Quantum chromodynamics, resonances, and the Riemann-Hilbert problem

Mark Paris

GWU collaborators:
D. Arndt, B. Briscoe, I. Strakovsky, & R. Workman

Data Analysis Center
Center for Nuclear Studies
The George Washington University

Department of Physics
The George Washington University
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Motivation

- Data Analysis Center/Center for Nuclear Studies
  - SAID[a]: suite of programs to analyze $2 \longrightarrow 2$ & 3 body scattering and reaction data
  - Routines: database, fit, and analysis
  - Reactions: $\pi N \longrightarrow \pi N, \pi\pi N$; $KN \longrightarrow KN$; $NN \longrightarrow NN$; $\pi d \longrightarrow \pi d$; $\pi d \longrightarrow pp$; $\gamma N \longrightarrow \pi N, \eta N, \eta' N, KY$; $eN \longrightarrow e\pi N$
- Current studies
  - Meson-nucleon reactions[b]
  - Electromagnetic meson production: photo- & electro-production

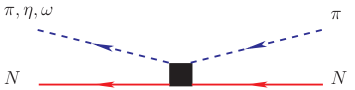
[a] Web: `http://gwdac.phys.gwu.edu/`
ssh: `ssh -C -X said@said.phys.gwu.edu [passwordless]`

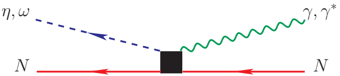
[b] In our terminology, we sometimes use 'reaction' to include elastic scattering.

Objective: Learn about QCD

- Strongly interacting
- Infinitely many degrees-of-freedom
- Non-linear

$\Rightarrow$ QCD is a challenging theory to solve





$\langle$final state$|$initial state$\rangle$

Outline

Outline

Quantum physics

Quantum mechanics

- Dynamical variables $\{x(t), p(t)\} \rightarrow [\hat{x}, \hat{p}] = i\hbar$ – ‘uncertainty’ principle (HUP)
- Quantum “weirdness” – position and velocity not definite simultaneously
- Wave-particle duality
  - wave $\leftrightarrow$ continuum properties in propagation
  - particle $\leftrightarrow$ energy exchanged discretely
- **Fixed number of particles**

Quantum field theory

- Dynamical variables $\rightarrow$ fields: $\phi_\alpha(t, \mathbf{r})$
- Predicts antiparticles: same mass, spin; opposite charge(s)
- Arises inevitably if:
  - Local: ‘action at a distance’ isn’t allowed
  - Poincaré ($\subset$ Lorentz xform) invariant: relativistic
  - Cluster decomposition: distant experiments are not correlated
  - CPT invariance
- **Variable number of particles** necessitated: HUP & Lorentz invariance

Outline

The “strong force”

Empirical considerations

- Strong & short ranged compared to electromagnetic, weak, and gravity
- Quarks (and gluons) aren’t directly observed
- Hadrons interact weakly at small momenta

The "strong force"

Empirical considerations

- Strong & short ranged compared to electromagnetic, weak, and gravity $\rightarrow$ Mass gap
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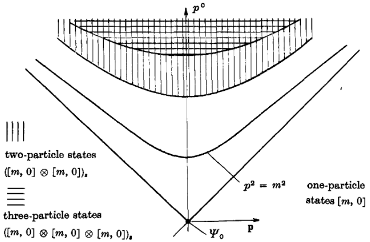
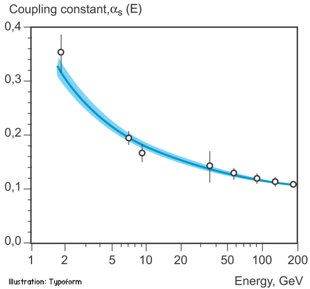


$$E(\mathbf{p}) = \sqrt{|\mathbf{p}|^2 + m^2}$$

The "strong force"

Empirical considerations

- Strong & short ranged compared to electromagnetic, weak, and gravity
  $\longrightarrow$ Mass gap
- Quarks (and gluons) aren't directly observed
  $\longrightarrow$ Color confinement
- Hadrons interact weakly at small momenta



Indication of confinement from perturbation theory. The running strong coupling constant α_s as a function of the energy, E at which it is measured.

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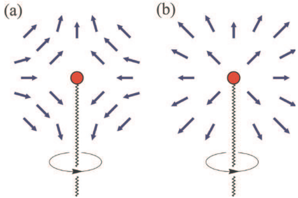

Non-perturbative confinement via monopoles. Possible monopole configurations.

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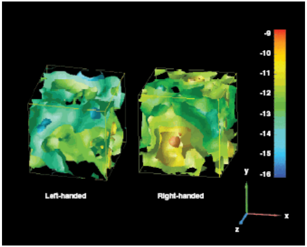
  $\longrightarrow$ Chiral symmetry breaking



Wave functions of chiral fermions on the lattice.

Gauge theory & QCD

Gauge principle [Weyl,Lee,Yang]

- All four fundamental forces are governed by the gauge principle
  - Electromagnetic: phase invariance
  - Weak: broken non-Abelian symmetry
  - Strong: color invariance
  - Gravity: diffeomorphism invariance
- Invariance under some local symmetry transformations, *eg.*
  1-dim Abelian symmetry electrodynamics:

$$\psi(x) \rightarrow e^{i\varphi(x)}\psi(x) \qquad \Rightarrow D_\mu \psi(x) = [\partial_\mu - ieA_\mu(x)]\psi(x)$$

  $A_\mu(x)$, the four-vector electromagnetic potential, 'compensates' for possible changes in the phase and has its own dynamics
- QCD
  - Internal quantum number "color": R, G, B
  - Invariance under local changes of color
  - 'Compensating' field are **gluons**, $G^A_\mu(x)$ – come in 8 types
- Gauge fields are massless, vector bosons[1]

[1] Unless the ground state of the theory breaks the gauge symmetry as in, the Higgs mechanism in the electroweak sector.

Outline

Classical atomic resonance

Dispersion characteristics of (low-density) dielectrics: Classical EOM for an electron ($e > 0$) bound harmonically within a non-conducting material

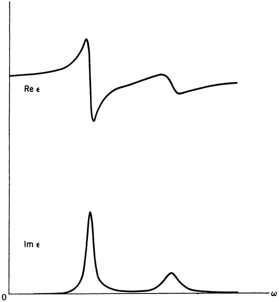
$$-\frac{e}{m}\mathbf{E}(t,\mathbf{r}) = \ddot{\mathbf{r}}(t) + \gamma\dot{\mathbf{r}}(t) + \omega_0^2\mathbf{r}(t)$$

$$\mathbf{E}(t) \sim e^{-i\omega t}$$

$$\epsilon(\omega) = 1 + \frac{4\pi N e^2}{m}\underbrace{\frac{1}{\omega_0^2 - \omega^2 - i\omega\gamma}}_{\textit{Response function}}$$

Response function

- Re $\epsilon(\omega)$: related to phase velocity ($v = \frac{c}{\mathrm{Re}\sqrt{\mu\epsilon}}$)
- Im $\epsilon(\omega) \neq 0$: energy dissipation EM wave $\rightarrow$ medium



The dielectric constant as a function of frequency.

Quantum atomic resonance

Resonance fluoresence

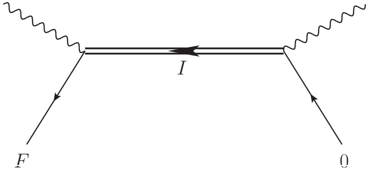
$$i\hbar\frac{\partial\psi(t)}{\partial t} = [H_{\rm free} + H_{\rm int}]\psi(t)$$

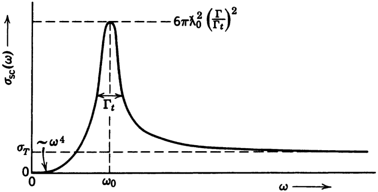
$$\psi(t) = \sum_k c_k(t)u_k(\mathbf{r})e^{-iE_k t}$$

$$\dot{c}_m(t) = -i\sum_k \langle m|H_{\rm int}|k\rangle e^{i(E_m-E_k)t}c_k(t)$$

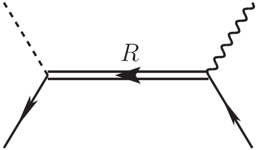
$$\dot{c}_I(t) = -i\langle I|H_{\rm int}|0\rangle c_0 e^{i(E_I-E_0)t} - \frac{\Gamma_I}{2}c_I(t)$$

$$|c_I|^2 = \underbrace{\frac{\langle I|H_{\rm int}|0\rangle}{(E_I-E_0-\omega)^2+\Gamma_I^2/4}}_{Breit-Wigner\ \ response\ \ fn.}$$

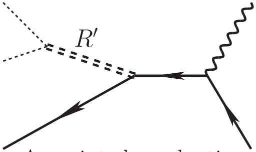




Hadronic[2] resonance



Formation



Associated production

[2] Necessarily quantum.

