Unified description of hadro- and photoproduction amplitudes

Mark Paris

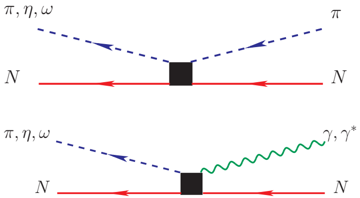
DAC members:
W. Briscoe, I. Strakovsky, & R. Workman
with special thanks to Dick Arndt 1933/01/03 – 2010/04/10

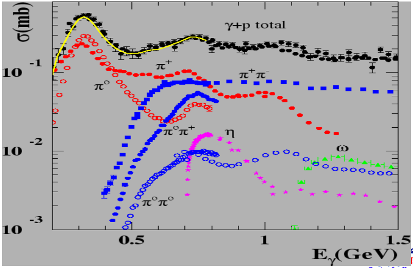
Data Analysis Center
Center for Nuclear Studies
The George Washington University

European Centre for Theoretical Studies in Nuclear Physics and Related Areas
Workshop on Amplitude Analysis & Hadron Spectroscopy
Villazzano, Italy
25 January 2011

Overview of SAID

- Data Analysis Center/Center for Nuclear Studies
  - SAID[a]: suite of programs to analyze $2 \to 2$ & 3 body data
  - Routines: database, fit, and analysis
  - Dedicated effort: analyze/interpret the terabytes of experimental data issuing from Bonn, JLab, Lund, Mainz, ...
- Reactions: $\pi N \to \pi N, \eta N, \ldots$; $\gamma N \to \pi N, \eta N, \omega N, \ldots$
- Objectives: **model independent amplitudes**; unified hadro- & electo-prod; study resonances & QCD
- Uses
  - Verify models vs. data
  - Experimental planning
  - Simulations/event gen: Astrophysics; Nuclear reactions; Detector design/calibration





[a] Web: `http://gwdac.phys.gwu.edu/`
ssh: `ssh -X said@said.phys.gwu.edu` `[passwordless]`

Outline

1. Observables & amplitudes
   - Hadroproduction
   - Photoproduction
   - Experiments & Data
2. Formalism
   - Models & parametrizations
   - Unitarity
   - Parametrizations
3. Results $\gamma N \to \pi N, \eta N$
   - Exploratory study
   - Pion Photoproduction

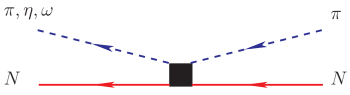
Outline

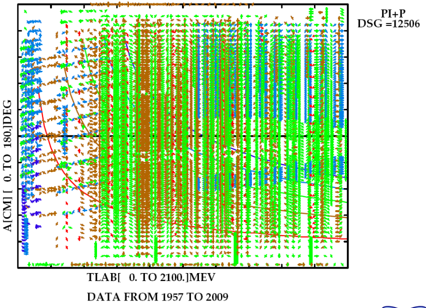
1. Observables & amplitudes
   - Hadroproduction
   - Photoproduction
   - Experiments & Data
2. Formalism
   - Models & parametrizations
   - Unitarity
   - Parametrizations
3. Results $\gamma N \to \pi N, \eta N$
   - Exploratory study
   - Pion Photoproduction

Hadronic observables

$\pi N \to \pi N, \eta N$

- How much info req'd? $\to$ Count states
  $$\prod_{i=1}^{4}(2\lambda_i+1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{1}_{\pi}$$
  $= 4$ spin combinations
- Parity reduces by factor 2
  $$\langle \lambda'_N | T | \lambda_N \rangle = \langle -\lambda'_N | T | -\lambda_N \rangle$$
  2 $\mathbb{C}$ amplitudes $\to$ 4 $\mathbb{R}$ real $\mathcal{O}_p(\theta; W)$
- 4 $\mathbb{R}$ observables: $\mathcal{O}_p(\theta; W)$
  $\to \frac{d\sigma}{d\Omega}$ – unpol. diff. cross section
  $\to P$ – recoil nucleon transverse pol.
  $\to R$ – spin rot. $\parallel$ $\mathbf{p}_f$
  $\to A$ – spin rot. $\perp$ $\mathbf{p}_f$
- Constraint: $P^2 + R^2 + A^2 = 1$
  $\to$ **3** indep. obs's
  $\to$ Overall phase **fixed by unitarity**

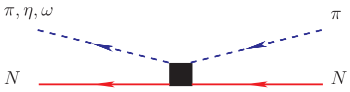


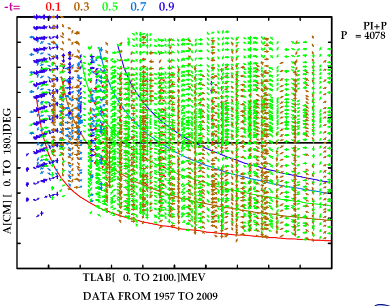


Hadronic observables

$\pi N \to \pi N, \eta N$

- How much info req'd? → Count states
  $$\prod_{i=1}^{4}(2\lambda_i+1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{1}_{\pi}$$
  $= 4$ spin combinations
- Parity reduces by factor 2
  $$\langle \lambda'_N | T | \lambda_N \rangle = \langle -\lambda'_N | T | -\lambda_N \rangle$$
  2 $\mathbb{C}$ amplitudes → 4 $\mathbb{R}$ real $\mathcal{O}_p(\theta; W)$
- 4 $\mathbb{R}$ observables: $\mathcal{O}_p(\theta; W)$
  → $\frac{d\sigma}{d\Omega}$ – unpol. diff. cross section
  → P – recoil nucleon transverse pol.
  → R – spin rot. $\parallel$ $\mathbf{p}_f$
  → A – spin rot. $\perp$ $\mathbf{p}_f$
- Constraint: $P^2 + R^2 + A^2 = 1$
  → **3** indep. obs's
  → Overall phase **fixed by unitarity**

