R-matrix analysis of reactions in the ^{9}B compound system

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Outline

- **Motivation: ENDF/ENSDF evaluations, ^{9}B in BBN/^{7}Li destruction**
- **R-matrix formalism: T-matrix/observables, EM channels, EDA code**
- **Summary of ^{9}B data: DCS, σ**
- **Analysis: χ^2/N_{data}, resolution broadening**
  - →Used earlier 3-channel evaluation by G. Hale, added capture channel
- **Resonance stucture: implications for BBN**
- **Summary, findings & future work**

Motivation

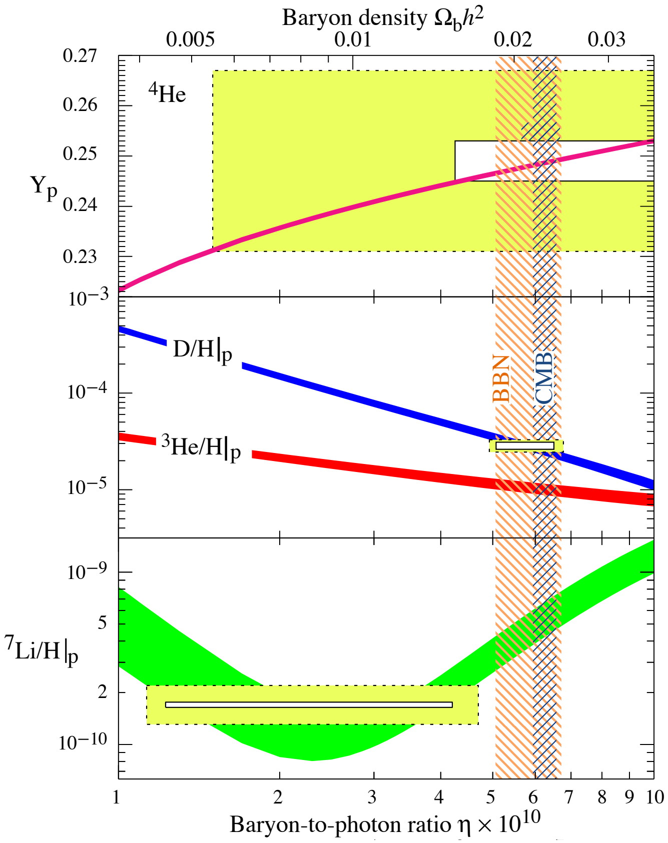
- **Cross section evaluation & resonance structure**
  - → *Nucl. Phys. A745, 155, 2004(2011)*

E_x [a] (MeV ± keV)	$J^\pi; T$	$\Gamma_{\text{c.m.}}$ (keV)	Decay
16.024 ± 25	$T = (\frac{1}{2})$	180 ± 16	
16.71 ± 100 [h]	$(\frac{5}{2}^+); (\frac{1}{2})$		
17.076 ± 4	$\frac{1}{2}^-; \frac{3}{2}$	22 ± 5	$(\gamma, {}^3\text{He})$
17.190 ± 25		120 ± 40	$\text{p, d, } {}^3\text{He}$
17.54 ± 100 [h,i]	$(\frac{7}{2}^+); (\frac{1}{2})$		
17.637 ± 10 [i]		71 ± 8	$\text{p, d, } {}^3\text{He}, \alpha$

- **Astrophysical applications**
  - → Big bang nucleosynthesis
    - Nuclear physics solution to ^7Li predicted overproduction problem? *(cf. Hoyle)*
    - Details next slide.
- **Purpose within Los Alamos Nat. Lab programmatic**
  - → Continue the R-matrix program for various end-users
  - → Ongoing/upcoming analysis releases: ^{7}Be, ^{13}C *[G. Hale Tues. Session GA 2]*, ^{14}C, ^{17}O, …

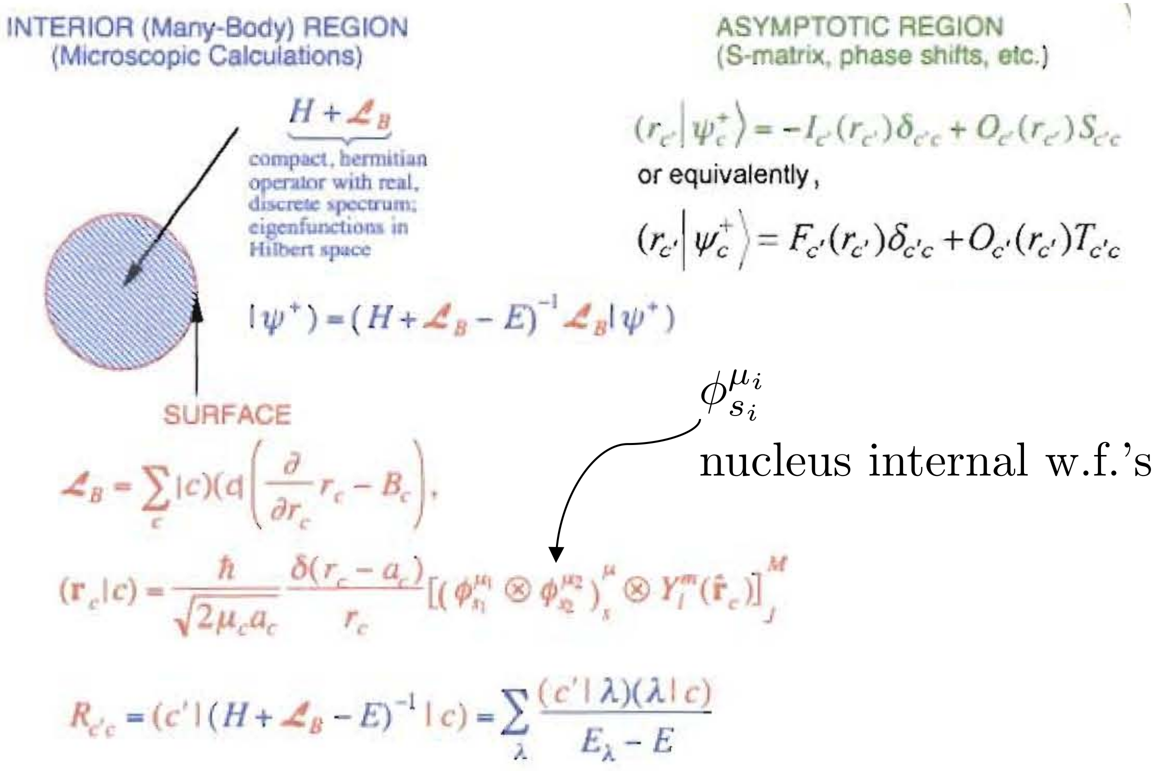
A nuclear physics solution to the BBN ^{7}Li problem?

