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LINEAR AND NONLINEAR METHODS FOR DETECTING CRACKS IN BEAMS

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ABSTRACT. *This paper presents experimental results from the vibration of a polycarbonate beam containing a crack that opens and closes during vibration. Several techniques were employed to detect and locate the crack making use of the nonlinearity. "Harmonic mode shapes" proved to be more sensitive to damage than conventional mode shapes. Instantaneous frequency and time-frequency methods also showed clear signatures for the crack. The results indicate that nonlinearities may provide increased capabilities for structural damage detection and location.*

1. INTRODUCTION

A desire to monitor a structure and detect damage at the earliest possible stage is pervasive throughout the civil, mechanical, and aerospace engineering communities. Investigators have applied modal analysis techniques to damage detection for more than 20 years. A recent review of this literature is given in [1]. The vast majority of the work has considered only linear types of structural damage, such as a local reduction in stiffness, a non-closing notch, or a change in geometry like removal of a member in a truss structure. Actual structural damage is often nonlinear, such as a fatigue crack that opens and closes. Only a handful of works have looked at making use of the nonlinearities for damage detection, e.g. [2-5]. It is hoped that the nonlinearity will impart some characteristic to the vibration signal that could result in a damage detection technique with improved sensitivity and/or spatial resolution.

This paper presents experimental results from the vibration of a beam containing an opening and closing crack. Several methods of examining the data are presented including conventional modal analysis compared to extraction of "harmonic mode shapes." Methods using instantaneous frequencies and time frequency transforms, which have only recently been applied to damage detection [6-7], are also applied. All the results presented here are based on measured responses only. No measurements of the input force or excitation were taken because a damage detection algorithm to be implemented in the field will likely need to work with only response measurements.

2. EXPERIMENTAL SETUP

A polycarbonate beam was constructed by bonding three pieces together in order to form a beam with an opening and closing crack. Figure 1 shows a finite element prediction for the first eigenmode, which is shown just to illustrate the geometry. It is noted that the bonding was not done especially well and so in addition to the crack there is a delaminated region near the crack. An uncracked beam was constructed by bonding together the two halves, in order to restrict the difference between the two beams to the crack region. The final dimensions of the beams were 61 cm (24 in.) long by 5.1 cm (2.0 in.) wide by 1.21 cm (0.475 in.) thick. The crack was located 24.9 cm (9.8 in.) from one end and the delaminated region extended about 3 cm (1.2 in.) towards the closer end and 4.8 cm (1.9 in.) towards the other end. The beam was constructed from pieces of equal thickness, so the crack penetrates to half the beam thickness.

For testing, about 1.3 cm (0.5 in.) of the end of the beam was clamped into a vise. Eight Endevco model 2250A-10 accelerometers (0.4 grams each) were mounted using wax along the centerline of the beam starting at 1.27 cm (0.5 in.) from the free end and spaced 7.62 cm (3.0 in) apart. The cracked beam was tested alternately with both ends clamped in the vice, making two effective crack locations: 23.6 cm (9.3 in.) and 34.8 cm (13.7 in.) from the clamp. The beams were excited by displacing the end of the beam by 1-2 cm and then releasing it, i.e., step relaxation. This excited predominantly the fundamental mode. In the case of the cracked beam, the experiments were repeated with the accelerometers on the same side as the crack and on the opposite side, to see if this had any affect on the ability to detect the crack.

Measurements included raw time histories, power spectra and cross power spectra. The spectra calculated using an exponential window and averaged over 10 runs. The sampling rate gave 4096 points in 8 seconds.

3. RESULTS — FREQUENCY DOMAIN

Figure 2 shows a portion of the time history for the accelerometer nearest the free end on the cracked beam. The closing event itself is apparent as the acceleration approaches its minimum. The crack closing well after the acceleration has passed through zero indicates an initial gap which must be overcome. The initial gap is visually apparent and varies from 0.2 mm to 0.3 mm (0.008 in. to 0.012 in.) along the length of the crack.

The trace clearly shows an asymmetry associated with the crack being closed for part of the cycle. This behavior can be approximated as being bilinear. For the portion of the cycle in which the crack is closed, the beam vibrates linearly with the modal properties of the uncracked beam. For the portion of the cycle with the crack open, the beam also vibrates linearly, but with the modal properties of the beam with reduced local stiffness due to the crack. These regions are referred to here as the "crack-open" and "crack-closed" half-cycles, although due to the initial gap the crack is closed for less than a full half-cycle.