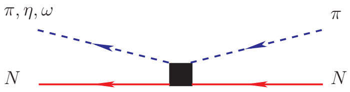


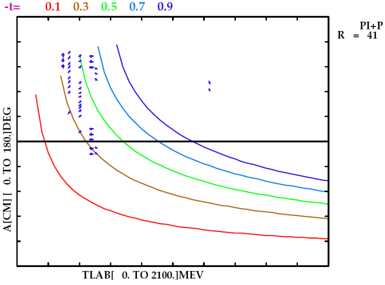


Hadronic observables

$\pi N \longrightarrow \pi N, \eta N$

- How much info req'd? $\rightarrow$ Count states
  $\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{1}_{\pi}$
  $= 4$ spin combinations
- Parity reduces by factor 2
  $$\langle \lambda'_N | T | \lambda_N \rangle = \langle -\lambda'_N | T | -\lambda_N \rangle$$
  2 $\mathbb{C}$ amplitudes $\rightarrow$ 4 $\mathbb{R}$ real $\mathcal{O}_p(\theta; W)$
- 4 $\mathbb{R}$ observables: $\mathcal{O}_p(\theta; W)$
  $\rightarrow$ $\frac{d\sigma}{d\Omega}$ – unpol. diff. cross section
  $\rightarrow$ P – recoil nucleon transverse pol.
  $\rightarrow$ R – spin rot. $\parallel$ $\mathbf{p}_f$
  $\rightarrow$ A – spin rot. $\perp$ $\mathbf{p}_f$
- Constraint: $P^2 + R^2 + A^2 = 1$
  $\rightarrow$ **3** indep. obs's
  $\rightarrow$ Overall phase **fixed by unitarity**





Hadronic observables

$\pi N \to \pi N, \eta N$

- How much info req'd? → Count states

  $\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{1}_{\pi}$

  $= 4$ spin combinations

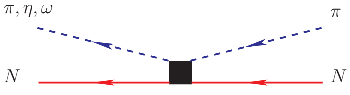
- Parity reduces by factor 2

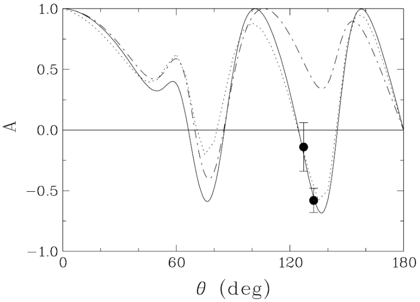
  $$\langle \lambda'_N | T | \lambda_N \rangle = \langle -\lambda'_N | T | -\lambda_N \rangle$$

  2 $\mathbb{C}$ amplitudes → 4 $\mathbb{R}$ real $\mathcal{O}_p(\theta; W)$

- 4 $\mathbb{R}$ observables: $\mathcal{O}_p(\theta; W)$
  → $\frac{d\sigma}{d\Omega}$ – unpol. diff. cross section
  → P – recoil nucleon transverse pol.
  → R – spin rot. $\parallel$ $\mathbf{p}_f$
  → A – spin rot. $\perp$ $\mathbf{p}_f$

- Constraint: $P^2 + R^2 + A^2 = 1$
  → **3** indep. obs's
  → Overall phase **fixed by unitarity**





What a unitary param.
does for you: no re-fit

Outline

Photoproduction observables

$\gamma N \to \pi N, \eta N$

- State count $\rightarrow$ *NB*: $\lambda_\gamma = \frac{1}{2}$

  $\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{2}_{\gamma}$

  $= 8$ spin combinations
- Parity reduces by factor 2

  $\rightarrow \lambda = \lambda_\gamma - \lambda_N$

  $$\langle \lambda'_N | T | \lambda \rangle = \langle -\lambda'_N | T | -\lambda \rangle$$

  4 $\mathbb{C}$ amplitudes $\rightarrow$ 8 $\mathbb{R}$ real $\mathcal{O}_p(W)$
- 16 $\mathbb{R}$ observables: *eg.*

  $\rightarrow \frac{d\sigma}{d\Omega}$ – unpol diff cross section

  $\rightarrow P$ – recoil transverse asymm

  $\rightarrow \Sigma$ – lin pol photon beam asymm

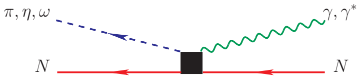
  $\rightarrow T$ – longitudinal target asymm

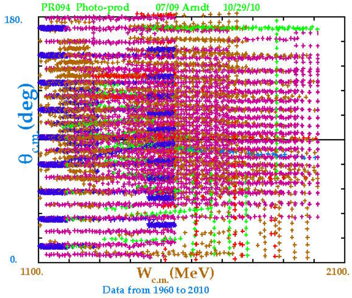
  $\rightarrow E$ – beam-target helicity asymm
- Constraints: *eg.*

  $-P^2 + \Sigma^2 + T^2 + E^2 + F^2 + G^2 = 1$

  $FG + \Sigma T = P + EH$

  $\rightarrow$ **8** indep. obs's [discrete ambiguity]



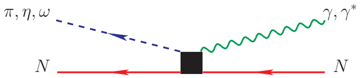


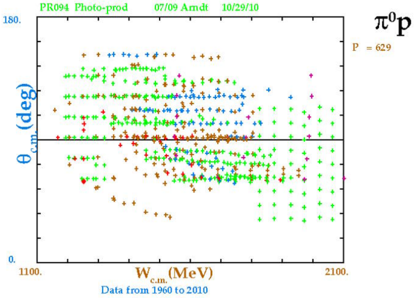
Photoproduction observables

$\gamma N \to \pi N, \eta N$

- State count $\to$ *NB*: $\lambda_\gamma = \frac{1}{2}$

  $$\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{2}_{\gamma}$$
  $= 8$ spin combinations
- Parity reduces by factor 2
  $\to \lambda = \lambda_\gamma - \lambda_N$
  $$\langle \lambda'_N | T | \lambda \rangle = \langle -\lambda'_N | T | -\lambda \rangle$$
  4 $\mathbb{C}$ amplitudes $\to$ 8 $\mathbb{R}$ real $\mathcal{O}_p(W)$
- 16 $\mathbb{R}$ observables: *eg.*
  $\to \frac{d\sigma}{d\Omega}$ – unpol diff cross section
  $\to P$ – recoil transverse asymm
  $\to \Sigma$ – lin pol photon beam asymm
  $\to T$ – longitudinal target asymm
  $\to E$ – beam-target helicity asymm
- Constraints: *eg.*
  $$-P^2 + \Sigma^2 + T^2 + E^2 + F^2 + G^2 = 1$$
  $$FG + \Sigma T = P + EH$$
  $\to$ **8** indep. obs's [discrete ambiguity]