- **Primordial nucleosynthesis**
  - →Probes early universe w/in standard model
  - →Big-bang nucleosynthesis: D,^{4}He,^{7}Li abundances
  - →D,^{4}He abundances agree with theo/expl uncertainties
  - →At η_{wmap} (CMB) ^{7}Li/H$|_{BBN}$ ~ (2.2−4.2)*^{7}Li/H$|_{halo*}$
  - →Discrepancy ~ 4.5−5.5σ→ the “Li problem”
- **Resonant destruction ^{7}Li (Hoyle-type solution)**
  - →Prod. mass 7 “well understood”; destruction not
  - →Cyburt & Pospelov *arXiv:0906.4373; IJMPE, 21(2012)*
    - ^{7}Be(d,p)αα & ^{7}Be(d,γ)^{9}B resonant enhancement
    - Identify ^{9}B $E_{5/2+}$≈16.7 MeV≈E_{thr}(d+^{7}Be)+200 keV
      - *Near threshold*
    - (E_r,Γ_d)≈(170−220,10−40) keV solve Li problem
  - →Chakraborty, Fields & Olive *PRD83, 063006 (2011)*
    - More general approach: A=8,9,10 & 11
    - Identify as possibly important: ^{9}B, ^{10}B, ^{10}C
  - →‘Large’ widths
    - Both conclude “large channel radius” required



<u>NB</u>: both approaches assume validity of TUNL-NDG tables

R-matrix formalism



INTERIOR (Many-Body) REGION
(Microscopic Calculations)

$$\underbrace{H+\mathcal{L}_B}$$
compact, hermitian operator with real, discrete spectrum; eigenfunctions in Hilbert space

$$|\psi^+)=(H+\mathcal{L}_B-E)^{-1}\mathcal{L}_B|\psi^+)$$

SURFACE

ASYMPTOTIC REGION
(S-matrix, phase shifts, etc.)

$$(r_{c'}|\psi_c^+\rangle=-I_{c'}(r_{c'})\delta_{c'c}+O_{c'}(r_{c'})S_{c'c}$$

or equivalently,

$$(r_{c'}|\psi_c^+\rangle=F_{c'}(r_{c'})\delta_{c'c}+O_{c'}(r_{c'})T_{c'c}$$

$\phi_{s_i}^{\mu_i}$
nucleus internal w.f.'s

$$\mathcal{L}_B=\sum_c|c)(c|\left(\frac{\partial}{\partial r_c}r_c-B_c\right),$$

$$(\mathbf{r}_c|c)=\frac{\hbar}{\sqrt{2\mu_c a_c}}\frac{\delta(r_c-a_c)}{r_c}[(\phi_{s_1}^{\mu_1}\otimes\phi_{s_2}^{\mu_2})_s^{\mu}\otimes Y_l^m(\hat{\mathbf{r}}_c)]_J^M$$

$$R_{c'c}=(c'|(H+\mathcal{L}_B-E)^{-1}|c)=\sum_\lambda\frac{(c'|\lambda)(\lambda|c)}{E_\lambda-E}$$

- Bloch operator $\mathcal{L}_B=\sum_c|c)(c|\left[\frac{\partial}{\partial r_c}r_c-B_c\right]$ ensures Hermiticity of Hamiltonian restricted to internal region

- R-matrix theory: unitary, multichannel parametrization of (not just resonance) data
- Interior/Exterior regions
  - →Interior: strong interactions
  - →Exterior: Coulomb/non-polarizing interactions
  - →Channel surface

$$\mathcal{S}_c : r_c = a_c \qquad \mathcal{S}=\sum_c \mathcal{S}_c$$

- R-matrix elements
  - →Projections on channel surface functions $(\mathbf{r}_c|c)$ of Green's function
    $$G_B=[H+\mathcal{L}_B-E]^{-1}$$
  - →Boundary conditions
    $$B_c=\frac{1}{u_c(a_c)}\frac{du_c}{dr_c}\Big|_{r_c=a_c}$$
- E-M channels
  - →Next slide

Electromagnetic channels

- **One-photon sector of Fock space**
  - Photon ‘wave function’

$$\mathbf{A}_{\mathbf{k}}(\mathbf{r}) = \left(\frac{2}{\pi\hbar c}\right)^{1/2} \sum_{jm} i^j \sum_{\lambda',\lambda=e,m,0} \mathbf{Y}^{(\lambda')}_{jm}(\hat{\mathbf{r}})\, u^j_{\lambda'\lambda}(r)\, \mathbf{Y}^{(\lambda)}_{jm}(\hat{\mathbf{k}}) \cdot \chi$$

  - Radial part

$$u^j_{ee} = -[f'_j(\rho) + t^j_{ee} h^{+\prime}_j(\rho)] \qquad u^j_{0e} = -\frac{\sqrt{j(j+1)}}{\rho}[f_j(\rho) + t^j_{e0} h^{+}_j(\rho)]$$

$$u^j_{mm} = \quad [f_j(\rho) + t^j_{mm} h^{+}_j(\rho)] \qquad u^j_{0m} = u^j_{me} = u^j_{em} = 0$$

  - Photon channel surface functions

$$(\mathbf{r}_c|c) = \left(\frac{\hbar c}{2\rho_\gamma}\right)^{1/2} \frac{\delta(r_\gamma - a_\gamma)}{r_\gamma} \left[\phi_{s\nu} \otimes \mathbf{Y}^{(e,m)}_{jm}(\hat{\mathbf{r}}_\gamma)\right]_{JM}$$