Figure 3 shows the power spectra for the free end accelerometer on both a cracked and an uncracked beam. The cracked beam shows a frequency shift for the fundamental frequency and the presence of several harmonics. Note that one of the harmonics overlays the second bending mode. However, the second mode was barely excited so this peak appears to be predominantly due to the harmonic.

Table 1 gives the frequencies for the first two bending modes and the harmonics in between for the different tests. Frequencies are given to the resolution of the power spectra, 0.125 Hz. No effort was made to interpolate for finer resolution. For the harmonics, x4 refers to the harmonic at 4 times the fundamental frequency and 2-1 refers to a harmonic at the frequency of the second mode minus the frequency of the first mode.

	Uncracked Beam	Crack at 23.6 cm	Crack at 34.8 cm
Mode 1	7.25 Hz	6.0 Hz	6.88 Hz
Harm. (x2)		11.88 Hz	13.63 Hz
Harm. (x4)		23.75 Hz	27.25 Hz
Harm. (2-1)			31.0 Hz
Harm. (x5)		29.63 Hz	34.13 Hz
Mode 2	40.5 Hz	35.5 Hz	37.75 Hz
Harm. (x6)		35.5 Hz	40.88 Hz

Table 1. Measured Frequencies

Mode shapes were calculated from the magnitudes of the peaks in the cross power spectra. The cross power spectra were taken relative to the accelerometer closest to the free end. Such a procedure for identifying mode shapes has been previously applied to nonlinear systems, e.g., [8]. The same procedure was applied also to harmonic peaks resulting in "harmonic mode shapes."

Figure 4 shows shapes of the fundamental mode for the uncracked beam and both of the cracked configurations. Lines on the plot show the location of the crack for the two configurations. Although the cracked mode shapes show a change, they do not clearly indicate the location of the crack. Also, the same figure shows the mode shapes for the first harmonic, at twice the fundamental frequency. The data points indicate sensor locations. The harmonic mode shapes very clearly indicate the location of the crack, to within the sensor resolution.

Figures 5 and 6 show the mode shapes for the second bending mode for the two different crack locations. In this case the cracked mode shapes do indicate the location of the crack to within the sensor resolution. Also plotted are the mode shapes for several higher harmonics. The harmonics indicate the position of the crack more clearly than the cracked mode shapes, but not as clearly as those in figure 4. In general, as the order of the harmonics increase, their ability to locate damage decreases. It is also worth noting that the harmonic at twice the fundamental frequency mimics the shape of the fundamental mode while the others mimic the shape of the second bending mode. This occurs in spite of the fact that all these harmonics occur before the second bending mode.

Figure 7 shows the uncracked and harmonic mode shapes for tests with the accelerometers on the side of the beam opposite the crack. The harmonic at twice the fundamental mode does a poor job of locating the crack relative to the case with the accelerometer on the same side as the crack (see Fig. 4). But the higher harmonics still locate the crack to within sensor resolution. Overall, there is clearly a decrease in ability to locate the crack with the accelerometers on the opposite side of the beam.

The power spectrum for the uncracked beam in Figure 3 shows a peak at about 22 Hz that could be a harmonic. This peak only appears in the accelerometer closest to the free end and could indicate some local nonlinearity.

4. RESULTS — HALF-CYCLE FREQUENCY

A very simple approach was used to identify the bilinear vibration of the cracked beam in the time domain. The time history signal (e.g., Fig. 2) was examined to locate the times at which the acceleration passed through zero. The time between these crossings, corresponding to approximately a half period of vibration, was used to calculate a frequency for the half cycle. This was only possible because the beams responded almost purely in mode 1 when excited using step relaxation.

Figure 8 shows this "half-cycle frequency" for the cracked beam. The accelerometer in this case is located 14.5 cm from the crack towards the clamp. Figure 9 shows the same plot for the same accelerometer location on the uncracked beam. The bilinear vibration of the beam is evident in Fig. 8.

