape measurement, one must make assumptions about the reference state. The simplest assumption would be that the cut surface was originally flat and that the measured shape equals the deformations plus a possible planar reference offset. That assumption turns out to be overly restrictive for the contour method. For example, in the case of a crooked cut, averaging the contours measured on the two halves removes the effect of a crooked cut.

The real assumption for the contour method is that the cut is finite width with respect to the state of the body prior to any cutting. Figure 8 illustrates the issue. As the cutting proceeds, stresses relax and the material at the tip of the cut deforms. The material at the cut tip that was originally  $w$  wide has stretched. However, the cut will still be only  $w$  wide, which means that the cut width has been reduced when measured relative to the original state of the body. The effect is greatly reduced by securely restraining the part during cutting to minimize deformations. The errors can generally be kept under 5%. Also, because the effect is elastic, it can be approximated using FEM and then the error corrected.

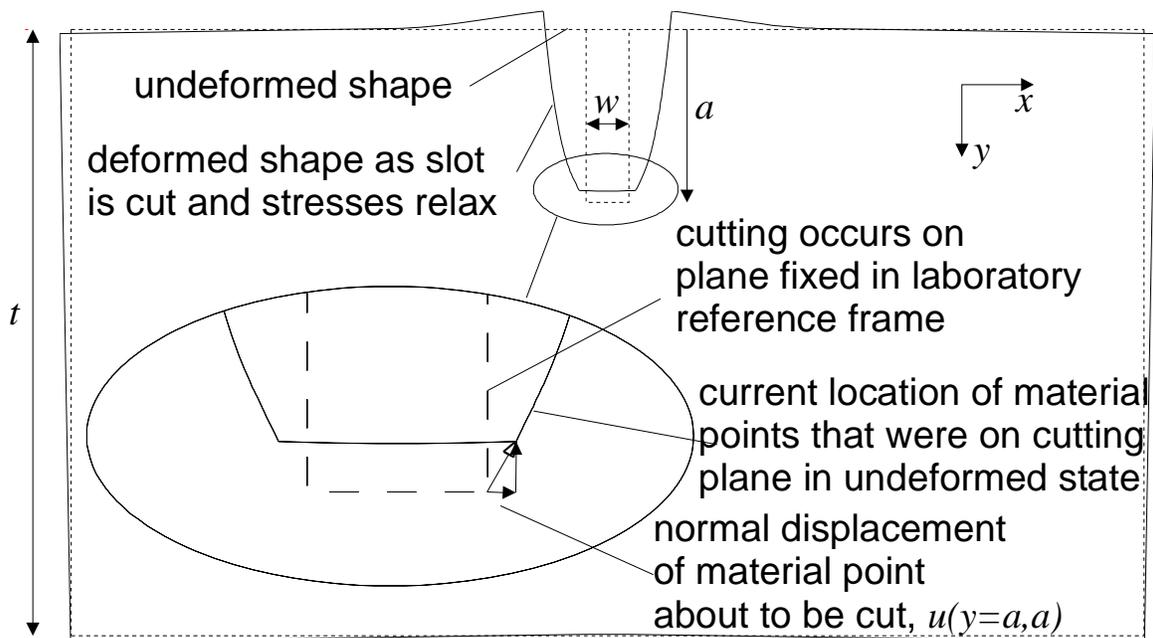


Figure 8. As cutting proceeds, the material at the tip of the cut deforms from stress relief. This changes the width of the cut relative to the original state of the body and causes errors.

## CONCLUSION

The contour method determines a 2-D map of residual stresses using a direct calculation to get stress from the measured surface contours. Many conventional methods require an inverse calculation and only get a 1-D stress profile. The contour method accomplishes this advance by departing from the traditional experimental mechanics approach. Conventional methods measure deformations on pre-existing free surfaces that are by definition remote from the location of complete stress relief. The contour method takes data on the surface created by the cut. On this surface, the stress is fully relaxed. Because that surface is not accessible prior to the cut, it is not possible to measure a reference state. Instead, the contour method measures the shape of the free surface. Without the reference state, inferring deformation from a measured shape requires assumptions that are unfamiliar for experimental mechanics. These new assumptions provide theoretical and experimental challenges. The challenges open up possibilities to further improve the accuracy and reliability of the contour method.

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## REFERENCES

- [1] Withers PJ (2007) Residual Stress and Its Role in Failure. Reports on Progress in Physics 70:2211-2264.
- [2] Prime MB (2001) Cross-Sectional Mapping of Residual Stresses by Measuring the Surface Contour after a Cut. Journal of Engineering Materials and Technology 123:162-168.
- [3] Prime MB, Sebring RJ, Edwards JM, Hughes DJ, Webster PJ (2004) Laser Surface-Contouring and Spline Data-Smoothing for Residual Stress Measurement. Experimental Mechanics 44:176-184.