Outline

Background/non-resonant vs. resonant

Folklore

Setup:

- target at rest in the lab
- projectile impinges upon the target with energy E_L
- interact over (very) short range [neglect, eg. *Coulomb*]
- scattering elastically or inelastically, receding to infinity

Qualitatively:

Non-resonant

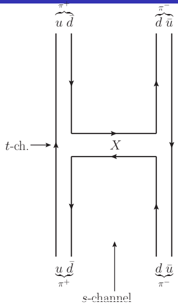
The target–projectile system interact via an attractive force, remaining in proximity for a time, τ all the while retaining their individual identities, then move off to infinity.

Resonant

The target–projectile system amalgamate to form a compound state, completely losing their individual identities in the process, existing for a time τ as a metastable state. This compound state may decay into particles whose species are identical to or distinct from the target–projectile species.

Background/non-resonant vs. resonant

QCD



- QCD degrees-of-freedom: quarks & gluons
- Observables are function(al)s of $\langle 0|T\{A_1(x_1)\cdots A_n(x_n)\}|0\rangle$
- Consider the quark "dual diagram"
  - Quarks propagate forward in time – 'up'
  - Antiquarks propagate backward in time – 'up'
  - Gluon field is implicit and ubiquitous – imagine gluon field describing a membrane spanning quark lines
- "Channels": a single quark diagram describes several processes at the hadronic level
  - s-channel: $\pi^+\pi^- \rightarrow \rho^0 \rightarrow \pi^+\pi^-$
  - t-channel: $\pi^+\pi^- \rightarrow \pi^+\pi^-\rho^0 \rightarrow \pi^+\pi^-$
- **Fact**: non-resonant and resonant are model dependent concepts
  Query: Are these useful concepts? And to what extent?

Background/non-resonant vs. resonant

QCD

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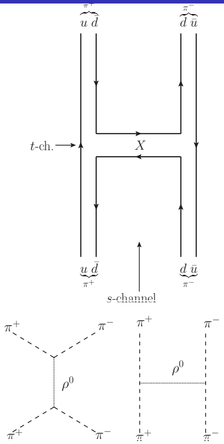


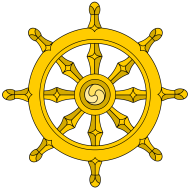
Outline

Current algebra/Eightfold Way

[Gell-mann, 1961; Ne'eman, 1961]

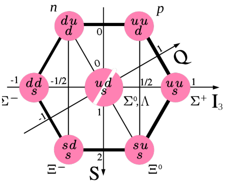
The 1950's proliferation of strongly interacting particles under the pejorative, "Particle Zoo," drove some fairly serious folks to humor:

- **J.R. Oppenheimer's lament**: *'The Nobel Prize should be given to the physicist who did* not *discover a particle.'*
- **W. Pauli's other career**: [*To Leon Lederman*] *'If I could remember the names of these particles I would have gone into botany.'*



The apparent chaos of the 100's of known strongly interacting particles was brought to order by M. Gell-Mann, *The Eightfold Way*, *Symmetries of Baryons and Mesons*, *Phys. Rev.* ***125****, 1962* **without explicit reference to quarks**.

- Goldberger-Treiman relation from PCAC: $\frac{f_\pi g_{\pi NN}}{m_N} = g_A$
- Adler-Weisberger relation:
  $g_A^{-2} = 1 + \frac{2m_N^2}{\pi g_{\pi NN}^2} \int_{\nu_0}^{\infty} \frac{d\nu}{\nu} \left[\sigma_{\pi^- p} - \sigma_{\pi^+ p}\right]$



Group theoretic quark model

Gell-Mann, 1964; Zweig, 1964

The *Eightfold Way* as an *irreducible* representation (the octet **8**) of the global symmetry group $SU(3)_{Flavor}$

Generators of $SU(3)_{Flavor}$

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- Hermitian, traceless, 3×3 complex matrices
- Why 8 generators?
  $$\underbrace{3 \times 3}_{\#elements} \times \underbrace{2}_{real+imag} - \underbrace{9}_{H^\dagger = H} - \underbrace{1}_{traceless} = 8$$
- Top row: isospin!
- First two matrices of each column: raising and lowering operators
- Third column: λ_3 & λ_8 diagonal

  [Cartan subalgebra]

  Use these to classify states...

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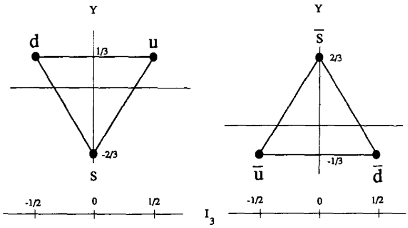
Quarks/Antiquarks

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix} = \text{“}\mathbf{3}\text{”}$$

$$I_3 = \tfrac{1}{2}\lambda_3 = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$Y = \frac{1}{\sqrt{3}}\lambda_8 = \frac{1}{3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$Q = I_3 + \frac{Y}{2} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix}$$



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Mesons

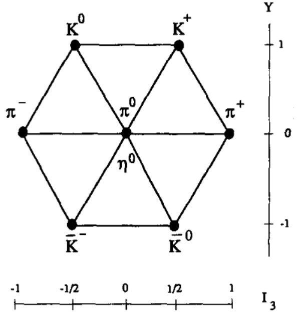
$$M = q \otimes \bar{q} = \mathbf{3} \otimes \mathbf{3} = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{3}(2u\bar{u}-d\bar{d}-s\bar{s}) & u\bar{d} & u\bar{s} \\ d\bar{u} & \frac{1}{3}(2d\bar{d}-u\bar{u}-s\bar{s}) & d\bar{s} \\ s\bar{u} & s\bar{d} & \frac{1}{3}(2s\bar{s}-u\bar{u}-d\bar{d}) \end{pmatrix}$$

$$+ \underbrace{\frac{1}{3}(u\bar{u}+d\bar{d}+s\bar{s})}_{singlet}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix} + |singlet\rangle$$

$$= \mathbf{8} \oplus \mathbf{1} = 9 \text{ states}$$



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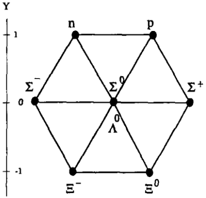
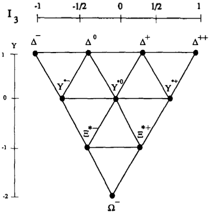


Baryons

$$B = q \otimes q \otimes q = \mathbf{3} \otimes \mathbf{3} \otimes \mathbf{3}$$
$$= \mathbf{10} \oplus \mathbf{8} \oplus \mathbf{8} \oplus \mathbf{1} = 27 \text{ states}$$

$$\mathbf{8} = \begin{pmatrix} \frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}}\Sigma^0 + \frac{1}{\sqrt{6}}\Lambda^0 & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda^0 \end{pmatrix}$$

$$\mathbf{10} = \{\Delta, \Sigma, \Xi, \Omega\}$$



Outline

Scattering & reactions

Definitions

- Target: a particle (elementary or composite) in the lab rest frame
- Projectile: a particle (elementary or composite) which impinges on the target
- Initial state: target and projectile at (effectively) infinite separation = non-interacting
- Final state: daughter particles (any number) at infinite separation
- Reaction channel: *n* particles where $n \geq 1$ in an initial state; *eg.* $e^-e^-, e^+N, \pi N, \gamma N, \pi\pi N, \ldots$