The effect diminishes as the beam rings down since the amplitude of vibration is no longer sufficient to close the crack. The effect is also dependent upon accelerometer location. For accelerometers farther from the crack than in Figure 8, the difference between the carck-open and crack-closed frequencies is smaller. This suggests that this effect may be useful in locating a crack as well as detecting it.

It should be noted that these plots do not display the form expected by the authors. The frequency for the crack-closed half of the cycle (higher frequency) would be expected to decay as the vibration amplitude decays because the crack has a finite gap and takes a certain amplitude to close. This is observed. However, the frequency for the crack-open half of the cycle would be expected to stay constant since the crack is open for this entire half cycle irrespective of vibration amplitude. Yet this frequency also changes in time as the beam rings down.

A possible explanation for this observation exists. The time history measurements were taken in AC coupling mode, which uses a high pass filter with a rolloff starting at about 2 Hz. The acceleration trace with the opening and closing crack would have unequal amplitudes for the two halves of the cycle, giving a non zero average acceleration or DC offset. In removing this offset, the high pass filter changes the zero crossing times slightly.

5. RESULTS — INSTANTANEOUS FREQUENCY

Another attempt to resolve the crack opening and closing from the time history data is afforded by instantaneous frequency techniques [9]. The instantaneous frequency is defined as the time derivative of the phase angle of the analytic signal divided by 2π. The analytic signal is computed from the raw signal by keeping only positive frequency components. A real signal will have a Fourier transform symmetric between positive and negative frequencies. The analytic signal zeros the negative frequency components of the signal. The inverse Fourier transform of the raw signal with zero negative frequency components, and consequently the analytic signal, will necessarily be complex. Hence it makes sense to talk about the phase angle and its derivative. For example the analytic signal of a sine wave is a complex exponential.

$$\sin(2\pi ft) \rightarrow ie^{-i2\pi ft}. \tag{1}$$

It is easy to see that if the signal has only a single frequency component the instantaneous frequency does in fact give the frequency of the signal. Interpreting the meaning of the instantaneous phase angle becomes more difficult when the signal has more than a single frequency component. This problem is analogous to interpreting the mean of a multi-modal distribution.

To alleviate this problem, the raw signal for both the cracked and uncracked runs was first low-pass filtered to eliminate frequency components not near the fundamental. A fifth order type II Chebyshev low-pass filter with 60 dB of attenuation and a cutoff of approximately 50 Hz was selected. It should be noted that the instantaneous frequency is very sensitive to the filter parameters. If too low a cutoff frequency is used then the harmonics are filtered out and no bilinear behavior is observed. If the cutoff frequency is chosen too high, then additional modes are included and interpretation becomes difficult.

Figure 10 shows the results of applying the filter discussed in the preceding paragraph to both the cracked and uncracked data runs and then computing the instantaneous frequency. The graphs show several interesting effects. For the cracked run, after an initial transient we see a large approximately square wave oscillating between 5.75 Hz and 7.5 Hz. These can be interpreted as the bilinear frequencies of the beam correspond to the crack open and closed respectively. Also notice how the lower frequency is of shorter time duration corresponding to an initial gap which must be overcome. The upper frequency of the crack is approximately the same as the mean of the instantaneous frequency for the uncracked beam.

There are several departures from the theoretically expected results which deserve comment. The theoretically expected curve for the uncracked beam should be a flat line. The expected curve for the cracked beam should be a square wave. In figure 10 we see a significant ripple for the uncracked frequency and departures from squareness for the cracked frequency. The artifacts can easily be attributed to two causes. The first is the inclusion of higher modes and noise which pass through the filter. The second in the case of the cracked beam is true high order harmonic response being removed by the filter.

6. RESULTS — WIGNER-VILLE DISTRIBUTION

The data was also analyzed using the Wigner-Ville distribution (WVD) [10]. The WVD is a method of determining the energy density of a signal as both a function of time and frequency. The WVD is defined by

$$W_x(t, f) = \int_{-\infty}^{\infty} e^{-2\pi i f\tau}\, x\left(t + \tau/2\right) x^*\left(t - \tau/2\right) d\tau . \tag{2}$$

Because it is bilinear in the signal, the WVD has cross terms, or artifacts, that show spurious frequency content. For example if a signal has two frequency components at f_1 and f_2, then the WVD shows an oscillatory artifact at the mean of the two frequencies. The explanation for the artifact is that when a sine wave is modulated by another sine wave, as the bilinear product in the definition of the WVD necessitates, the result has frequency content at the sum and difference frequencies. In practice the artifacts may be easy to recognize because they typically oscillate in magnitude between positive and negative values.