Photoproduction observables

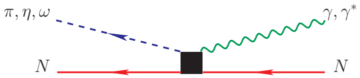
$\gamma N \to \pi N, \eta N$

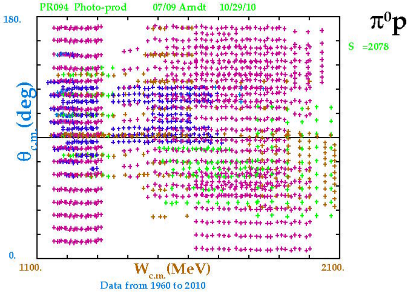
- State count → *NB*: $\lambda_\gamma = \frac{1}{2}$

  $\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{2}_{\gamma}$
  $= 8$ spin combinations
- Parity reduces by factor 2
  $\to \lambda = \lambda_\gamma - \lambda_N$

  $$\langle \lambda'_N | T | \lambda \rangle = \langle -\lambda'_N | T | -\lambda \rangle$$

  4 $\mathbb{C}$ amplitudes → 8 $\mathbb{R}$ real $\mathcal{O}_p(W)$
- 16 $\mathbb{R}$ observables: *eg.*
  $\to \frac{d\sigma}{d\Omega}$ – unpol diff cross section
  $\to P$ – recoil transverse asymm
  $\to \Sigma$ – lin pol photon beam asymm
  $\to T$ – longitudinal target asymm
  $\to E$ – beam-target helicity asymm
- Constraints: *eg.*
  $-P^2 + \Sigma^2 + T^2 + E^2 + F^2 + G^2 = 1$
  $FG + \Sigma T = P + EH$
  → **8** indep. obs's [discrete ambiguity]





Photoproduction observables

$\gamma N \to \pi N, \eta N$

- State count $\to$ *NB*: $\lambda_\gamma = \frac{1}{2}$

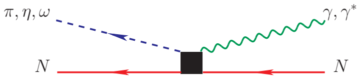
  $$\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{2}_{\gamma}$$
  $= 8$ spin combinations
- Parity reduces by factor 2
  $\to \lambda = \lambda_\gamma - \lambda_N$

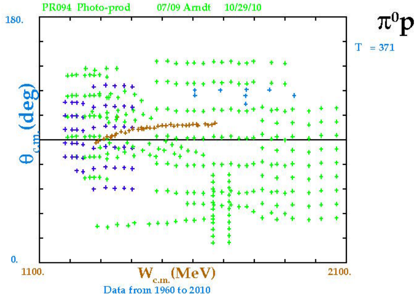
  $$\langle \lambda'_N | T | \lambda \rangle = \langle -\lambda'_N | T | -\lambda \rangle$$

  4 $\mathbb{C}$ amplitudes $\to$ 8 $\mathbb{R}$ real $\mathcal{O}_p(W)$
- 16 $\mathbb{R}$ observables: *eg.*
  $\to \frac{d\sigma}{d\Omega}$ – unpol diff cross section
  $\to P$ – recoil transverse asymm
  $\to \Sigma$ – lin pol photon beam asymm
  $\to T$ – longitudinal target asymm
  $\to E$ – beam-target helicity asymm
- Constraints: *eg.*

  $$-P^2 + \Sigma^2 + T^2 + E^2 + F^2 + G^2 = 1$$
  $$FG + \Sigma T = P + EH$$

  $\to$ **8** indep. obs's [discrete ambiguity]





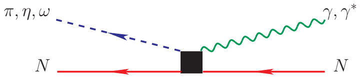
Photoproduction observables

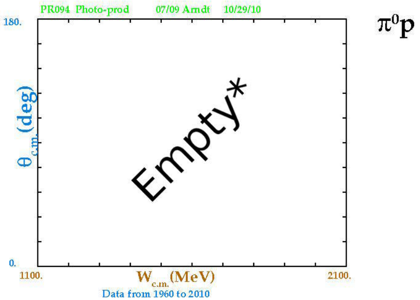
$\gamma N \longrightarrow \pi N, \eta N$

- State count $\rightarrow$ *NB*: $\lambda_\gamma = \frac{1}{2}$

  $$\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{2}_{\gamma}$$
  $= 8$ spin combinations
- Parity reduces by factor 2
  $\rightarrow \lambda = \lambda_\gamma - \lambda_N$

  $$\langle \lambda'_N | T | \lambda \rangle = \langle -\lambda'_N | T | -\lambda \rangle$$
  4 $\mathbb{C}$ amplitudes $\rightarrow$ 8 $\mathbb{R}$ real $\mathcal{O}_p(W)$
- 16 $\mathbb{R}$ observables: *eg.*
  $\rightarrow \frac{d\sigma}{d\Omega}$ – unpol diff cross section
  $\rightarrow P$ – recoil transverse asymm
  $\rightarrow \Sigma$ – lin pol photon beam asymm
  $\rightarrow T$ – longitudinal target asymm
  $\rightarrow E$ – beam-target helicity asymm
- Constraints: *eg.*
  $$-P^2 + \Sigma^2 + T^2 + E^2 + F^2 + G^2 = 1$$
  $$FG + \Sigma T = P + EH$$
  $\rightarrow$ **8** indep. obs's [discrete ambiguity]





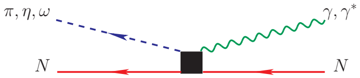
*H. Iwamoto (GWU) thesis
Forthcoming, Jan. 2011