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- Elastic: kinetic energy conserved
  - Moller scattering: $e^-e^- \rightarrow e^-e^-$
  - Bhabha scattering: $e^-e^+ \rightarrow e^-e^+$
  - Rayleigh/Thompson scattering: $e^- {}^N_Z A(i) \rightarrow e^- {}^N_Z A(i)$
  - Compton scattering: $\gamma e^- \rightarrow \gamma e^-$, $\gamma p \rightarrow \gamma p$, $\gamma A \rightarrow \gamma A$ (nucleus $A = d, {}^3He, \ldots, \ldots$
  - πN scattering: $\pi^0 p \rightarrow \pi^0 p$ (neutral), $\pi^+ p \rightarrow \pi^+ p$ (charged)
  - Gold scattering: $Au\ Au \rightarrow Au\ Au$

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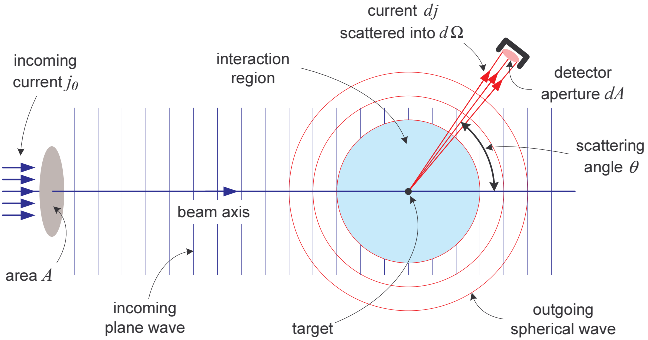
  - Gold scattering: $Au\,Au \to Au\,Au$
- Inelastic
  - Electron-positron annihilation: $e^-e^+ \to \gamma\gamma$, hadrons
  - Raman scattering: $e^- {}^N_Z A(i) \to e^- {}^N_Z A(f)$, $i \neq f$
  - πN scattering: $\pi^- p \to \pi^0 n$ (charge exchange)
  - Meson π-production: $\pi^- p \to \eta p, \omega p, \ldots$
  - Meson photoproduction: $\gamma p \to \pi^0 p, \pi^+ n, \eta p, \omega p, \pi^+\pi^- p, \ldots$

Scattering & reactions

Experimental setup

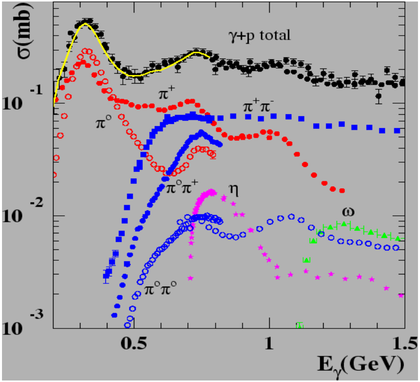


[Figure courtesy H. Haberzettl]

Resonance production

- Photoproduction exhibits strong resonance signature (bumps) in all channels
- Single meson production falls-off $E_\gamma \sim 750$ MeV, $W \sim 1500$ MeV
- Coupled-channel treatment absolute necessity
- Aside: energies
  E_γ photon lab energy [experiment]
  W total COM energy [calculations]

$$W = [m_N^2 + 2m_N E_\gamma]^{1/2}$$
$$\approx m_N + E_\gamma - \frac{E_\gamma^2}{4m_N}$$
$$E_\gamma = \frac{W^2 - m_N^2}{2m_N}$$
$$= \frac{1}{2}(1 + \frac{W}{m_N})[W - m_N]$$



Outline

Formalism

Scattering matrix S

- In/Out states & Scattering matrix

$$\Psi_\alpha^\pm = \begin{cases} \text{In state } \alpha \text{ before scattering} \\ \text{Out state } \alpha \text{ after scattering} \end{cases} \qquad \alpha = \{\mathbf{p}_i, \lambda_i, \text{ channel}, \ldots\}$$

$$S_{\alpha\beta} = \langle \Psi_\alpha^- | \Psi_\beta^+ \rangle \equiv (\Psi_\alpha^-, \Psi_\beta^+) \qquad \longrightarrow \text{probability amplitude}$$

- Time-translation invariance ($\subset$ of Poincaré)

$$e^{-iHt}\Psi_\alpha^\pm = e^{-iE_\alpha t}\Psi_\alpha^\pm \qquad E_\alpha = E_{\alpha,1} + E_{\alpha,2} + \cdots$$

Generalized Schrödinger equation

$$E_\alpha \Psi_\alpha^\pm = H\Psi_\alpha^\pm = [H_0 + H_{\text{int}}]\Psi_\alpha^\pm \qquad [E_\alpha - H_0]\Psi_\alpha^\pm = H_{\text{int}}\Psi_\alpha^\pm$$

- Inverting $[E_\alpha - H_0] \to [E_\alpha - H_0 \pm i\epsilon]^{-1}$ with **boundary conditions**

$$\Psi_\alpha^+ \to 0 \text{ after scattering} \qquad \Psi_\alpha^- \to 0 \text{ before scattering}$$

Relativistic Lippmann-Schwinger equation

$$\Psi_\alpha^\pm = \Phi_\alpha + \frac{1}{E_\alpha - H_0 \pm i\epsilon} H_{\text{int}}\Psi_\alpha^\pm \qquad H_0\Phi_\alpha = E_\alpha\Phi_\alpha$$

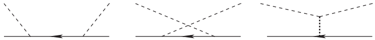
Formalism

Lippmann-Schwinger equation

- L-S equation

$$\Psi_\alpha^\pm = \Phi_\alpha + G_0(E_\alpha)V\Psi_\alpha^\pm$$

- Interaction mechanisms $\pi N \to \pi N$



- Iteration

$$\Psi_\alpha^\pm = \Phi_\alpha + G_0(E_\alpha)V\Phi_\alpha + G_0(E_\alpha)V\Phi_\alpha G_0(E_\alpha)V\Phi_\alpha + \cdots$$

Definitions:

$\Psi_\alpha^\pm$	exact w.f.
Φ_α	homogeneous w.f.
$G_0(E_\alpha) = \frac{1}{E_\alpha - H_0 \pm i\epsilon}$	propagator
$V \equiv H_{\text{int}}$	interaction

Formalism

Transition matrix T

- Define **free propagator** G_0 and **exact propagator** G which have singularities (denominator→zero) in the spectrum of H_0 or H

$$G_0^{-1}(E_\alpha) = E_\alpha - H_0 \pm i\epsilon \qquad G^{-1}(E_\alpha) = E_\alpha - H \pm i\epsilon$$

$$G^{-1} = G_0^{-1} - V \qquad G = G_0 + G_0 V G$$

- Rewrite L-S

$$\Psi_\alpha^\pm = [1 + G^\pm V]\Phi_\alpha \qquad \Psi_\alpha^- = \Psi_\alpha^+ + (G^- - G^+)V\Phi_\alpha$$

- S matrix

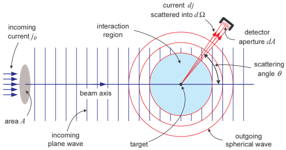
$$\begin{aligned} S_{\alpha\beta} &= (\Psi_\alpha^-, \Psi_\beta^+) \\ &= (\Psi_\alpha^+, \Psi_\beta^+) + ([G^- - G^+]V\Phi_\alpha, \Psi_\beta^+) \\ &= \delta_{\alpha\beta} + 2\pi i \delta(E_\alpha - E_\beta)(\Phi_\alpha, V\Psi_\alpha^+)\ \mathcal{R}\text{–}\mathcal{H}!! \\ &= \delta_{\alpha\beta} + 2\pi i \delta(E_\alpha - E_\beta) T_{\alpha\beta}^+ \qquad T_{\alpha\beta}^+ = -(\Phi_\alpha, V\Psi_\beta^+) \end{aligned}$$

Observables → Amplitudes

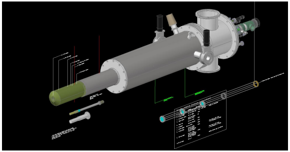
Differential cross section $1+2 \to 1'+2'$ (exclusive)

$$\frac{d\sigma}{d\Omega} = \frac{\text{\# particles scattered into } (\theta,\phi)}{\text{unit time} \cdot \text{incident flux}}$$

$$= \frac{(4\pi)^2}{k^2}\rho_{1'2'}(k')\rho_{12}(k)\left|T_{\lambda_{1'}\lambda_{2'},\lambda_1\lambda_2}(k',k;W)\right|^2$$



- Complete set of measurements: # ampls. = $\prod_i N(\lambda_i)$
- Need twice ($\mathbb{C} \to \mathbb{R}$) number of observables, modulo symmetries (C, P, T) & discrete ambiguities
- Polarized particles
- New experiments (FROST, HD-ICE)
- Upcoming complete measurement $\gamma\vec{p} \to K^+\vec{\Lambda}$
- Unitarity requires *multi-channel* data, *eg.* $\gamma N \to \pi N$, $\gamma N \to \pi\pi N$, $\gamma N \to \eta N$, ...