The problem of artifacts becomes worse if the signal is multi component or broad band because artifacts appear at all possible sum and difference frequencies. The artifacts also appear between positive and negative frequency components of real signals. For this reason it is also typical to use the analytic signal, which was described in the section on instantaneous frequency methods. The analytic signal zeros the negative frequency components and thereby eliminates many of the spurious cross terms in the WVD. A final property of the WVD which should be mentioned is that the distribution can be negative. This is an unfortunate property since we would like an energy density to be positive definite. This is not specific to the WVD. All time frequency methods suffer from analogous if not identical pathologies such as spurious cross terms and negativity [10].

Figures 11 and 12 show the WVD for the cracked and uncracked beams respectively. Both figures show the exponential ring-down in the energy of the beam. The uncracked beam shows frequency content primarily at the first bending mode, consistent with the excitation of the beam being pulled and released which excites only the fundamental mode. In contrast, even though the cracked beam was also pulled and released, the WVD clearly shows frequency content at harmonic frequencies of the fundamental. The uncracked beam does show some minor harmonic content. Possible sources of this are any nonlinearity in the system or sensor. We could easily attribute the harmonics to an imperfect mount of the beam in the vise.

Ideally, we would like to see the frequency change for the different half cycles of the bilinear response. Figure 11 does show periodic modulations but it is difficult to determine if they are artifacts or true indications of the crack opening and closing.

7. CONCLUSIONS

In this paper, experimental data from a beam with an opening and closing crack was analyzed with the goal of detecting and locating the crack. The main goal was to explore the possibility of using the nonlinearity for improved damage detection capability over linear based methods. The authors realize that the laboratory conditions employed do not translate necessarily translate to field applications.

The “harmonic mode shapes” were qualitatively demonstrated to give improved sensitivity and spatial resolution over conventional mode shapes.

Several time domain and time-frequency domain techniques were employed in an effort to resolve the opening and closing behavior of the crack. A half-cycle frequency was able to resolve the different frequencies of the crack-open and crack-closed portions of the vibration cycle. A more sophisticated instantaneous frequency based on the analytic signal was also employed. This too resolved the opening and closing behavior, but also showed some frequency oscillation in the uncracked data. In both cases, the effects varied depending on sensor location, indicating the possibility of locating the crack. Both of these methods relied heavily on the fact that the beam was vibrating almost purely in the fundamental mode. Also, the results from both methods differed somewhat from the expected behavior for a bilinear oscillation.

The Wigner-Ville distribution was also applied to the experimental data. It clearly showed energy content at the harmonic frequencies for the cracked beam. Ideally, the frequency change for the different half-cycles in the cracked beam would be apparent. There were some periodic modulations in the results that may indicate the crack opening and closing. No definite conclusion was drawn.

The general conclusion to be drawn is that detection and location of a structural nonlinearity may provide more accurate damage identification than detection and localization of linear structural perturbations.

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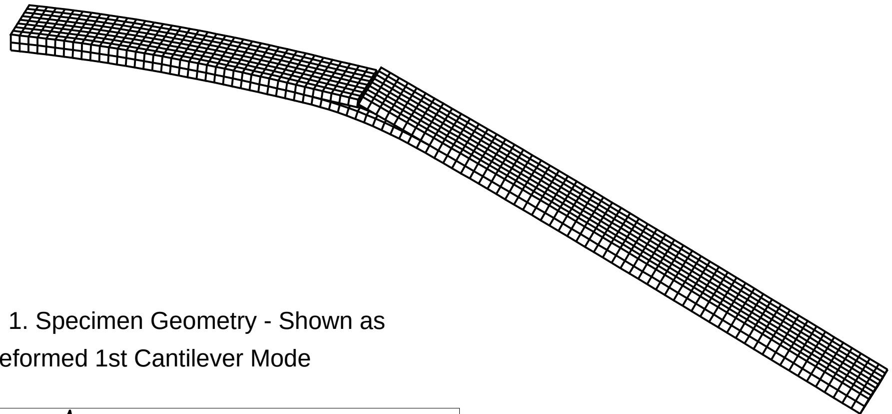