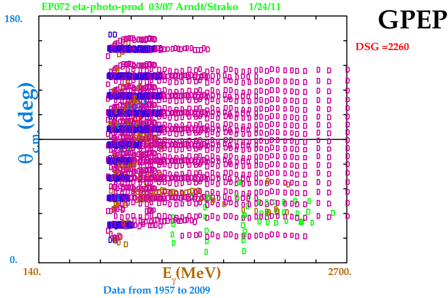
CNS Center for Nuclear Studies

Photoproduction observables

$\gamma N \to \pi N, \eta N$

- State count $\to$ *NB*: $\lambda_\gamma = \frac{1}{2}$
  $$\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{2}_{\gamma}$$
  $= 8$ spin combinations
- Parity reduces by factor 2
  $\to \lambda = \lambda_\gamma - \lambda_N$
  $$\langle \lambda'_N | T | \lambda \rangle = \langle -\lambda'_N | T | -\lambda \rangle$$
  4 $\mathbb{C}$ amplitudes $\to$ 8 $\mathbb{R}$ real $\mathcal{O}_p(W)$
- 16 $\mathbb{R}$ observables: *eg.*
  $\to \frac{d\sigma}{d\Omega}$ – unpol diff cross section
  $\to P$ – recoil transverse asymm
  $\to \Sigma$ – lin pol photon beam asymm
  $\to T$ – longitudinal target asymm
  $\to E$ – beam-target helicity asymm
- Constraints: *eg.*
  $$-P^2 + \Sigma^2 + T^2 + E^2 + F^2 + G^2 = 1$$
  $$FG + \Sigma T = P + EH$$
  $\to$ **8** indep. obs's [discrete ambiguity]

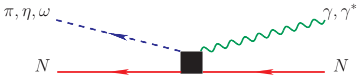


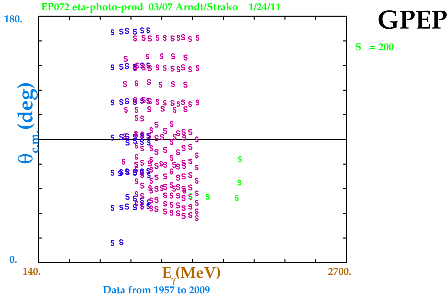


Photoproduction observables

$\gamma N \longrightarrow \pi N, \eta N$

- State count $\rightarrow$ *NB*: $\lambda_\gamma = \frac{1}{2}$
  $$\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{2}_{\gamma}$$
  $= 8$ spin combinations
- Parity reduces by factor 2
  $\rightarrow \lambda = \lambda_\gamma - \lambda_N$
  $$\langle \lambda'_N | T | \lambda \rangle = \langle -\lambda'_N | T | -\lambda \rangle$$
  4 $\mathbb{C}$ amplitudes $\rightarrow$ 8 $\mathbb{R}$ real $\mathcal{O}_p(W)$
- 16 $\mathbb{R}$ observables: *eg.*
  $\rightarrow \frac{d\sigma}{d\Omega}$ – unpol diff cross section
  $\rightarrow P$ – recoil transverse asymm
  $\rightarrow \Sigma$ – lin pol photon beam asymm
  $\rightarrow T$ – longitudinal target asymm
  $\rightarrow E$ – beam-target helicity asymm
- Constraints: *eg.*
  $$-P^2 + \Sigma^2 + T^2 + E^2 + F^2 + G^2 = 1$$
  $$FG + \Sigma T = P + EH$$
  $\rightarrow$ **8** indep. obs's [discrete ambiguity]





Photoproduction observables

$\gamma N \to \pi N, \eta N$

- State count $\to$ *NB*: $\lambda_\gamma = \frac{1}{2}$

  $$\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{2}_{\gamma}$$
  $$= 8 \text{ spin combinations}$$

- Parity reduces by factor 2

  $\to \lambda = \lambda_\gamma - \lambda_N$

  $$\langle \lambda'_N | T | \lambda \rangle = \langle -\lambda'_N | T | -\lambda \rangle$$

  4 $\mathbb{C}$ amplitudes $\to$ 8 $\mathbb{R}$ real $\mathcal{O}_p(W)$

- 16 $\mathbb{R}$ observables: *eg.*

  $\to \frac{d\sigma}{d\Omega}$ – unpol diff cross section

  $\to P$ – recoil transverse asymm

  $\to \Sigma$ – lin pol photon beam asymm

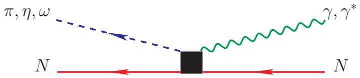
  $\to T$ – longitudinal target asymm

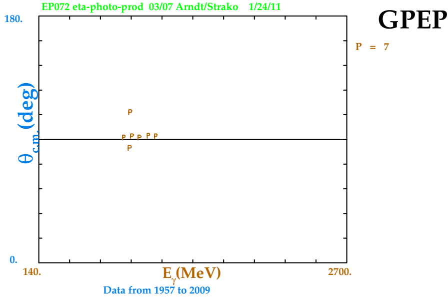
  $\to E$ – beam-target helicity asymm

- Constraints: *eg.*

  $$-P^2 + \Sigma^2 + T^2 + E^2 + F^2 + G^2 = 1$$
  $$FG + \Sigma T = P + EH$$

  $\to$ **8** indep. obs's [discrete ambiguity]

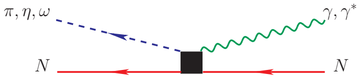


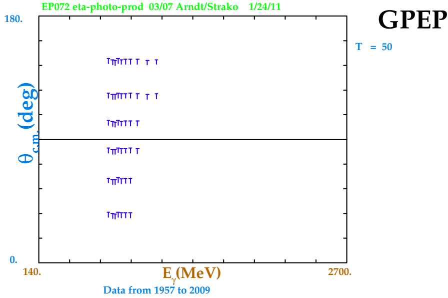


Photoproduction observables

$\gamma N \to \pi N, \eta N$

- State count $\to$ *NB*: $\lambda_\gamma = \frac{1}{2}$
  $\prod_{i=1}^{4}(2\lambda_i + 1) = \underbrace{2}_{N'} \cdot \underbrace{1}_{\pi'} \cdot \underbrace{2}_{N} \cdot \underbrace{2}_{\gamma}$
  $= 8$ spin combinations
- Parity reduces by factor 2
  $\to \lambda = \lambda_\gamma - \lambda_N$
  $$\langle \lambda'_N | T | \lambda \rangle = \langle -\lambda'_N | T | -\lambda \rangle$$
  4 $\mathbb{C}$ amplitudes $\to$ 8 $\mathbb{R}$ real $\mathcal{O}_p(W)$
- 16 $\mathbb{R}$ observables: *eg.*
  $\to \frac{d\sigma}{d\Omega}$ – unpol diff cross section
  $\to P$ – recoil transverse asymm
  $\to \Sigma$ – lin pol photon beam asymm
  $\to T$ – longitudinal target asymm
  $\to E$ – beam-target helicity asymm
- Constraints: *eg.*
  $-P^2 + \Sigma^2 + T^2 + E^2 + F^2 + G^2 = 1$
  $FG + \Sigma T = P + EH$
  $\to$ **8** indep. obs's [discrete ambiguity]