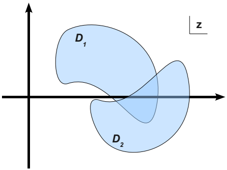
Outline

The analytic continuation

The miracle of complex numbers

- Complex analytic functions (*holomorphic, regular*)
  - All derivatives exist everywhere in open domain, $\mathcal{D}$
  - Derivatives independent of direction (*Cauchy-Riemann eqs.*)
  - Harmonicity: $u_{xx} + u_{yy} = 0$ and $v_{xx} + v_{yy} = 0$ [Laplace]
- Analytic continuation - AC
  - Analytic function in $\mathcal{D}$ uniquely determined by values on a domain or along an 1-dim curve
  - $f_1(z)$ analytic in $\mathcal{D}_1$ and $f_1(z) = f_2(z)$ in $\mathcal{D}_1 \cap \mathcal{D}_2 \Rightarrow$, then there *may be* $f_2(z)$ analytic in $\mathcal{D}_2$; if so, **unique**.
- Contrast with real functions
  - Analytic $f_1(x)$ on $a < x < b$ and $f_1(x) = f_2(x)$ doesn't imply $f_2(x)$ is unique (if it exists)
- Cauchy-Gorsat [Green's/Stoke's theorem+*C-R*]

$$\oint_C dz\, f(z) = 0 \qquad \left[\oint_C d\mathbf{l} \cdot \mathbf{A}(\mathbf{x}) = \int d^2\mathbf{S} \cdot \nabla \times \mathbf{A}(\mathbf{x})\right]$$



Poles & resonances

'Toy' model: 1-D scattering

Scattering from a finite square well

$$V(x) = \begin{cases} 0 & -\frac{a}{2} \geq x, \\ -V_0 & -\frac{a}{2} \leq x \leq \frac{a}{2} \\ 0 & x \geq \frac{a}{2} \end{cases}$$

$$\psi_1(x) = e^{ipx} + Re^{-ipx}$$

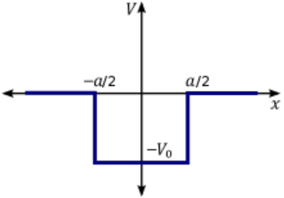
$$\psi_2(x) = Ae^{i\bar{p}x} + Be^{-i\bar{p}x}$$

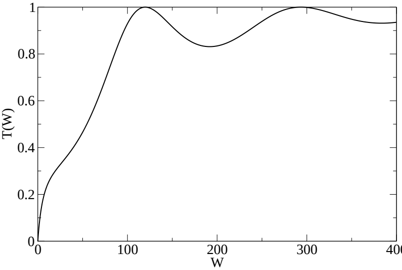
$$\psi_3(x) = Se^{ipx}$$

$$p = \sqrt{2mW} \qquad \bar{p} = \sqrt{2m(W + V_0)} \quad W > 0$$

$$S(E)e^{ipa} = \frac{1}{\cos \bar{p}a - \frac{i}{2}\left[\frac{p}{\bar{p}} + \frac{\bar{p}}{p}\right] \sin \bar{p}a}$$

$$T(E) = |S(W)|^2 = \frac{1}{1 + \frac{V_0^2}{4E(E+V_0)} \sin^2 \bar{p}a}$$





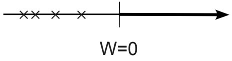
Analytic structure of S

Bound states, resonances, & poles

Bound states: $W < 0$

$$\psi_1(x) = e^{\kappa x} \qquad \psi_2(x) = A \begin{pmatrix} \cos \\ \sin \end{pmatrix} \bar{p}x \qquad \psi_3(x) = \pm e^{-\kappa x}$$

$$x < -a/2 \qquad -a/2 \leq x \leq a/2 \qquad a/2 < x$$



$$\kappa = \sqrt{-2mW} > 0, W \leq 0$$

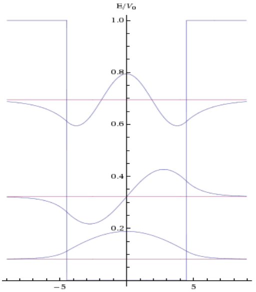
$$S(E)e^{ipa} = \frac{1}{\cos \bar{p}a - \frac{i}{2}\left[\frac{p}{\bar{p}} + \frac{\bar{p}}{p}\right] \sin \bar{p}a}$$

Denominator zeros→bound states when $W < 0$

$$\tan \frac{\bar{p}a}{2} = \frac{\kappa}{\bar{p}}$$

$$\tan \frac{\bar{p}a}{2} = -\frac{\bar{p}}{\kappa}$$

$$p = i\kappa.$$



Analytic structure of S

Bound states, resonances, & poles

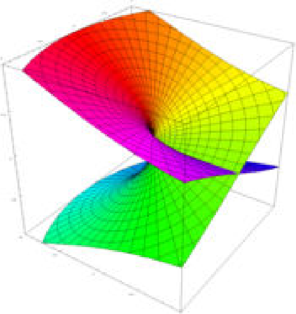
Riemann surface

$$p = \sqrt{2mW} \qquad\qquad W \in \mathbb{C}$$

$$\sqrt{W} = |W|^{\frac{1}{2}} e^{i\theta/2}$$

$$\theta = \begin{cases} 0 \le \theta < 2\pi & \text{`upper' sheet} \\ 2\pi \le \theta < 4\pi & \text{`lower' sheet} \end{cases}$$

$$\text{Disc } p \equiv p(W + i\epsilon) - p(W - i\epsilon)$$
$$= \sqrt{2m|W|}[e^{i\cdot 0/2} - e^{i\cdot 2\pi/2}] = 2\sqrt{2m|W|}$$



Riemann surface representation of the function $\sqrt{W}$. The complex–W plane is horizontal. The vertical axis gives the imaginary part of the function.

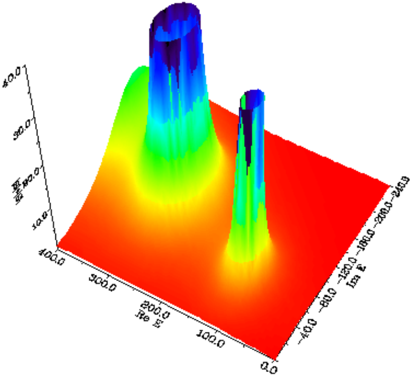
Analytic structure of S

Bound states, resonances, & poles

Given $T(W) = \mathrm{Re}\, T(W) + i \mathrm{Im}\, T(W)$, for $W > 0$ consider AC in $z = W + i \mathrm{Im}\, z$

$$S(E)e^{ipa} = \frac{1}{\cos \bar{p}a - \frac{i}{2}\left[\frac{p}{\bar{p}} + \frac{\bar{p}}{p}\right] \sin \bar{p}a}$$