Fig. 1. Specimen Geometry - Shown as Deformed 1st Cantilever Mode

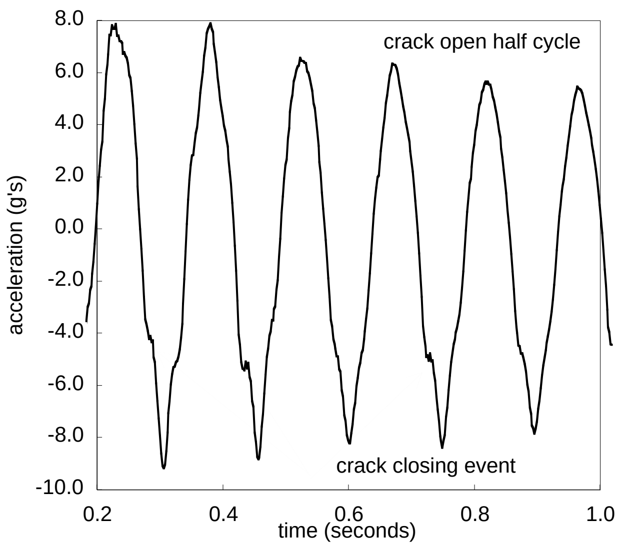

Fig. 2. Time History of Acceleration on Cracked Beam, Crack Closing is Evident

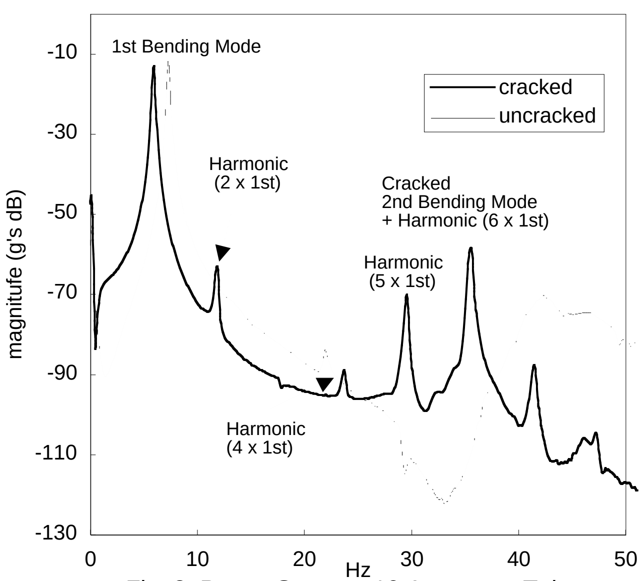

Fig. 3. Power Spectra, 10 Averages, Taken Near Free End

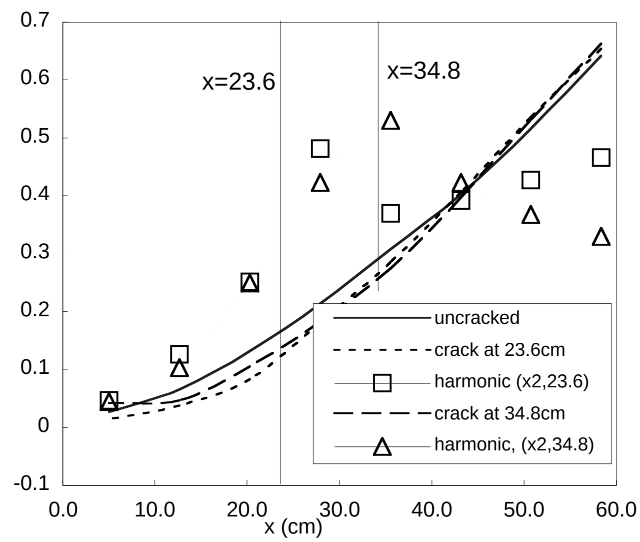

Fig. 4. Normalized Mode 1 Shapes - Uncracked, Cracked and Harmonic at Twice the First Mode