    - Photon ‘mass’: $\hbar k_\gamma / c$

  - R-matrix definition preserved

$$(c'|\psi) = \sum_c R^B_{c'c}(c|\frac{\partial}{\partial r_c} r_c - B_c|\psi)$$

$$\mathbf{T} = \rho^{1/2}\mathbf{O}^{-1}\mathbf{R}_{\mathbf{L}}\mathbf{O}^{-1}\rho^{1/2} - \mathbf{F}\mathbf{O}^{-1}$$

$$\mathbf{R}_L = [\mathbf{R}_B^{-1} - \mathbf{L} + \mathbf{B}]^{-1}$$

$$\mathbf{L} = \rho\mathbf{O}'\mathbf{O}^{-1}$$

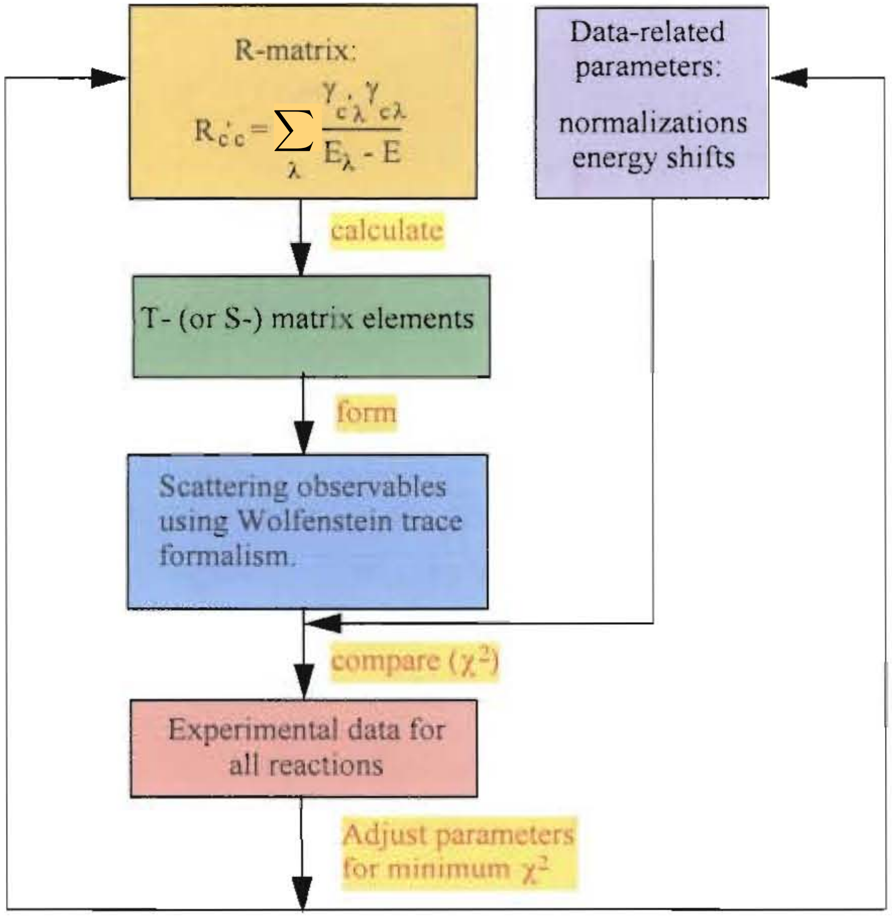
$$F = \mathrm{Im}\ \mathbf{O}$$

Implementation in EDA

- **EDA = Energy Dependent Analysis**
  - →Adjust E_λ & $\gamma_{c\lambda}$
- **Any number of two-body channels**
  - →Arbitrary spins, masses, charges (incl. mass zero)
- **Scattering observables**
  - →Wolfenstein trace formalism
- **Data**
  - →Normalization
  - →Energy shifts
  - →Energy resolution/spread
- **Fit solution**

$$\chi^2_{EDA} = \sum_i \left[\frac{nX_i(\mathbf{p}) - R_i}{\delta R_i}\right]^2 + \left[\frac{nS - 1}{\delta S/S}\right]^2$$

- **Covariance determined**



R-matrix:

$$R_{c'c} = \sum_\lambda \frac{\gamma_{c'\lambda}\gamma_{c\lambda}}{E_\lambda - E}$$

Data-related parameters:

normalizations
energy shifts

calculate

T- (or S-) matrix elements

form

Scattering observables using Wolfenstein trace formalism

compare (χ^2)

Experimental data for all reactions

Adjust parameters for minimum χ^2

Summary of included ^{9}B data

- **^{6}Li+^{3}He elastic** *Buzhinski et.al., Izv. Rossiiskoi Akademii Nauk, Ser.Fiz., Vol.43, p.158 (1979)*
  - → Differential cross section
  - → 1.30 MeV < E(^{3}He) < 1.97 MeV
- **^{6}Li+^{3}He $\rightarrow$ p+^{8}Be*** *Elwyn et.al., Phys. Rev. C 22, 1406 (1980)*
  - → Integrated cross section
  - → Quasi-two-body, excited-state averaged final channel
  - → 0.66 MeV < E(^{3}He) < 5.00 MeV
- **^{6}Li+^{3}He $\rightarrow$ d+^{7}Be** *D.W. Barr & J.S. Gilmore, unpublished (1965)*
  - → Integrated cross section
  - → 0.42 MeV < E(^{3}He) < 4.94 MeV
- **^{6}Li+^{3}He $\rightarrow \gamma$+^{9}B** *Aleksic & Popic, Fizika 10, 273-278 (1978)*
  - → Integrated cross section
  - → 0.7 MeV < E(^{3}He) < 0.825 MeV
  - → New to ^{9}B analysis
- **Data for future evaluation**
  - → Separate ^{8}Be* states
    - 2^{+}@200 keV [16.9 MeV], 1^{+}@650 keV [17.6 MeV], 1^{+}@1.1 MeV[18.2 MeV]
  - → n+^{8}B: E_{thresh}(^{3}He) = 3 MeV
  - → Simultaneous analysis with ^{9}Be mirror system

All data from
EXFOR/CSISRS
database (in
C4 format)

R-matrix configuration in EDA code

- Hadronic channels (in blue, not included)