- [4] Schajer GS (2001) "Residual Stresses: Measurement by Destructive Testing." Encyclopedia of Materials: Science and Technology, Elsevier, 8152–8158.
- [5] DeWald AT, Hill MR (2006) Multi-Axial Contour Method for Mapping Residual Stresses in Continuously Processed Bodies. *Experimental Mechanics* 46:473-490.
- [6] Pagliaro P, 2008, "Mapping Multiple Residual Stress Components Using the Contour Method and Superposition," Ph.D. Dissertation, Università degli Studi di Palermo, Palermo.
- [7] Bueckner HF (1973) "Field Singularities and Related Integral Representations." *Mechanics of Fracture G. C. Sih, ed.*, 239-314.
- [8] Cheng W, Finnie I, Gremaud M, Prime MB (1994) Measurement of near-Surface Residual-Stresses Using Electric-Discharge Wire Machining. *Journal of Engineering Materials and Technology-Transactions of the ASME* 116:1-7.
- [9] Shin SH (2005) Fem Analysis of Plasticity-Induced Error on Measurement of Welding Residual Stress by the Contour Method. *Journal of Mechanical Science and Technology* 19:1885-1890.
- [10] Kelleher J, Prime MB, Buttle D, Mummery PM, Webster PJ, Shackleton J, Withers PJ (2003) The Measurement of Residual Stress in Railway Rails by Diffraction and Other Methods. *Journal of Neutron Research* 11:187-193.
- [11] Johnson G, 2008, "Residual Stress Measurements Using the Contour Method," Ph.D. Dissertation, University of Manchester.
- [12] Prime MB, Newborn MA, Balog JA (2003) Quenching and Cold-Work Residual Stresses in Aluminum Hand Forgings: Contour Method Measurement and Fem Prediction. *Materials Science Forum* 426-432:435-440.
- [13] Kartal ME, Liljedahl CDM, Gungor S, Edwards L, Fitzpatrick ME (2008) Determination of the Profile of the Complete Residual Stress Tensor in a VPPA Weld Using the Multi-Axial Contour Method. *Acta Mat.* 56:4417-4428.
- [14] Pagliaro P, Prime MB, Clausen B, Lovato ML, Zuccarello B (accepted) Known Residual Stress Specimens Using Opposed Indentation. *Journal of Engineering Materials and Technology*
- [15] Withers PJ, Turski M, Edwards L, Bouchard PJ, Buttle DJ (2008) Recent Advances in Residual Stress Measurement. *The International Journal of Pressure Vessels and Piping* 85:118-127.
- [16] Bouchard PJ (2007) Code Characterisation of Weld Residual Stress Levels and the Problem of Innate Scatter. *International Journal of Pressure Vessels and Piping* 85:152-165.
- [17] Kartal M, Turski M, Johnson G, Fitzpatrick ME, Gungor S, Withers PJ, Edwards L (2006) Residual Stress Measurements in Single and Multi-Pass Groove Weld Specimens Using Neutron Diffraction and the Contour Method. *Materials Science Forum* 524/525:671-676.
- [18] Zhang Y, Ganguly S, Edwards L, Fitzpatrick ME (2004) Cross-Sectional Mapping of Residual Stresses in a VPPA Weld Using the Contour Method. *Acta Mat.* 52:5225-5232.
- [19] Evans A, Johnson G, King A, Withers PJ (2007) Characterization of Laser Peening Residual Stresses in Al 7075 by Synchrotron Diffraction and the Contour Method. *Journal of Neutron Research* 15:147-154.
- [20] Richter-Trummer V, Tavares SMO, Moreira P, de Figueiredo MAV, de Castro P (2008) Residual Stress Measurement Using the Contour and the Sectioning Methods in a Mig Weld: Effects on the Stress Intensity Factor. *Ciência & Tecnologia dos Materiais* 20:114-119.
- [21] Schajer GS (1988) Measurement of Non-Uniform Residual-Stresses Using the Hole-Drilling Method .1. Stress Calculation Procedures. *Journal of Engineering Materials and Technology* 110:338-343.
- [22] Schajer GS, Steinzig M (2005) Full-Field Calculation of Hole Drilling Residual Stresses from Electronic Speckle Pattern Interferometry Data. *Experimental Mechanics* 45:526-532.
- [23] Nelson DV, Makino A, Schmidt T (2006) Residual Stress Determination Using Hole Drilling and 3D Image Correlation. *Experimental Mechanics* 46:31-38.
- [24] Ya M, Dai FL, Lu J (2003) Study of Nonuniform in-Plane and in-Depth Residual Stress of Friction Stir Welding. *Journal of Pressure Vessel Technology-Transactions of the ASME* 125:201-208.