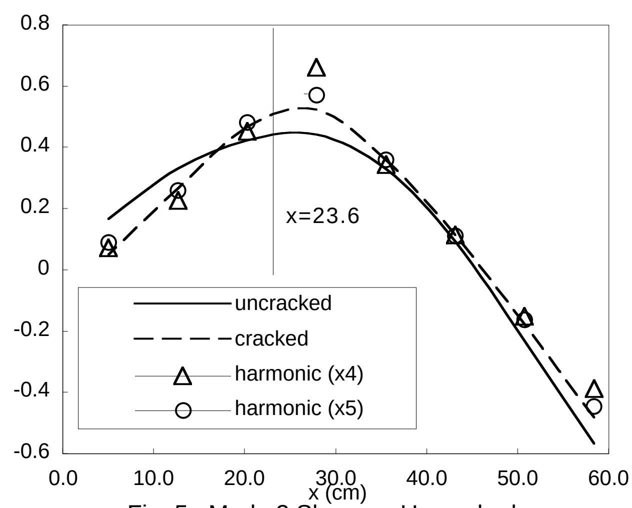

Fig. 5. Mode 2 Shapes - Uncracked, Cracked, and Harmonics, Crack at 23.6 cm

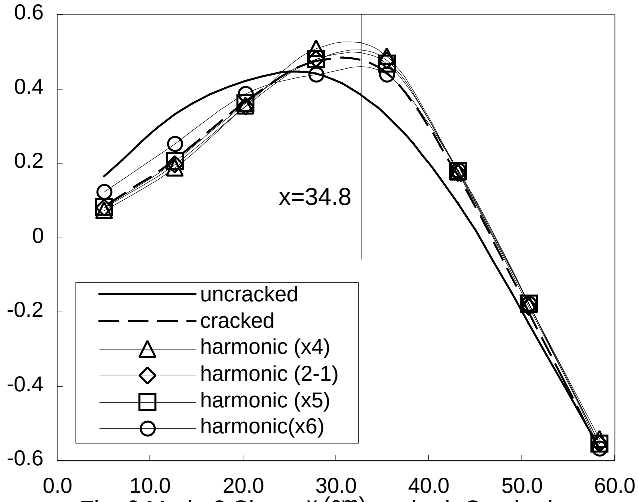

Fig. 6 Mode 2 Shapes - Uncracked, Cracked, and Harmonics, Crack at 34.8cm

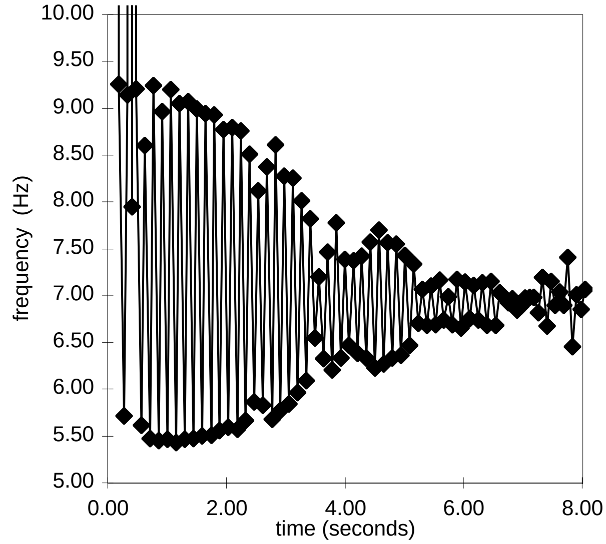