$A_1A_2^\pi$	$^3\mathrm{He}^6\mathrm{Li}^+$(1)		$p^8\mathrm{Be}^{*+}$(2)		$d^7\mathrm{Be}^-$(3)		
S / ℓ	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{3}{2}$	$\frac{1}{2}$
0	$^4S_{3/2}$	$^2S_{1/2}$	$^6S_{5/2}$	$^4S_{3/2}$	$^6S_{5/2}$	$^4S_{3/2}$	$^2S_{1/2}$
1	$^4P_{5/2,3/2,1/2}$	$^2P_{3/2,1/2}$	$^6P_{7/2,5/2,3/2}$	$^4P_{5/2,3/2,1/2}$	$^6P_{7/2,5/2,3/2}$	$^4P_{5/2,3/2,1/2}$	$^2P_{3/2,1/2}$
2	$^4D_{7/2,5/2,3/2,1/2}$	$^2D_{5/2,3/2}$	$^6D_{9/2,7/2,5/2,3/2,1/2}$	$^4D_{7/2,5/2,3/2,1/2}$	$^6D_{9/2,7/2,5/2,3/2,1/2}$	$^4D_{7/2,5/2,3/2,1/2}$	$^2D_{5/2,3/2}$
E_{thr}(CM, MeV)	16.6		16.7				16.5

Electromagnetic channel: $\gamma + {}^9B \rightarrow E_1^{3/2}, M_1^{5/2}, M_1^{3/2}, M_1^{1/2}, E_1^{5/2}, E_1^{1/2}$

<u>Full model space</u>: *state number; channel pair; LS; J; channel radius [fm]*

```
 1     1 4s 3/2      7.50000000f          20     1 4p 1/2      7.50000000f
 2     1 4d 3/2      7.50000000f          21     1 2p 1/2      7.50000000f
 3     1 2d 3/2      7.50000000f          22     2 4p 1/2      5.50000000f
 4     2 4s 3/2      5.50000000f          23     3 2s 1/2      7.00000000f
 5     3 6p 3/2      7.00000000f          24     4 M1 1/2     50.00000000f
 6     3 4p 3/2      7.00000000f          25     1 4d 7/2      7.50000000f
 7     3 2p 3/2      7.00000000f          26     3 6p 7/2      7.00000000f
 8     4 E1 3/2     50.00000000f          27     1 4d 5/2      7.50000000f
 9     1 4p 5/2      7.50000000f          28     1 2d 5/2      7.50000000f
10     2 6p 5/2      5.50000000f          29     2 6s 5/2      5.50000000f
11     2 4p 5/2      5.50000000f          30     3 6p 5/2      7.00000000f
12     3 6s 5/2      7.00000000f          31     3 4p 5/2      7.00000000f
13     4 M1 5/2     50.00000000f          32     4 E1 5/2     50.00000000f
14     1 4p 3/2      7.50000000f          33     1 4d 1/2      7.50000000f
15     1 2p 3/2      7.50000000f          34     1 2s 1/2      7.50000000f
16     2 6p 3/2      5.50000000f          35     3 4p 1/2      7.00000000f
17     2 4p 3/2      5.50000000f          36     3 2p 1/2      7.00000000f
18     3 4s 3/2      7.00000000f          37     4 E1 1/2     50.00000000f
19     4 M1 3/2     50.00000000f          38     2 6p 7/2      5.50000000f
```

Analysis result: resonance structure

Ex(MeV)	Jpi	Gamma(keV)	Er(MeV)	ImEr(MeV)	E(3He)	Strength
16.46539	1/2-	768.46	-.1369	-0.3842	-0.2054	0.06 weak
17.11317	1/2-	0.14	0.5109	-0.6771E-04	0.7664	1.00 strong
17.20115	5/2-	871.63	0.5989	-0.4358	0.8984	0.40 weak
17.28086	3/2-	147.78	0.6785	-0.0739	1.0178	0.77 strong
17.66538	5/2+	33.33	1.0631	-0.0167	1.5947	0.98 strong
17.83619	7/2+	2036.21	1.2339	-1.0181	1.8509	0.15 weak
17.84773	3/2-	42.52	1.2454	-0.0213	1.8681	0.97 strong
18.04821	3/2+	767.11	1.4459	-0.3836	2.1689	0.54 weak
18.42292	1/2+	5446.32	1.8206	-2.7232	2.7309	0.03 weak
18.67716	1/2-	10278.41	2.0749	-5.1392	3.1124	0.15 weak
19.60923	3/2-	1478.22	3.0069	-0.7391	4.5104	0.52 weak

S-matrix pole & residue *Hale, Brown, Jarmie PRL 59 ‘87*

$$\mathcal{E}_{\lambda'\lambda} = E_\lambda \delta_{\lambda'\lambda} - \sum_c \gamma_{c\lambda'}[L_c(E) - B_c]\gamma_{c\lambda}$$

$$E_0 = E_r - i\Gamma/2 \qquad \text{residue: } i\rho_0\rho_0^T$$

NB: no strong resonance seen ~100 keV of ^{3}He+^{6}Li threshold

$$\text{strength} = \frac{1}{\Gamma}\rho_0^\dagger\rho_0 = \frac{1}{\Gamma}\sum_c \Gamma_c$$

$$\rho_{0c} = \left(\frac{2k_{0c}a_c}{N}\right)^{1/2} \mathcal{O}_c^{-1}(k_{0c}a_c)\sum_\lambda(\lambda|\mu_0)$$

$$N = \sum_{\lambda'\lambda}(\lambda|\mu_0)(\lambda'|\mu_0)\left[\delta_{\lambda'\lambda} + \sum_c \gamma_{c\lambda'}\frac{\partial L_c}{\partial E}\bigg|_{E=E_0}\gamma_{c\lambda}\right]$$

$$L_c = r_c\frac{\partial \mathcal{O}_c}{\partial r_c}\mathcal{O}_c^{-1}\bigg|_{r_c=a_c}$$