- Denominator zeros on the second sheet→resonances



Analytic structure of S

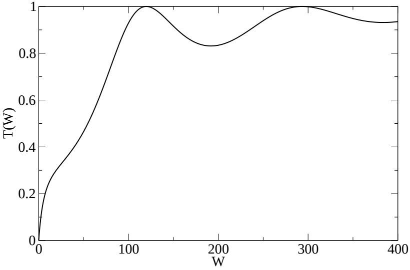
Bound states, resonances, & poles

Given $T(W) = \mathrm{Re}\, T(W) + i \mathrm{Im}\, T(W)$, for $W > 0$ consider AC in $z = W + i\mathrm{Im}\, z$

$$S(E)e^{ipa} = \frac{1}{\cos \bar{p}a - \frac{i}{2}\left[\frac{p}{\bar{p}} + \frac{\bar{p}}{p}\right] \sin \bar{p}a}$$

$$T(E) = |S(W)|^2 = \frac{1}{1 + \frac{V_0^2}{4E(E+V_0)} \sin^2 \bar{p}a}$$

$$\bar{p}a = n\pi \rightarrow E_n = n^2 \frac{\pi^2}{2ma^2} - V_0$$



- Denominator zeros on the second sheet→resonances

Analytic structure of S

Bound states, resonances, & poles

Given $T(W) = \mathrm{Re}\, T(W) + i \mathrm{Im}\, T(W)$, for $W > 0$ consider AC in $z = W + i \mathrm{Im}\, z$

$$S(E)e^{ipa} = \frac{1}{\cos \bar{p}a - \frac{i}{2}\left[\frac{p}{\bar{p}} + \frac{\bar{p}}{p}\right] \sin \bar{p}a}$$

$$T(E) = |S(W)|^2 = \frac{1}{1 + \frac{V_0^2}{4E(E+V_0)} \sin^2 \bar{p}a}$$

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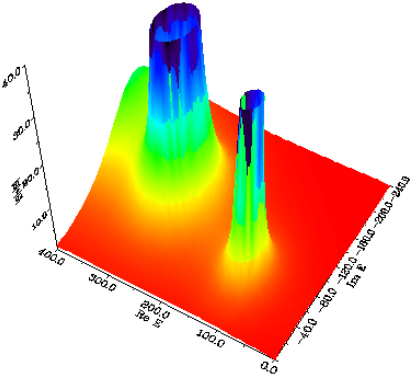
- Denominator zeros on the second sheet→resonances

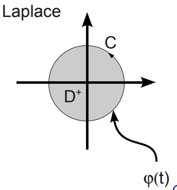


Outline

The $\mathcal{R}$iemann–$\mathcal{H}$ilbert problem

Properly: 'The scalar $\mathcal{R}$–$\mathcal{H}$ method'

- Reconstruction of complex *sectionally holomorphic* function given boundary data
- Diverse applications in math . . .
  - Find $f(z) = u(z) + iv(z)$ given $\alpha(z(t))u(z(t)) + \beta(z(t))v(z(t)) = \gamma(z(t))$ on a curve C [Poisson problem on circle $\alpha = 1, \beta = 0$]
  - Solve (singular) linear integral equations
  - Solve partial differential equations
  - Integral transforms (generalized Fourier transforms)
  - Solve "Fuchsian" system diff. eqs. via representation of monodromy group on the punctured Riemann sphere
  - . . .
- . . . & physics
  - Elasticity: Laplace boundary value prob. on D^+
  - Hydrodynamics: non-linear Korteweg-deVries (KdV) equation, $u_t + u_{xxx} + uu_x = 0$ shallow water *soliton* waves
  - Electrostatics: find surface density on $C \Rightarrow$ constant potential
  - Hadronic physics: discontinuity data from unitarity
  - Renormalization group: *Connes & Kreimer* showed that renorm. is equivalent to solving an $\mathcal{R}$–$\mathcal{H}$ problem
  - . . .



Unitarity

- Unitarity $\leftrightarrow$ Conservation of probability

$$|\Psi^+_\beta\rangle = \sum_\alpha |\Psi^-_\alpha\rangle\langle\Psi^-_\alpha|\Psi^+_\beta\rangle = \sum_\alpha |\Psi^-_\alpha\rangle S_{\alpha\beta}$$

$$\langle\Psi^+_\gamma|\Psi^+_\beta\rangle = \delta_{\gamma\beta} \text{ Proof: L-S equation}$$

$$= \sum_\alpha \langle\Psi^+_\gamma|\Psi^-_\alpha\rangle S_{\alpha\beta} = \sum_\alpha S^*_{\alpha\gamma} S_{\alpha\beta}$$

$$\delta_{\gamma\beta} = [S^\dagger S]_{\gamma\beta}$$

- Unitarity constraint on T

$$S^\dagger S = SS^\dagger = 1 \qquad S = 1 + 2i\rho T$$

$$T^+ - T^- = 2iT^-\rho T^+$$

$$T^{-\,-1} - T^{+\,-1} = 2i\rho$$

$$\text{Disc } T^{-1} = -2i\rho$$

Unitarity $\leftrightarrow$ analytic structure

$$\langle\alpha|\left\{T^+ - T^- = 2iT^+\rho T^-\right\}|\beta\rangle \rightarrow T^+_{\alpha\beta} - T^-_{\alpha\beta} = 2i\sum_\sigma T^+_{\alpha\sigma}\rho_\sigma(W)T^-_{\sigma\beta}$$

$$\rightarrow \text{Im } T(W) = 2i\sum_\sigma T^+_{\alpha\sigma}(W)\rho_\sigma(W)T^-_{\sigma\beta}(W)$$

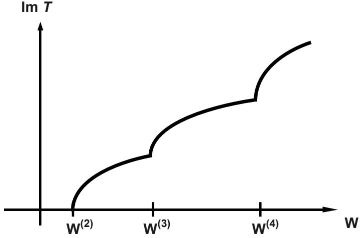
$$\rho^{(2)}_\sigma = \theta(W-(m_{\sigma,1}+m_{\sigma,2}))\mathcal{K}_2$$

$$\rho^{(3)}_\sigma = \theta(W-(m_{\sigma,1}+m_{\sigma,2}+m_{\sigma,3}))\mathcal{K}_3$$

$$\vdots$$

$$\rho^{(n)}_\sigma = \cdots$$

- 'Kinks' due to Heaviside-θ function, due to $\delta(E-H)$
- Non-analytic function? [*Eden (1952)*]
- Violation of Cauchy-Riemann conditions $\rightarrow$ **branch points**



Unitarity $\leftrightarrow$ analytic structure

$$\langle\alpha|\left\{T^+ - T^- = 2iT^+\rho T^-\right\}|\beta\rangle \to T^+_{\alpha\beta} - T^-_{\alpha\beta} = 2i\sum_\sigma T^+_{\alpha\sigma}\rho_\sigma(W)T^-_{\sigma\beta}$$

$$\to \text{Im}\ T(W) = 2i\sum_\sigma T^+_{\alpha\sigma}(W)\rho_\sigma(W)T^-_{\sigma\beta}(W)$$

$$\rho^{(2)}_\sigma = \theta(W - (m_{\sigma,1} + m_{\sigma,2}))\mathcal{K}_2$$

$$\rho^{(3)}_\sigma = \theta(W - (m_{\sigma,1} + m_{\sigma,2} + m_{\sigma,3}))\mathcal{K}_3$$

$$\vdots$$

$$\rho^{(n)}_\sigma = \cdots$$

- 'Kinks' due to Heaviside-θ function, due to $\delta(E - H)$
- Non-analytic function? [*Eden (1952)*]
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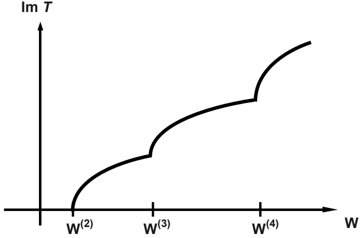
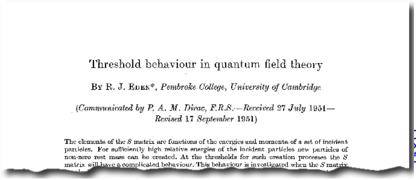



Threshold behaviour in quantum field theory

By R. J. Eden*, *Pembroke College, University of Cambridge*

(*Communicated by P. A. M. Dirac, F.R.S.—Received* 27 *July* 1951—*Revised* 17 *September* 1951)