Fig. 8 Half-Cycle Frequency - Cracked Beam

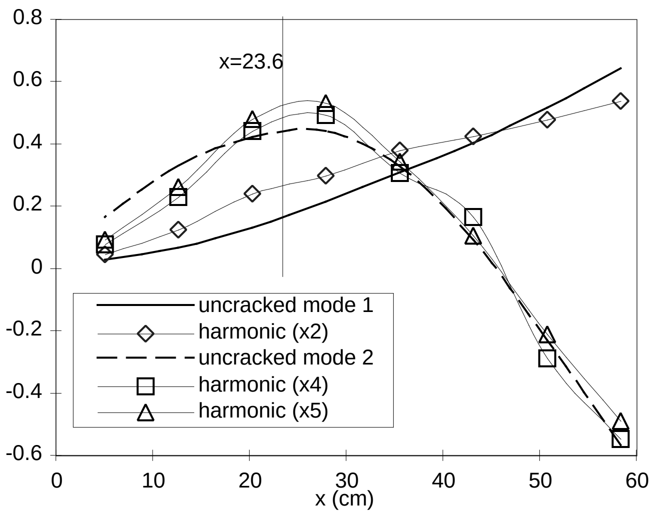

Fig. 7. Harmonic Mode Shapes for Accelerometers on Back Side

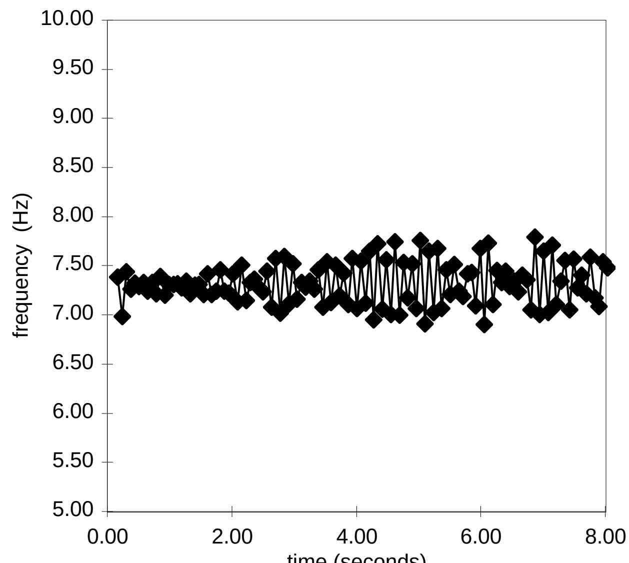

Fig. 9. Half Cycle Frequency - Uncracked Beam