Database

Tabular form

πN elastic data $W \lesssim 2.1$ GeV

Reaction	Data	χ^2
$\pi^+p \to \pi^+p$	13,344	27,242
$\pi^-p \to \pi^-p$	11,967	22,705
$\pi^-p \to \pi^0n$	2,933	6,091
$\pi^-p \to \eta n$	257	628
DRs	3,375	671
Total	31,876	57,241

- 1$^{\text{st}}$ generation ('57–'79)
  - used by CMB(79) and KH(84) analyses
  - $\sim$ 10k $\pi^\pm p$ each; 2k CXS
- 2nd generation $\rightarrow$ SAID fits
- $\sim$25% data is polarized
- expt ratings: 1–3*'s
- fits subject to data flagging

π photoproduction $W \lesssim 2.5$ GeV

Reaction	Data (Dpol)	χ^2
$\gamma p \to \pi^0 p$	13,759 (7 %)	30,853
$\gamma p \to \pi^+ n$	8,629 (9 %)	16,558
$\gamma n \to \pi^- p$	2,990 (4 %)	5,651
$\gamma n \to \pi^0 n$	148 (0 %)	372
Total	25,526	52,423

- Ukai & Nakamura ('85)
- $\sim$10k before '90
- 85% *bremsstrahlung* before '94
- tagged photon data
- limited coverage, polarization data
- dearth of neutron data

Outline

1. Observables & amplitudes
   - Hadroproduction
   - Photoproduction
   - Experiments & Data
2. Formalism
   - Models & parametrizations
   - Unitarity
   - Parametrizations
3. Results $\gamma N \longrightarrow \pi N, \eta N$
   - Exploratory study
   - Pion Photoproduction

Data summary

Motivation for why we attempt a unified approach

Hadronic

- $\pi N \longrightarrow \pi N$: complete over 'reasonably good' kinematic range
- $\pi N \longrightarrow \eta N$: $\frac{d\sigma}{d\Omega}$ only, essentially (a few *P*-data at higher energies)

Photoproduction

- $\gamma N \longrightarrow \pi N$: unpolarized and single polarization only
- $\gamma N \longrightarrow \eta N$: unpolarized and very few single polarization

Forthcoming

- Hadronic: expectation is low [J-PARC?]
- Photoproduction: expectation is for (nearly) complete measurement

Upshot

- Current approaches fix hadronic when fitting electromagnetic; but EM is strongly dependent on hadronic subprocesses
- Require a **unified** (multichannel, unitary) parametrization approach
- Permits hadronic $\leftrightarrow$ electromagnetic processes to affect each other

Forthcoming polarization measurements

Pion and Eta Photoproduction Data on Nucleon

Wei Chen & Haiyan Gao, Duke U.: CLAS g10: DSG $\gamma n \to \pi^- p$
Daria Sokhan & Dan Watts, EU: CLAS g13: Σ $\gamma n \to \pi^- p$
Paul Mattione & Dan Carman, JLab: CLAS g13: DSG $\gamma n \to \pi^- p$
Mike Dugger, ASU: CLAS g8b: Σ $\gamma p \to \pi^+ n$, $\gamma p \to \pi^0 p$
Patrick Collins, CUA: CLAS g8b: Σ $\gamma p \to \eta p$
Hideko Iwamoto & Bill Briscoe, GW: CLAS g9a: E $\gamma p \to \pi^0 p$
Steffen Strauch, SCU: CLAS g9a: E $\gamma p \to \pi^+ n$
Steffen Strauch, SCU: CLAS g9b: T, H, F, & P $\gamma p \to \pi^0 p$
Brian Morisson & Mike Dugger, ASU: CLAS g9a: E & G $\gamma p \to \eta p$
Wei Chen & Haiyan Gao, Duke U.: CLAS g12: DSG $\gamma p \to \pi^+ n$
Arthur Sabintsev & Bill Briscoe, GW: CLAS g9b: H, F, & P $\gamma p \to \pi^+ n$
Jo McAndrew & Dan Watts, EU, CLAS g9a: G $\gamma p \to \pi^+ n$,$\gamma p \to \pi^0 p$
Reinhard Beck, Bonn U.: CB-ELSA: GDH $\gamma p \to \pi^0 p$,$\gamma p \to \eta p$
Derek Glazier & Dan Watts, EU: MAMI-B: C_x & C_z $\gamma p \to \pi^0 \eta p$
David Hornidge, MTA & Sergey Prakhov, UCLA: CB@MAMI-C: DSG & Σ $\gamma p \to \pi^0 p$
Berhan Demissie & Bill Briscoe, GW: CB@MAMI-C: DSG $\gamma n \to \pi^0 n$
Evie Downie, GW/Mainz U.: CB@MAMI-C: F $\gamma p \to \pi^0 p$
Kevin Fissum, GW/Lund U.: MAX-lab: $\gamma p \to \pi^+ n$
Kevin Fissum, GW/Lund U. & Bill Briscoe, GW: MAX-lab: $\gamma n \to \pi^- p$,$\gamma n \to \pi^0 n$
Andy Sandorfi, JLab: BNL: E & G $\gamma p \to \pi^+ n$
Andy Sandorfi, JLab & Franz Klein, CUA: JLab g14: E $\gamma n \to \pi^- p$
Slava Kuznetsov, INR: GRAAL: DSG $\gamma n \to \eta n$
Carlo Shaerf, INFN: GRAAL: DSG $\gamma n \to \eta n$
Berndt Krusche, Basel U.: CB-ELSA: DSG & Σ $\gamma n \to \eta n$
Berndt Krusche, Basel U.: CB@MAMI-C: DSG & Σ $\gamma n \to \eta n$
Hajime Shimizu, Tahoku U.: -LNS: DSG $\gamma n \to \eta n$

Please notify me of omissions

Pseudoscalar prod. observables

Unpolarized
$\frac{d\sigma}{d\Omega}$ - unpol. diff. x-section

Single spin observables
Σ - linear photon beam
T - long. target
P - recoil transverse