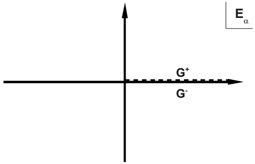
The elements of the S matrix are functions of the energies and moments of a set of incident particles. For sufficiently high relative energies of the incident particles new particles of non-zero rest mass can be created. At the thresholds for such creation processes the S matrix will have a complicated behaviour. This behaviour is investigated when the S matrix

$\mathcal{R}$iemann-$\mathcal{H}$ilbert

First blush

$$([G^- - G^+]V\Phi_\alpha, \Psi^+_\beta) = (\Phi_\alpha, V^\dagger[G^+ - G^-]\Psi^+_\beta) \qquad H\Psi^+_\beta = E_\alpha \Psi^+_\beta$$

Plemelj Formula:



$$\begin{aligned} G^\pm &= \frac{1}{E_\alpha - H \pm i\epsilon} \\ &= \frac{1}{E_\alpha - H} \mp i \lim_{\epsilon \to 0^+} \frac{\epsilon}{(E_\alpha - H)^2 + \epsilon^2} \\ &= \frac{1}{E_\alpha - H} \mp i\pi\delta(E_\alpha - H) \end{aligned}$$

$$[G^+ - G^-]\Psi^+_\beta = -2\pi i \delta(E_\alpha - E_\beta)\Psi^+_\beta$$

$$\to S_{\alpha\beta} = \delta_{\alpha\beta} + 2\pi i \delta(E_\alpha - E_\beta) T^+_{\alpha\beta} \qquad T^+_{\alpha\beta} = -(\Phi_\alpha, V\Psi^+_\beta)$$

- The scattering amplitude is proportional to the discontinuity in G across the real energy axis E_α: Disc $G = G^+ - G^- = 2\pi i \delta(E_\alpha - H)$
- Plemelj formula $\Rightarrow$ imaginary part gives *coupling to the continuum*
- *Sectionally holomorphic* function

Outline

Chew-Mandelstam approach

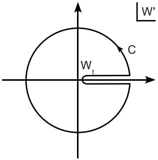
Discontinuity data from unitarity: Disc $T^{-1}(W) = \operatorname{Im} T(W) = -\rho(W)$

- Direct approach: Cauchy-integral representation or ‘dispersion relation’ [$W \in \mathbb{C}$]

  [neglecting left-hand cut, no subtractions]

$$T(W) = \frac{1}{2\pi i} \oint_C dW' \, \frac{T(W')}{W' - W}$$

$$T(W) = \int_{W_t}^{\infty} \frac{dW'}{\pi} \, \frac{\operatorname{Im} T(W')}{W' - W}$$



- Alternate approach: Chew-Mandelstam
  - Use Heitler K matrix

$$T^{-1} = \operatorname{Re} T^{-1} + \operatorname{Im} T^{-1} = K^{-1} - i\rho \qquad T = K + iK\rho T$$

  - Account for the cuts **exactly** ...

$$T^{-1} = K^{-1} - i\rho = K_{CM} - C \qquad \operatorname{Im} C = -\rho$$

  - ... and parameterize K_{CM}

$$K_{CM} = \sum_n c_n [W - W_t]^n$$

  - Parameters are fixed by fitting **scattering observables** (unpolarized diff. x-sec., pol. asymmetries, ...)

SAID: Scattering Analysis Interactive Database

πN elastic scattering and inelastic reactions

- Chi-squared per datum compared with model calculations

$$\chi^2(p) = \frac{1}{N_{data}} \sum_{i=1}^{N_{data}} \left[\frac{\Phi_{n(i)} y_i(p) - Y_i}{\Delta Y_i} \right]^2 + \frac{1}{N_{exp}} \sum_{n=1}^{N_{exp}} \left[\frac{\Phi_n - 1}{\Delta \Phi_n} \right]^2$$

χ^2/Data	SP06		FA02		KA84		EBAC		Gießen	
Reaction	Norm	UnNorm	Norm	UnNorm	Norm	UnNorm	Norm	UnNorm	Norm	UnNorm
$\pi^+p \to \pi^+p$	2.0	6.1	2.1	8.8	5.0	24.9	13.1	23.7	10.5	17.7
$\pi^-p \to \pi^-p$	1.9	6.2	2.0	6.6	9.1	51.9	4.9	16.0	12.1	34.1
$\pi^-p \to \pi^0n$	2.0	4.0	1.9	5.9	4.4	8.8	3.5	6.3	6.3	15.2
$\pi^-p \to \eta n$	2.5	9.6	2.5	10.5						

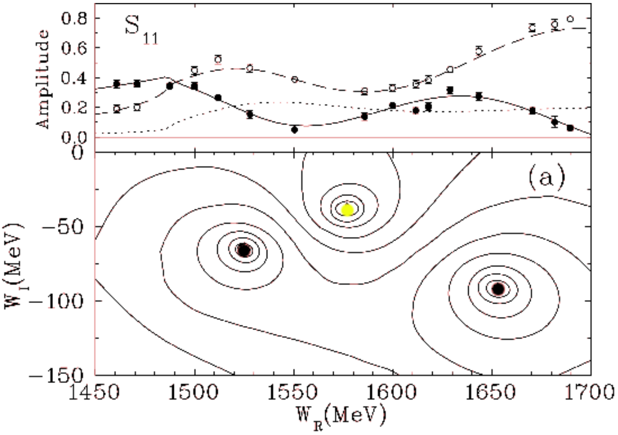
FA02 [R. Arndt *ea* Phys Rev C **69**, 035213 (2004)]
KA84 [R. Koch, Z Phys C **29**, 597 (1985)]
EBAC [B. Julia-Diaz *ea* Phys Rev C **76**, 065201 (2007)]
Gießen [V. Shklyar *ea* Phys Rev C **71**, 055206 (2005)]

CNS Center for Nuclear Studies

$\pi N \longrightarrow \pi N$

Analytic continuation

Spectroscopic notation: $L_{2I,2J}$ – L: rel. πN orb. ang. mom.; I: isospin; J: total intrinsic ang. mom.

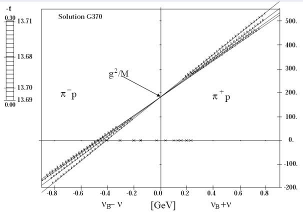


$\pi N \longrightarrow \pi N$ dispersion relations

πNN coupling; σ term

- The fit supplement with dispersion relation (DR) ‘pseudo-data’
- Solution method
  - Fit data via K_{CM}-matrix parameters
  - Evaluate forward/fixed-t DR’s, evaluate subtraction constants and include deviations from average as pseudo-data
  - Adjust real part of invariant amplitudes (and K_{CM} pars.) to minimize χ^2

Fixed-t DR



$$
\begin{aligned}
&(\nu_B \pm \nu)\,\{\mp \mathrm{Re}\, B_\pm(\nu, t) \\
&\pm \frac{\nu}{\pi} \int_{\nu_{\mathrm{th}}}^{\infty} \frac{d\nu'}{\nu'} \left[\frac{\mathrm{Im}\, B_+}{\nu' \mp \nu} + \frac{\mathrm{Im}\, B_-}{\nu' \pm \nu} \right] \Big\} \\
&= \frac{g^2}{M} + \tilde{B}(0, t)(\nu_B \pm \nu)
\end{aligned}
$$

$g = 13.69 \pm 0.07$ $\qquad$ $f = 0.0757 \pm 0.0004$

Outline

Effective field theory

Local, relativistic fields + canonical commutation relations $\longrightarrow$ correct analytics poss.