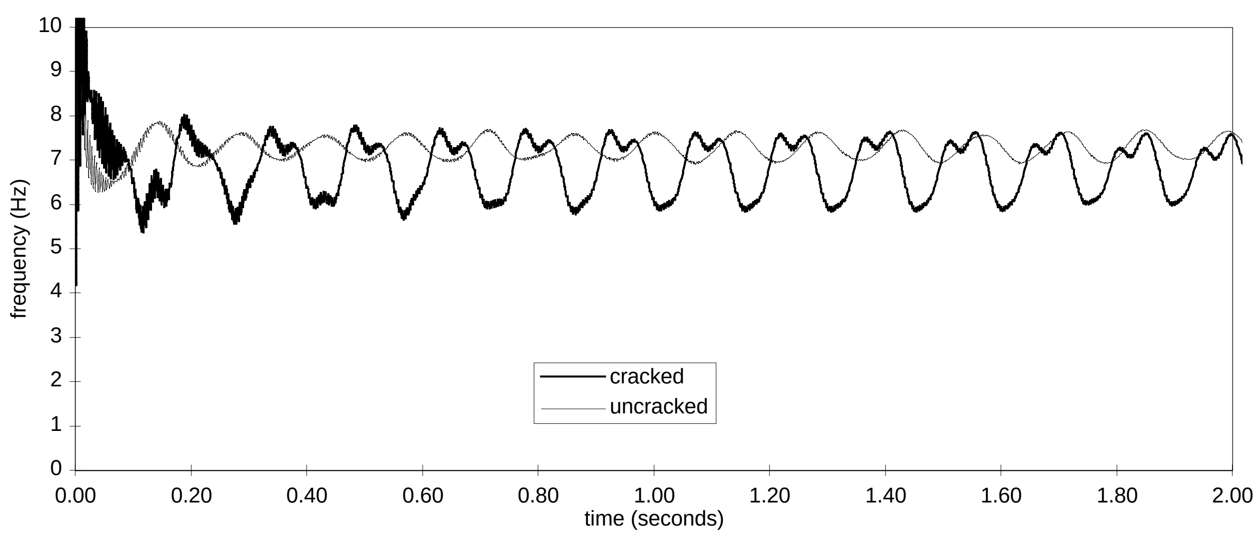

Fig. 10. Instantaneous Frequency

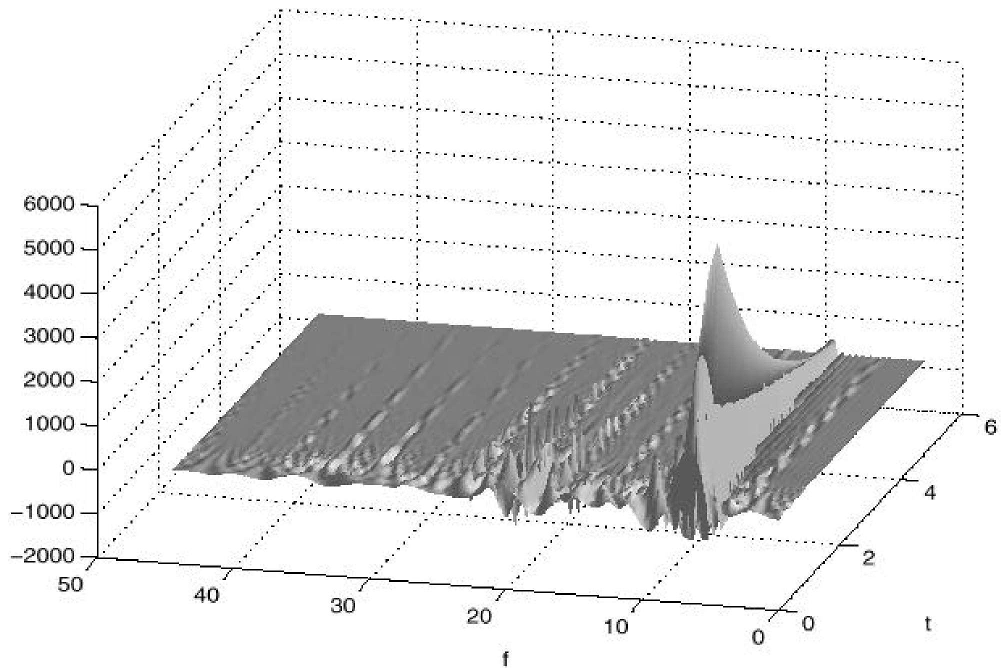

Fig. 11. Wigner-Ville Distribution of the Cracked Signal

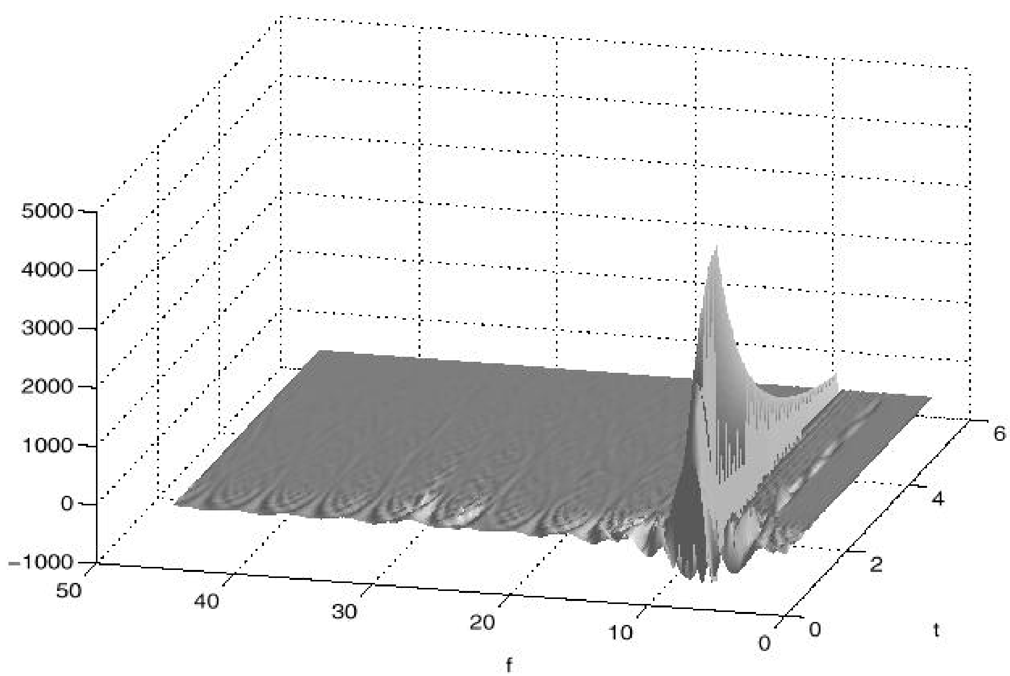

Fig. 12. Wigner-Ville Distribution of the Uncracked Signal