Double spin observables
Beam-target
E - circ. photon beam-long. target
F - circ. photon beam-trans. target
G - lin. photon-long. target
H - lin. photon-trans. target
Beam-recoil
C_x - circ. photon beam-trans. recoil
C_z - circ. photon beam-long. target
O_x - lin. photon-trans. target
O_z - lin. photon-long. target
Target-recoil
T_x - trans. target-trans. recoil
T_z - trans. target-long. target
L_x - long. target-trans. target
L_z - long. target-long. target

'Missing' resonances

For SAID, the 'missing resonance problem' is getting worse:

Summary of N* and Δ * Finding

[R. Arndt, WB, I. Strakovsky, R. Workman, Phys Rev C **74**, 045205 (2006)]

- Standard PWA reveals only wide Resonances, but not too wide (Γ < 500 MeV) and possessing not too small BR (BR > 4%)
- Standard PWA (by construction) tends to miss narrow Resonances with Γ < 30 MeV

- Our study does not support several N* and Δ * reported by PDG2006:
  - *** $\Delta(1600)P_{33}$, $N(1700)D_{13}$, $N(1710)P_{11}$, $\Delta(1920)P_{33}$
  - ** $N(1900)P_{13}$, $\Delta(1900)S_{31}$, $N(1990)F_{17}$, $\Delta(2000)F_{35}$, $N(2080)D_{13}$, $N(2200)D_{15}$, $\Delta(2300)H_{39}$, $\Delta(2750)I_{313}$
  - * $\Delta(1750)P_{31}$, $\Delta(1940)D_{33}$, $N(2090)S_{11}$, $N(2100)P_{11}$, $\Delta(2150)S_{31}$, $\Delta(2200)G_{37}$, $\Delta(2350)D_{35}$, $\Delta(2390)F_{37}$

- Our study does suggest several 'new' N* and Δ *:
  - **** $\Delta(2420)H_{311}$
  - *** $\Delta(1930)D_{35}$, $N(2600)I_{111}$ [BW, no pole]
  - ** $N(2000)F_{15}$, $\Delta(2400)G_{39}$
  - new $N(2245)H_{111}$ [CLAS ?]

1

Credit: W. Briscoe [N*2007 Plenary]

Outline

FAQ about SAID

Everything you wanted to know but were afraid to ask

- SAID is a *parametrization*, not a *model* (see next slide)
- There is no 'non-resonant' / 'resonant' separation $\implies$ we cannot provide a unique decomposition of the SAID amplitudes into "background" and "resonant" pieces
- Resonances arise 'dynamically,' in a sense, from the fit and the form of the parametrization
- "Fit to **data**" $\simeq$ a fit to scattering observables (not amplitudes nor phases); important for interference effects
- Analytic structure, as dictated by two-body and 'features' of three-body unitarity are correct; *eg.* Fixed-t dispersion relations are well-satisfied (and no re-fit was needed after DR 'pseudo-data' added)

Models vs. parametrizations

Attempted definition

Objective: determine the resonance spectrum of the nucleon

Definition context: What is the assumed particle content of the theory?

- **Model**: assume stable (π, η, ω, N) & unstable (σ, ρ, N^*, Δ^*, etc.) $\longrightarrow$ calculate observables $\longrightarrow$ adjust resonance contibution
- **Parametrization**: assume stable (π, η, ω, N) only $\longrightarrow$ fit data $\longrightarrow$ deduce resonance spectrum

πN models and parametrizations (no particular order)

Models	**Parametrizations**
Carnegie-Mellon Berkeley	Karlsruhe-Helsinki*
Jülich	MAID
Giessen	SAID
EBAC	
Chiral-Unitary	

Outline

Unitarity constraint on T

- S matrix definition

$$\begin{aligned} S_{\alpha\beta}(E) &= \langle \mathbf{k}_\alpha \alpha | S | \mathbf{k}_\beta \beta \rangle \\ &= \delta^{(3)}(\mathbf{k}_\alpha - \mathbf{k}_\beta)\delta_{\alpha\beta} + 2i\pi\delta(E_\alpha - E_\beta)\langle \mathbf{k}_\alpha \alpha | T | \mathbf{k}_\beta \beta \rangle \end{aligned}$$

- Unitarity constraint on T from $S^\dagger S = SS^\dagger = 1$

$$T_{\alpha\beta} - T^\dagger_{\alpha\beta} = 2\pi i \sum_\gamma \int d^3k_\gamma \, T^\dagger_{\alpha\gamma}\delta(E_\gamma - E_\beta) T_{\gamma\beta}$$

$$\begin{aligned} T_{\alpha\beta} - T^\dagger_{\alpha\beta} &= 2\pi i \sum_\gamma \int d\Omega_\gamma \int_0^\infty dk_\gamma \; T^\dagger_{\alpha\gamma}\delta[(k_\gamma^2 + m_{\gamma 1}^2)^{1/2} + (k_\gamma^2 + m_{\gamma 2}^2)^{1/2} - E_\beta] T_{\gamma\beta} \\ &= 2i \sum_\gamma \int d\Omega_\gamma \, T^\dagger_{\alpha\gamma} \rho_\gamma T_{\gamma\beta} \end{aligned}$$

$$\rho_\gamma = \theta(W - m_{\gamma +}) \frac{\pi \bar{k}_\gamma E_{\gamma 1} E_{\gamma 2}}{W}.$$

NB: Presence of Heaviside $\theta(W - m_{\sigma +}) \rightarrow$ threshold branch points

- "Maximal analyticity" $\Longrightarrow$ real branch points
- Branch points $\notin \mathbb{R}$ are **model dependent**

Kinematic singularity-free amplitudes

$$\frac{1}{2i}[T_{\alpha\beta} - T^*_{\alpha\beta}] = \mathrm{Im}\ T_{\alpha\beta}$$
$$= \sum_\sigma \int d\Omega_\sigma T^*_{\alpha\sigma} \theta(W - m_{\sigma+}) \rho_\sigma T_{\sigma\beta}$$

The kinematical singularities are removed from the unitarity constraint by considering $T'_{\alpha\beta} = \sqrt{\rho}_\alpha T_{\alpha\beta} \sqrt{\rho}_\beta$.