Hadronic interactions:

π, η, N, Δ:

$$L_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi} \bar{\psi}_N \gamma_\mu \gamma_5 \vec{\tau} \psi_N \cdot \partial^\mu \vec{\phi}_\pi,$$

$$L_{\pi N\Delta} = -\frac{f_{\pi N\Delta}}{m_\pi} \bar{\psi}^\mu_\Delta \vec{T} \psi_N \cdot \partial_\mu \vec{\phi}_\pi,$$

$$L_{\pi\Delta\Delta} = \frac{f_{\Delta\Delta\pi}}{m_\pi} \bar{\psi}_{\Delta\mu} \gamma^\nu \gamma_5 \vec{T}_\Delta \psi^\mu_\Delta \cdot \partial_\nu \vec{\phi}_\pi,$$

$$L_{\eta NN} = -\frac{f_{\eta NN}}{m_\eta} \bar{\psi}_N \gamma_\mu \gamma_5 \psi_N \partial^\mu \phi_\eta.$$

ρ:

$$L_{\rho NN} = g_{\rho NN} \bar{\psi}_N \left[\gamma_\mu - \frac{\kappa_\rho}{2m_N} \sigma_{\mu\nu} \partial^\nu\right] \vec{\rho}^\mu \cdot \frac{\vec{\tau}}{2} \psi_N,$$

$$L_{\rho N\Delta} = -i\, \frac{f_{\rho N\Delta}}{m_\rho} \bar{\psi}^\mu_\Delta \gamma^\nu \gamma_5 \vec{T} \cdot [\partial_\mu \vec{\rho}_\nu - \partial_\nu \vec{\rho}_\mu] \psi_N + [h.c.],$$

$$L_{\rho\Delta\Delta} = g_{\rho\Delta\Delta} \bar{\psi}_{\Delta\alpha} \left[\gamma^\mu - \frac{\kappa_{\Delta\Delta\rho}}{2m_\Delta} \sigma^{\mu\nu} \partial_\nu\right] \vec{\rho}_\mu \cdot \vec{T}_\Delta \psi^\alpha_\Delta,$$

$$L_{\rho\pi\pi} = g_{\rho\pi\pi} [\vec{\phi}_\pi \times \partial_\mu \vec{\phi}_\pi] \cdot \vec{\rho}^\mu,$$

$$L_{NN\rho\pi} = \frac{f_{\pi NN}}{m_\pi} g_{\rho NN}\, \bar{\psi}_N \gamma_\mu \gamma_5 \vec{\tau} \psi_N \cdot \vec{\rho}^\mu \times \vec{\phi}_\pi,$$

$$L_{NN\rho\rho} = -\frac{\kappa_\rho g^2_{\rho NN}}{8m_N} \bar{\psi}_N \sigma^{\mu\nu} \vec{\tau} \psi_N \cdot \vec{\rho}_\mu \times \vec{\rho}_\nu.$$

ω:

$$L_{\omega NN} = g_{\omega NN} \bar{\psi}_N \left[\gamma_\mu - \frac{\kappa_\omega}{2m_N} \sigma_{\mu\nu} \partial^\nu\right] \omega^\mu \psi_N,$$

$$L_{\omega\pi\rho} = -\frac{g_{\omega\pi\rho}}{m_\omega} \epsilon_{\mu\alpha\lambda\nu} \partial^\alpha \vec{\rho}^\mu \partial^\lambda \vec{\phi}_\pi \omega^\nu.$$

σ:

$$L_{\sigma NN} = g_{\sigma NN} \bar{\psi}_N \psi_N \phi_\sigma,$$

$$L_{\sigma\pi\pi} = -\frac{g_{\sigma\pi\pi}}{2m_\pi} \partial^\mu \vec{\phi}_\pi \partial_\mu \vec{\phi}_\pi \phi_\sigma.$$

Electromagnetic ints:

$\pi, \eta, N, \omega, \Delta, \rho, \sigma$:

$$L_{\gamma NN} = \bar{\psi}_N \left[\hat{e}_N \gamma^\mu - \frac{\hat{\kappa}_N}{2m_N} \sigma^{\mu\nu} \partial_\nu\right] \psi_N A_\mu,$$

$$L_{\gamma\pi\pi} = [\vec{\phi}_\pi \times \partial^\mu \vec{\phi}_\pi]_3 A_\mu,$$

$$L_{\gamma N\pi N} = \frac{f_{\pi NN}}{m_\pi} [\bar{\psi}_N \gamma^\mu \gamma_5 \vec{\tau} \psi_N) \times \vec{\phi}_\pi]_3 A_\mu,$$

$$L_{\gamma\rho\rho} = [(\partial^\mu \vec{\rho}^\nu - \partial^\nu \vec{\rho}^\mu) \times \vec{\rho}_\nu]_3 A_\mu,$$

$$L_{\gamma\rho\pi\pi} = -g_{\rho\pi\pi} [(\vec{\rho}^\mu \times \vec{\phi}_\pi) \times \vec{\phi}_\pi]_3 A_\mu,$$

$$L_{\gamma N\pi\Delta} = \frac{f_{\pi N\Delta}}{m_\pi} [(\bar{\psi}^\mu_\Delta \vec{T} \psi_N) \times \vec{\phi}_\pi]_3 A_\mu,$$

$$L_{\gamma N\rho N} = g_{\rho NN} \left[\frac{\kappa_\rho}{2m_N} \left(\bar{\psi}_N \frac{\vec{\tau}}{2} \sigma^{\nu\mu} \psi_N\right) \times \vec{\rho}_\nu\right]_3 A_\mu,$$

$$L_{\gamma N\Delta} = -i \bar{\psi}^\mu_\Delta \Gamma^{em,\Delta}_{\mu\nu} T_3 \psi_N A^\nu + (h.c.),$$

$$L_{\gamma\rho\pi} = \frac{g_{\rho\pi\gamma}}{m_\pi} \epsilon_{\alpha\beta\gamma\delta} \vec{\phi}_\pi \cdot (\partial^\gamma \vec{\rho}^\delta)(\partial^\alpha A^\beta),$$

$$L_{\gamma\omega\pi} = \frac{g_{\omega\pi\gamma}}{m_\pi} \epsilon_{\alpha\beta\gamma\delta} (\partial^\alpha A^\beta) \phi^3_\pi (\partial^\gamma \omega^\delta),$$

$$L_{\gamma\rho\eta} = \frac{g_{\rho\eta\gamma}}{m_\rho} \epsilon^{\mu\nu\alpha\beta} \partial_\mu \rho^3_\nu \partial_\alpha A_\beta \phi_\eta,$$

$$L_{\gamma\rho\sigma} = -\frac{g_{\rho\sigma\gamma}}{m_\rho} (\partial_\mu \rho^3_\nu)(\partial^\mu A^\nu - \partial^\nu A^\mu) \sigma,$$

$$L_{\gamma\Delta\Delta} = \bar{\psi}^\eta_\Delta \left(T^3_\Delta + \frac{1}{2}\right) \left[-\gamma^\mu g_{\eta\nu} + (g^\mu_\eta \gamma^\nu + g^\mu_\nu \gamma^\eta) + \frac{1}{3} \gamma_\eta \gamma^\mu \gamma_\nu\right] \psi^\nu_\Delta A_\mu.$$

CNS Center for Nuclear Studies

Outline

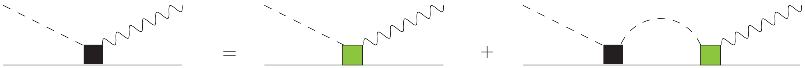
Dynamical model

Lagrangian density of preceeding page$\rightarrow$Hamiltonian density

$$H = \int d^3x\, \mathcal{H}(\mathbf{x}) = H_0 + H_{\mathrm{int}} \qquad H_{\mathrm{int}} = \sum_{M,B,B'} \Gamma_{MB,B'} + \sum_{M.M',M''} \Gamma_{MM',M''}$$