Analysis result: resonance structure

```
 Ex(MeV)        Jpi    Gamma(keV)  Er(MeV)    ImEr(MeV)     E(3He)     Strength
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17.66538        5/2+      33.33    1.0631     -0.0167       1.5947     0.98 strong
17.83619        7/2+    2036.21    1.2339     -1.0181       1.8509     0.15 weak
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```

TUNL-NDG/ENSDF parameters

E_x [a] (MeV ± keV)	$J^\pi; T$	$\Gamma_{\text{c.m.}}$ (keV)	Decay
16.024 ± 25	$T = (\frac{1}{2})$	180 ± 16	
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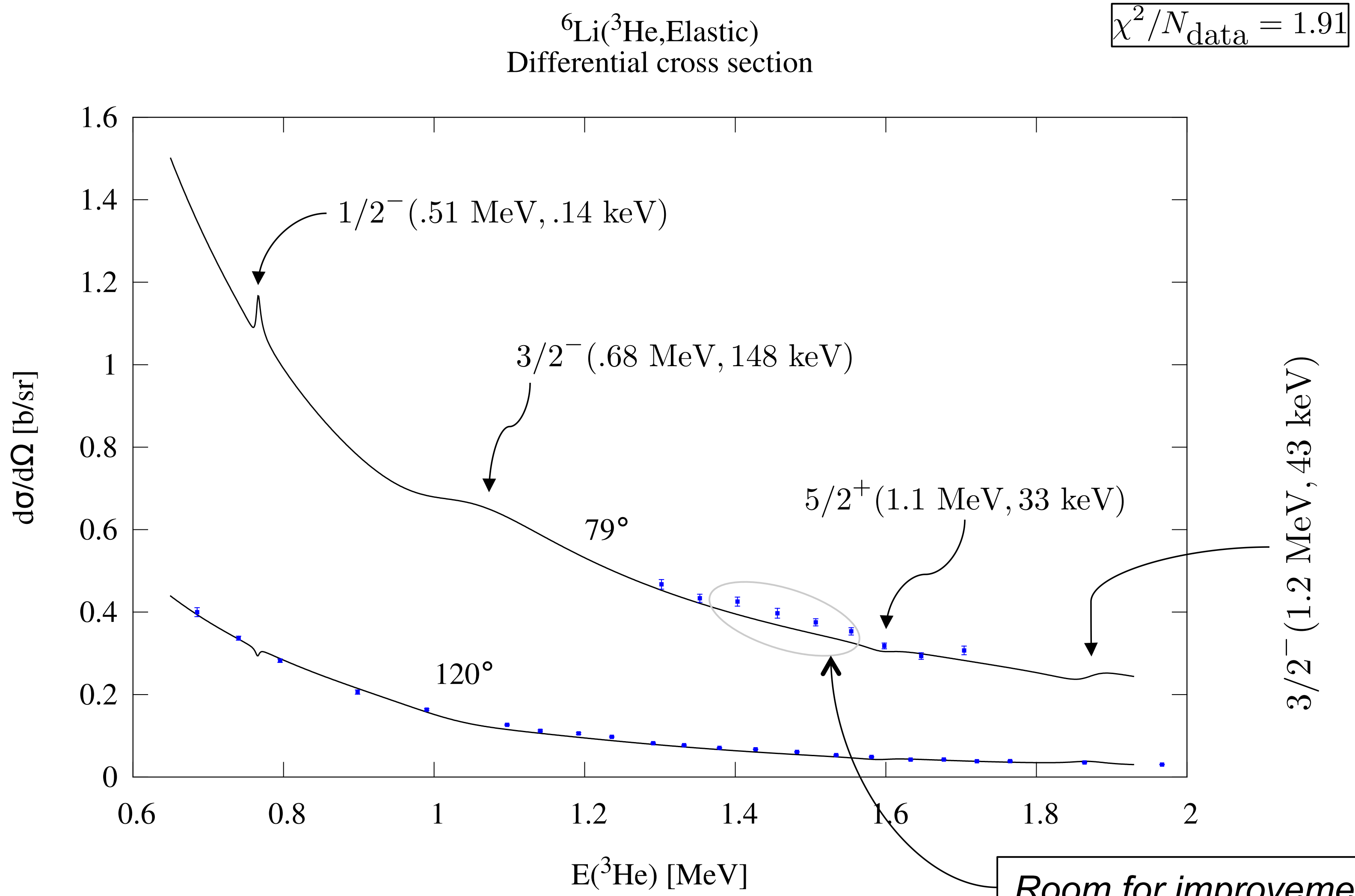
17.076 ± 4	$\frac{1}{2}^-; \frac{3}{2}$	22 ± 5	$(\gamma, {}^3\mathrm{He})$
17.190 ± 25		120 ± 40	p, d, ^{3}He
17.54 ± 100 [h,i]	$(\frac{7}{2}^+); (\frac{1}{2})$		