$$T'_{\alpha\beta} - T'^*_{\alpha\beta} = 2i \sum_\sigma T'^*_{\alpha\sigma} \theta(W - m_{\sigma+}) T'_{\sigma\beta}$$

where the $T'_{\alpha\beta}$ now represent the partial wave amplitudes. Casting this relation as a matrix equation

$$\frac{1}{2i}[T' - T'^*] = T'^* \theta(W - M_+) T',$$

where $M_{+,\alpha\sigma} = m_{\sigma+}\delta_{\alpha\sigma}$, and multiplying from the left by $[T'^*]^{-1}$ and from the right by T'^{-1} gives

$$\mathrm{Im}\ T'^{-1} = -\theta(W - M_+).$$

Unitarity

Branch points

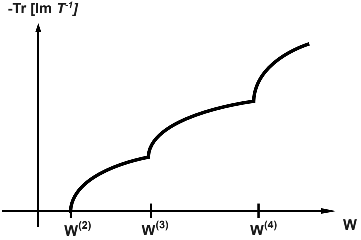
- Unitarity (conservation of probability) $\leftrightarrow$ Analyticity

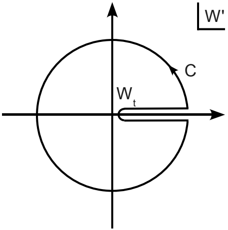
$$S = 1 + 2i\rho T$$

$$S^\dagger S = SS^\dagger = 1$$

$$T - T^\dagger = 2iT\rho T^\dagger \qquad \rho = \text{density of states}$$

$$\mathrm{Im}\, T^{-1}(W) = -\rho \qquad \mathrm{Disc}\ T^{-1} = -2i\rho$$





- Ignoring poles and unphysical branch points

Outline

Parametrizations

Complexity $\longrightarrow$ simplicity

Objective

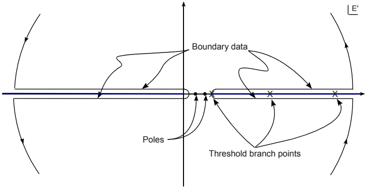
Determine simple functional form to reduce data observables to **model independent** amplitudes

Domain of analyticity

- Unitarity $\longrightarrow$ threshold branch points
- Bound-state poles, $\mathrm{Re}\, E = W < W_t$
- Large W behavior $\longrightarrow$ subtractions

Simplify or 'implement functionally'

- Stage 1: 'remove' branch points $\longrightarrow$ Heitler K matrix
- Stage 2: 'remove' poles $\longrightarrow$ Chew-Mandelstam K matrix



Heitler K-matrix form

'Removing' branch points

Unitarity constraint determines on-shell K uniquely

$$\begin{aligned} T^{-1} &= \mathrm{Re}\ T^{-1} + i\mathrm{Im}\ T^{-1} \\ &= K^{-1} - i\rho \end{aligned} \qquad \begin{aligned} \mathrm{Im}\, T^{-1} &= -\rho \\ \mathrm{Re}\, T^{-1} &\equiv K^{-1} \end{aligned}$$

Heitler equation

$$K^{-1} \times \{K^{-1} = T^{-1} + i\rho\} \times T^{-1}$$

$$\boxed{T = K + iK\rho T}$$

- $\mathrm{Im}\, T^{-1}$ saturates unitarity, determines branch points
- $\mathrm{Re}\, T^{-1} \equiv K^{-1}$ differentiable (analytic) physical region
- K meromorphic function of W, may possess poles

$$K = \frac{1}{T^{-1} + i\rho} = T\frac{1}{1 + i\rho T}$$

Chew-Mandelstam K-matrix form

'Removing' poles

- Motivation — Im $T^{-1} = -\rho$ [UCT]

$$\begin{aligned} T^{-1} &= K^{-1} - i\rho \\ &= \{K^{-1} - \mathrm{Re}\, C\} + \{\mathrm{Re}\, C - i\rho\} \\ &= K_{CM}^{-1} - C \end{aligned}$$

$[UCT] \implies \mathrm{Im}\, C = \rho$ 'disperse' it – next slide

- Chew-Mandelstam K_{CM} vs. K matrix

$$\begin{aligned} K^{-1} &= K_{CM}^{-1} - \mathrm{Re}\, C \\ K &= [1 - K_{CM}\mathrm{Re}\ C]^{-1} K_{CM} \end{aligned}$$

- For polynomial K_{CM}, in general, has Det$[1 - K_{CM}\mathrm{Re}\ C] = 0$
- No need for explicit poles in K_{CM}, but possible to include explicit poles
- K_{CM} poles are 'dressed'
- **Query**: Are K_{CM} poles related to bare poles of dynamical models (*eg.* $S_{11}(1535)$ & $P_{33}(1232)$)? (arXiv:1101.0621 w/R. Workman)

T and K poles

*R. Workman & MP Phys. Rev. C **79**, 038201 (2009)*

ℓ_{JT}	T poles		K poles	
S_{11}	$(1500, 50)$	$(1650, 40)$	1535	1675
P_{11}	$(1360, 80)$	$(1390, 80)^{\dagger}$	–	–
P_{13}	$(1665, 175)$		–	
D_{13}	$(1515, 55)$		–	
D_{15}	$(1655, 70)$		1760	
F_{15}	$(1675, 60)$	$(1780, 130)$	–	–

Table: Pole positions in complex energy plane of T and K matrix for the $\pi N \longrightarrow \pi N$ reaction from SAID (SP06) for isospin $T = \frac{1}{2}$ partial waves. Each T pole position is expressed in terms of its real and imaginary parts $(M_R, -\Gamma_R/2)$ in MeV. Only K matrix pole positions which satisfy 1.1 GeV $< W <$ 2.0 GeV are considered. †This pole is located on the second Riemann sheet.