Dynamical Lippmann-Schwinger equation

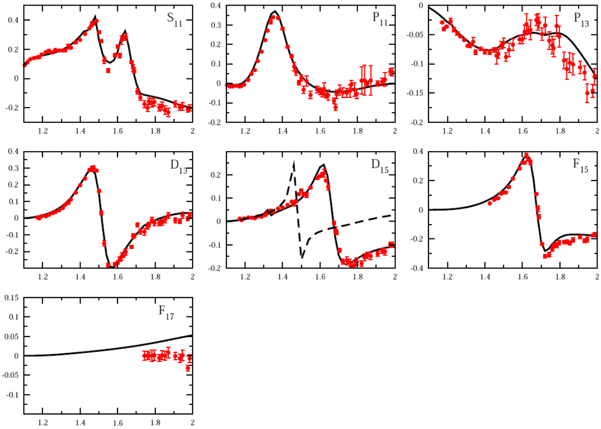
$$T = V + TG_0 V$$



Outline

Hadronic π and ω production

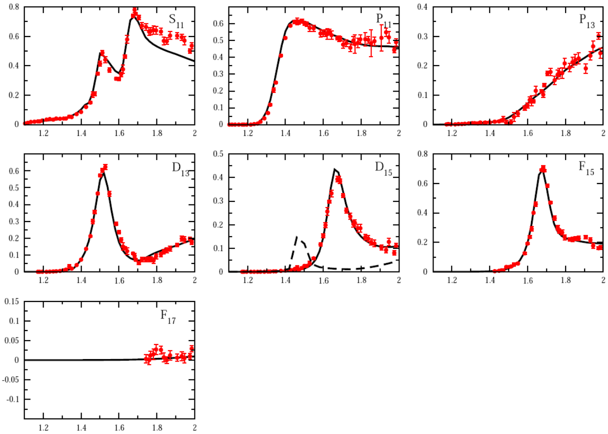
$\pi N \longrightarrow \pi N, \omega N$



Real part, isospin $1/2$

Hadronic π and ω production

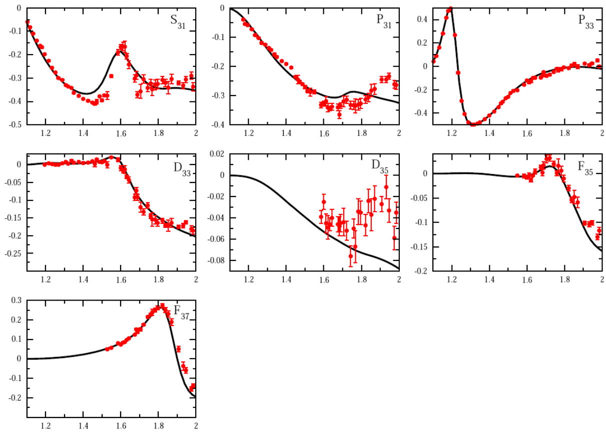
$\pi N \longrightarrow \pi N, \omega N$



Imag part, isospin 1/2

Hadronic π and ω production

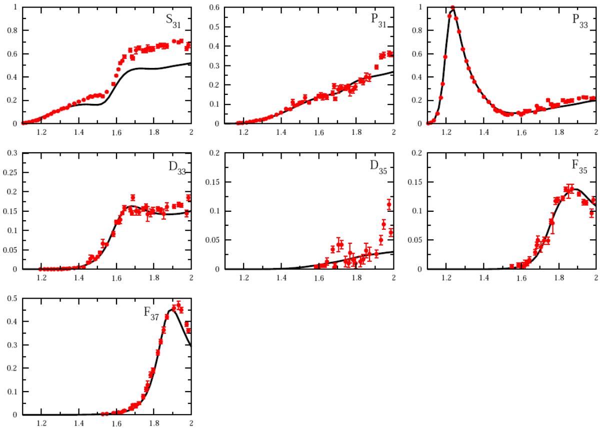
$\pi N \longrightarrow \pi N, \omega N$



Real part, isospin $3/2$

Hadronic π and ω production

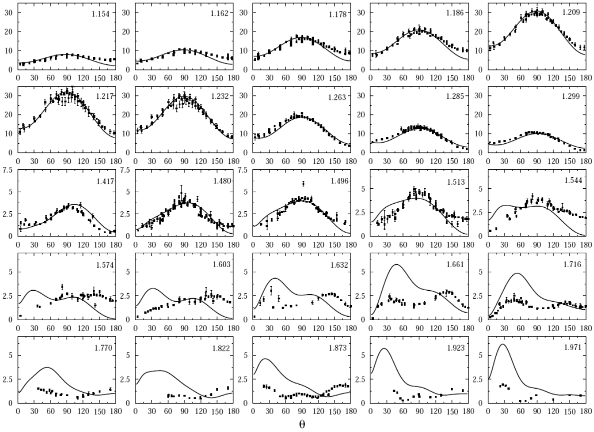
$\pi N \longrightarrow \pi N, \omega N$



Imag part, isospin 3/2

Photoproduction of π and ω production

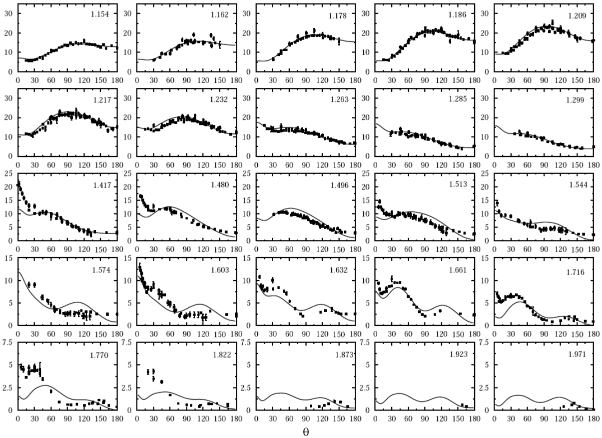
$\gamma N \longrightarrow \pi N, \omega N$



$$\frac{d\sigma}{d\Omega}\ \gamma p \longrightarrow \pi^0 p$$

Photoproduction of π and ω production

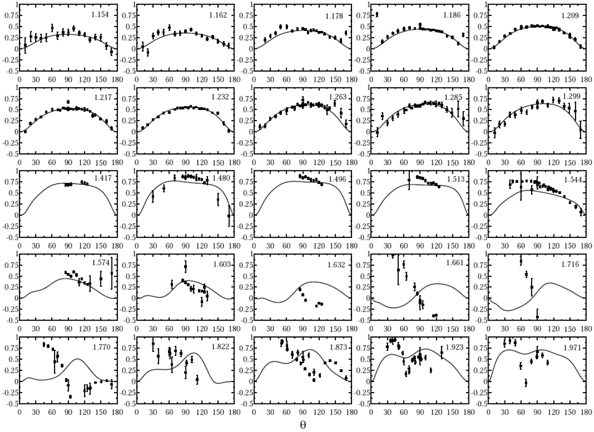
$\gamma N \longrightarrow \pi N, \omega N$



$$\frac{d\sigma}{d\Omega}\ \gamma p \longrightarrow \pi^+ n$$

Photoproduction of π and ω production

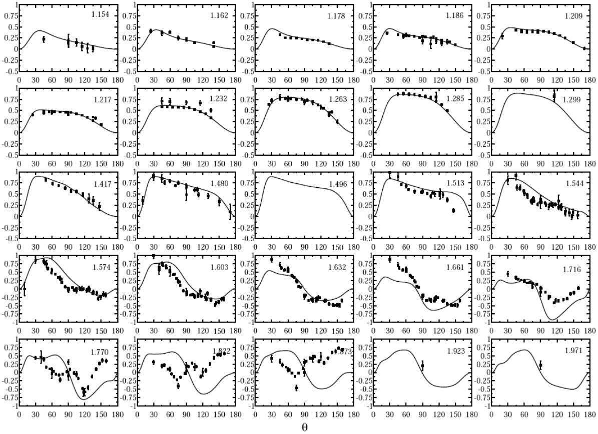
$\gamma N \longrightarrow \pi N, \omega N$



$\Sigma(W)\ \gamma p \longrightarrow \pi^0 p$

Photoproduction of π and ω production

$\gamma N \longrightarrow \pi N, \omega N$



$\Sigma(W)\ \gamma p \longrightarrow \pi^+ n$

Conclusion

- Non-perturbative QCD
  - Problem of mass in QCD requires detailed understanding of the hadroic spectrum
- Resonance
  - Signals onset of complex dynamics
- Scattering & reaction amplitudes
  - Comprehensive reaction theory required to make contact between theory and experiment
- Phenomenology
  - Provides an indispensable bridge between measured and calculated quantities
- Modeling
  - Necessarily challenging endeavor of *ab initio* calculations guided by/informs phenomenology

Dedication

To the memory of our friend and colleague, Dick Arndt, GWU Research Professor and Virginia Tech Emeritus Professor, who passed Saturday, April 10, 2010.