17.637 ± 10 [i]		71 ± 8	p, d, ^{3}He, α

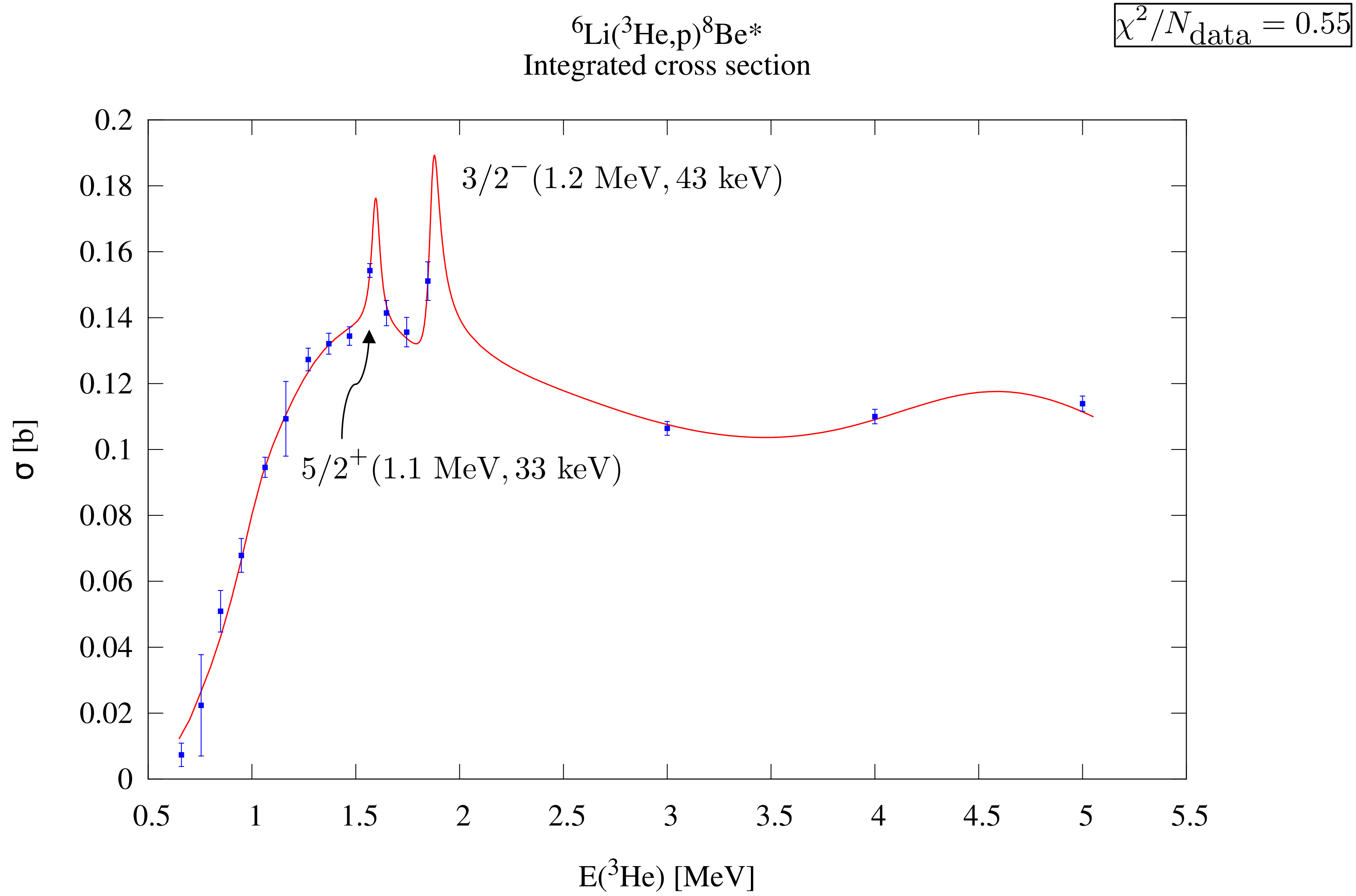
NB: no strong resonance seen ~100 keV of ^{3}He+^{6}Li threshold

Observable fit: ^{3}He+^{6}Li elastic DCS

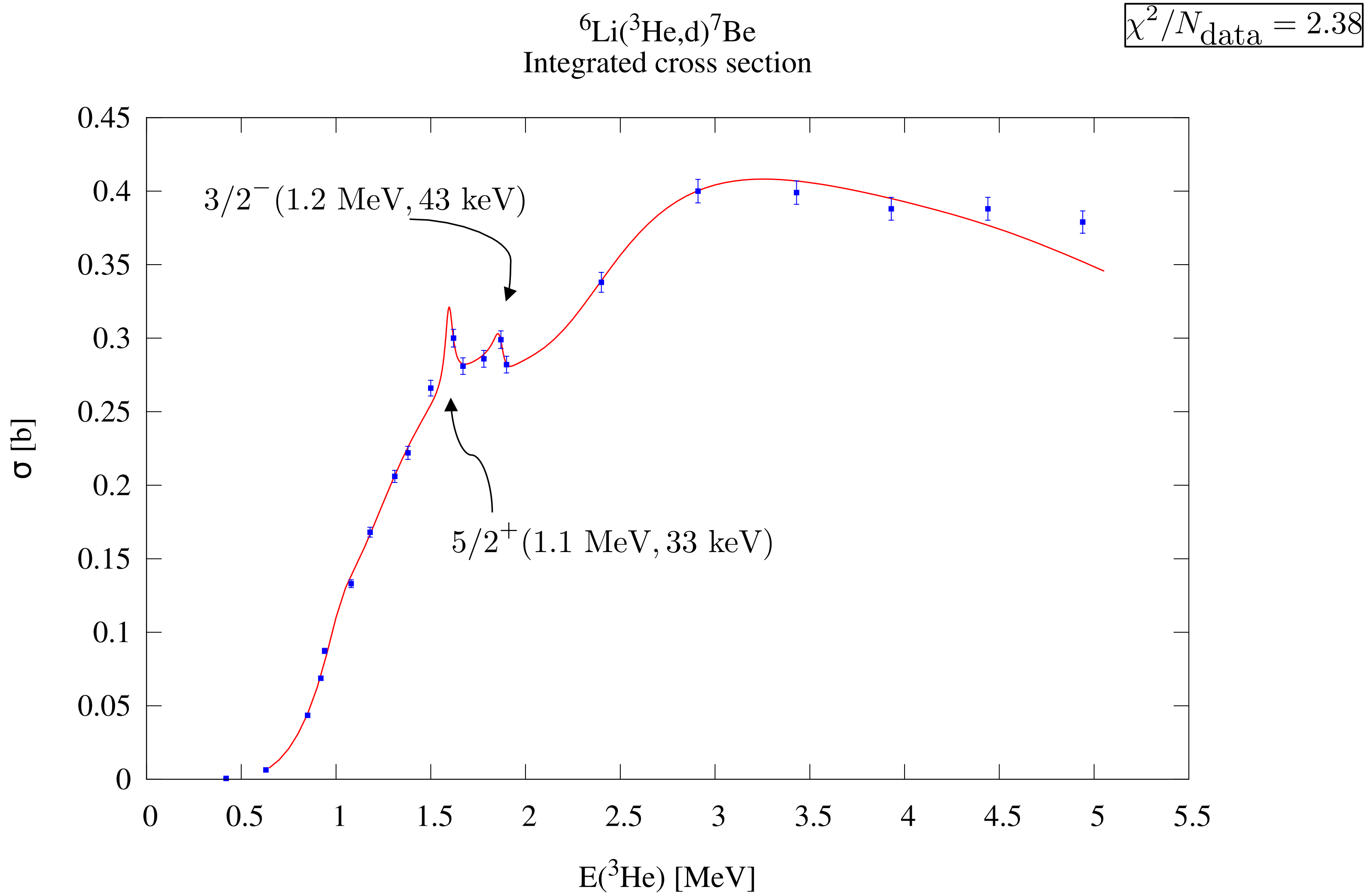


Room for improvement

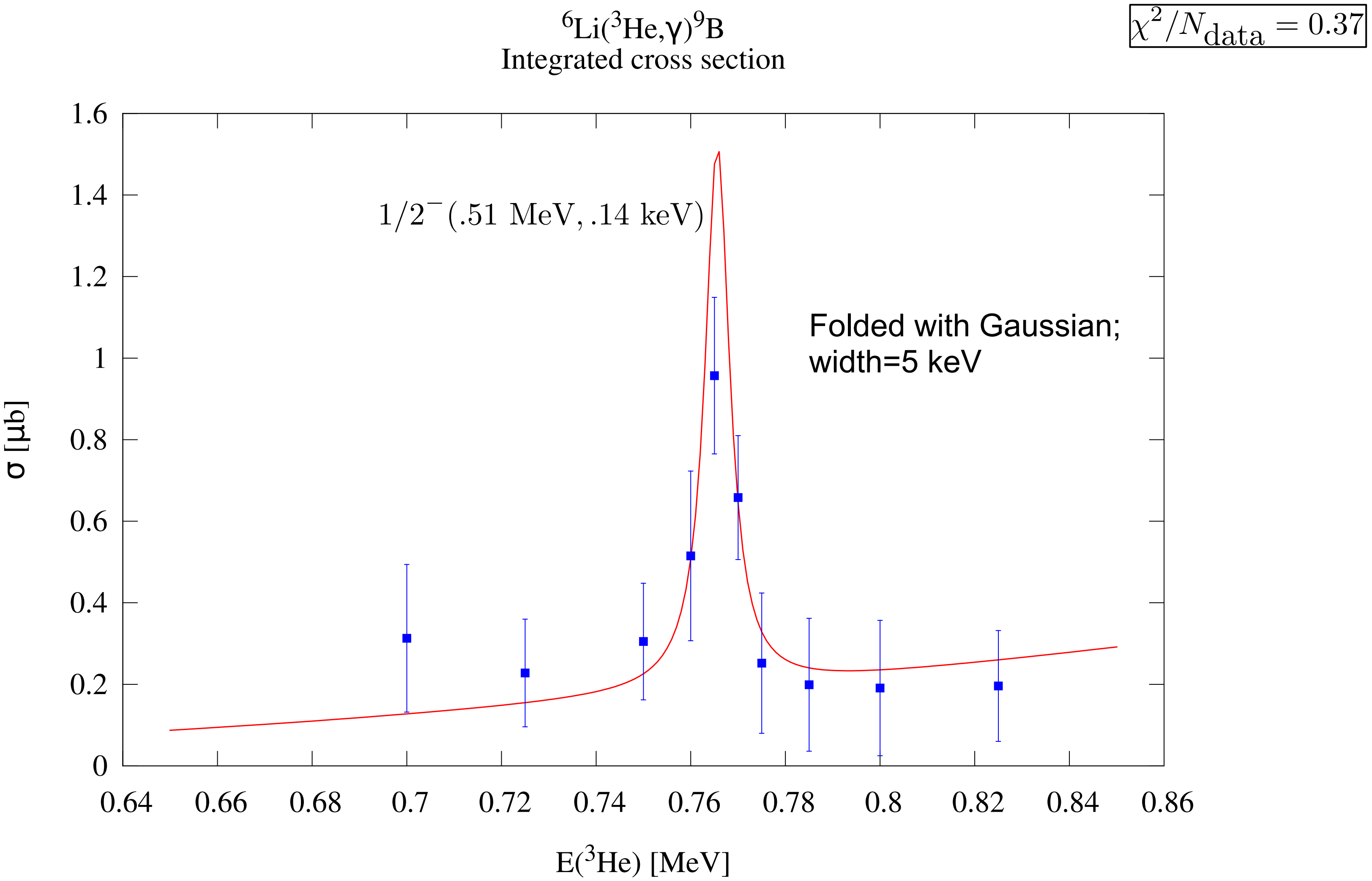
Observable fit: ^{6}Li(^{3}He,p)^{8}Be* integrated x-sec



Observable fit: ^{6}Li(^{3}He,d)^{7}Be integrated x-sec



Observable fit: $^{6}Li(^{3}He,\gamma)^{9}B$ integrated x-sec



Summary, findings & future work

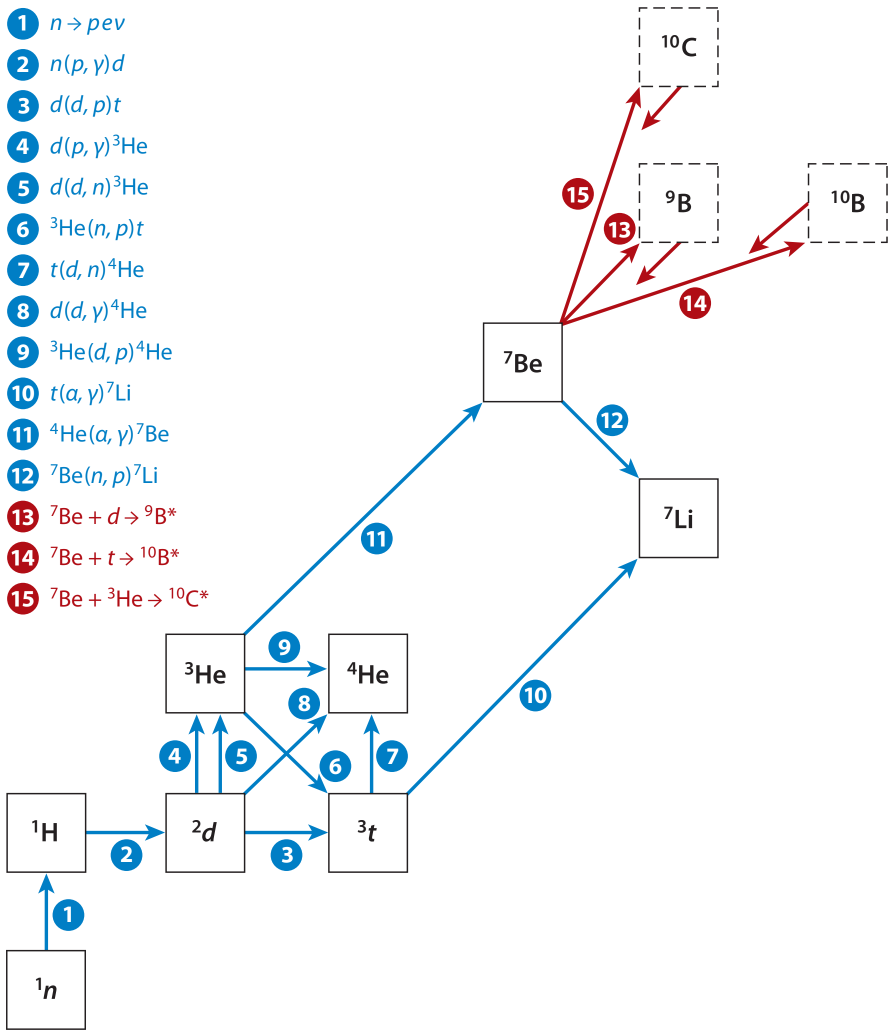
- Nuclear physics/microphysics explanations for the “Li problem” have been entertained
- There are no resonances in ^{9}B that reside within ~200 (~100) keV of the d+^{7}Be (^{3}He+^{6}Li) threshold with ‘large’ widths 10—40 keV
- This would appear to rule out scenarios considered by *Cyburt & Pospelov (2009)* and *Chakraborty, Fields & Olive(2011)* that low-lying, robust resonance in ^{9}B could explain the “Li problem”
- It may be worth emphasizing that other nuclear physics explanations, such as insufficiently accurate and/or precise analyses of “known” nuclear reactions, may still be considered for the resolution of the “Li problem”
- While very useful, the TUNL-NDG/ENSDF tables may not be definitive; unitary analyses are req. and sometimes lacking; TO DO: submit new ^{9}B analysis
- ***Need for dedicated, low-energy, high pol. facility***
- Improvements in the present analysis: more channels; incorporate p+^{8}Be* angular data; proper treatment three-body final states

Supplementary material

Additional slides follow

BBN reaction network (simplified)

- **Fields** *Annu. Rev. Nucl. Part. Sci. 2011. 61:47–68*



① $n \to p e \nu$

② $n(p,\gamma)d$

③ $d(d,p)t$

④ $d(p,\gamma)^3\mathrm{He}$

⑤ $d(d,n)^3\mathrm{He}$

⑥ $^3\mathrm{He}(n,p)t$

⑦ $t(d,n)^4\mathrm{He}$

⑧ $d(d,\gamma)^4\mathrm{He}$

⑨ $^3\mathrm{He}(d,p)^4\mathrm{He}$

⑩ $t(\alpha,\gamma)^7\mathrm{Li}$

⑪ $^4\mathrm{He}(\alpha,\gamma)^7\mathrm{Be}$

⑫ $^7\mathrm{Be}(n,p)^7\mathrm{Li}$

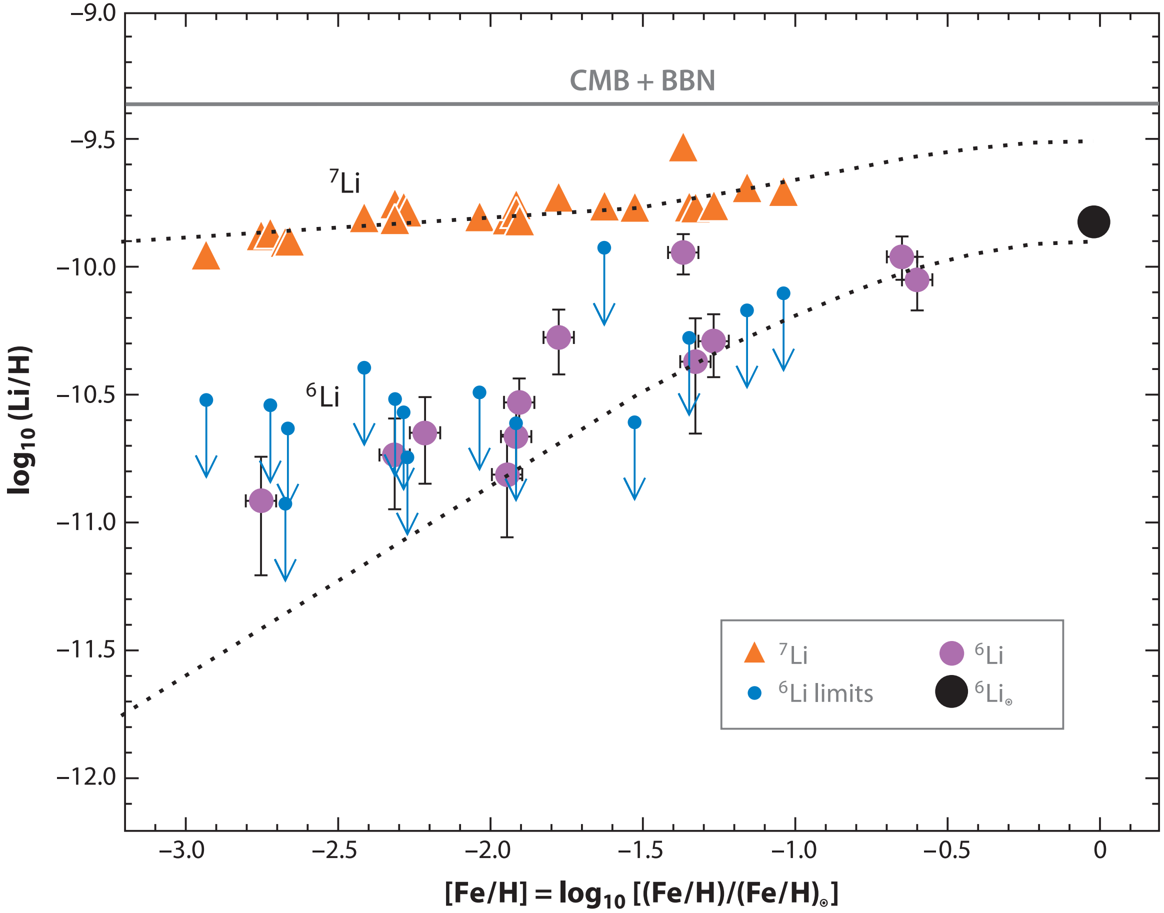
⑬ $^7\mathrm{Be} + d \to {}^9\mathrm{B}^*$

⑭ $^7\mathrm{Be} + t \to {}^{10}\mathrm{B}^*$

⑮ $^7\mathrm{Be} + {}^3\mathrm{He} \to {}^{10}\mathrm{C}^*$

Spite Plateau

- **Measurement of primordial 7Li from low-metallicity halo dwarf stars**



Asplund M, et al. Astrophys. J. 644:229 (2006)