Chew-Mandelstam function $C_\ell(W)$

Dispersion relation representation

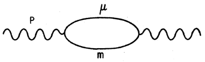


FIG. 1. Feynman graph representing the Chew-Mandelstam function for the scattering of stable particles with masses m and μ.

$$C_\ell(W) = \int_{W_t}^{\infty} \frac{dW'}{\pi} \frac{\mathrm{Im}\ C_\ell(W')}{W' - W} - \int_{W_t}^{\infty} \frac{dW'}{\pi} \frac{\mathrm{Im}\ C_\ell(W')}{W' - W_s}$$

$$\mathrm{Im}\ C_\ell(W) = \left(\frac{W - W_t}{W - W_s} \right)^{\ell + 1/2}$$

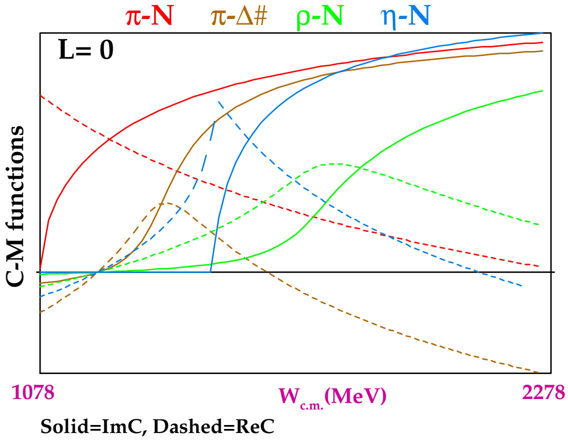
$$C_\ell(z) = \int_0^1 \frac{dx}{\pi} \frac{x^{\ell+1/2}}{x - z}$$

$$z = \frac{W - W_t}{W - W_s}$$

- Proper threshold behavior $\sim \sqrt{W - W_t}$
- Two-particle channel cut
- Unstable (quasi two-body) particle channels $W_t = \mathrm{Re}\ W_t + i\mathrm{Im}\ W_t$
- NO left-hand cut

Chew-Mandelstam function $C_\ell(W)$

Dispersion relation representation



N/D (Wiener-Hopf) approach

Relation to C-M approach

$$T(W) = D^{-1}(W)N(W)$$

$$\begin{aligned} \operatorname{Im} D(W) &= N(W)\operatorname{Im} T^{-1}(W) & \qquad \operatorname{Im} N(W) &= 0 & \qquad W &> m_i + m_t \\ \operatorname{Im} N(W) &= D(W)\operatorname{Im} T(W) & \qquad \operatorname{Im} D(W) &= 0 & \qquad W &< 0 \end{aligned}$$

$$D(W) = \sum_{i=1}^{n_p} D(W; W_i) - \frac{1}{\pi}\prod_{i=1}^{n_p}(W - W_i)\int_{W_t}^{\infty} dW' \frac{N(W')\rho(W')}{(W' - W)\prod_j (W' - W_j)}$$

$$N(W) = K\left\{\sum_i D(W; W_i) - \frac{1}{\pi}\prod_{i=1}^{n_p}(W - W_i)\int_{W_t}^{\infty} dW' \frac{N(W')\rho(W')}{(W' - W)\prod_j (W' - W_j)}\right\}$$

Chew-Mandelstam approximates N, neglecting left-hand cut

$$N(W) = \overline{K}(W)$$

$$\overline{K}_{\alpha\beta} = \sum_{n=0}^{n_{\alpha\beta}} c_{\alpha\beta,n}\overline{z}^{n}_{\alpha\beta}(W)$$

Energy-dependent solutions

Chew-Mandelstam K-matrix parametrization

Partial wave $T = \rho^{1/2} K_{CM} [1 - C K_{CM}]^{-1} \rho^{1/2}$ parametrized:

$$K_{CM} = \begin{pmatrix} K_{ee} & K_{ei} \\ K_{ei} & K_{ii} \end{pmatrix}$$

K_{ee}	$\pi N \to \pi N$	element
K_{ei}	$\pi N \to i$	vector, $i = 1, \ldots, N_{ch} - 1$
K_{ij}	$N_{ch} - 1 \times N_{ch} - 1$	matrix

Parametrization

$$K_{ee} = \sum_{n=0}^{5} p_n z^n + \frac{p_{r1}^2 + p_{r2}^2}{M_R - W} \qquad z = W - (m_\pi + m_N)$$

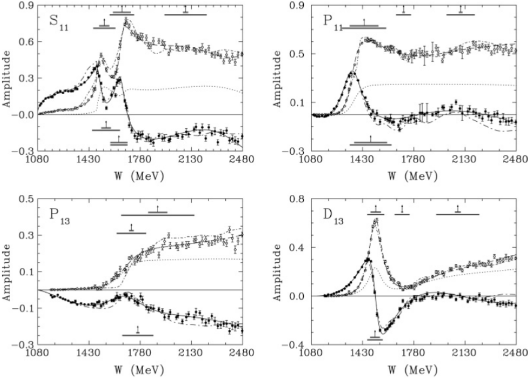
$$K_{ei} = \sum_{n=0}^{3} p_{n+5i} z^n + \frac{p_{r3} p_{r4}}{M_R - W} \qquad z = W - (2m_\pi + m_N)$$

$$K_{ij} = \delta_{ij} \left(p_{4+5i} + p_{5+5i} z + \frac{p_{r5}^2}{M_R - W} \right) \qquad z = W - (2m_\pi + m_N)$$

SAID partial waves

'SP06' Solution

Solution names: 'XX##', 'X###' — SP06 = Spring 2006, FA02 = Fall 2002, . . .



Each $\pi N \longrightarrow \pi N$ partial wave [shown here: S_{11}, P_{11}, P_{13}, D_{13}] shows

- energy-dependent
- single-energy (SE) solutions.

Single energy are solutions are sometimes called, confusingly, "energy-independent."

Models

Contrast models with parametrizations

Complementary approaches

Model: Assume unstable particle content, deduce observables
Parametrization: Deduce unstable particle content (amplitudes) from observable data

Dynamical equation

$$T = V + VG_0 T$$

- Hamiltonian $G_0 = [E - H_0 + i\epsilon]^{-1}$ – Lippmann-Schwinger
- Covariant $G_0 = [(k^2 - m_1^2 + i\epsilon)(k^2 - m_2^2 + i\epsilon)]^{-1}$

Nonresonant $+$ **Resonant = A model**

$$V = v + v^R \implies T = t + t^R$$

$v =$  $+ \cdots$ $\qquad v^R = \Gamma[E - H_0]^{-1}\Gamma$

$$t = [1 - vG_0]^{-1} v \qquad t^R = \overline{\Gamma}[E - M^{(0)} - \Sigma(E)]^{-1}\overline{\Gamma}$$

Consider poles. Ceci et.al. [*Phys. Lett. B 659 (2008) 228–233*] show the model dependence of the resonance contribution to T matrix poles

Outline

Unified hadro/photoproduction parametrization

Motivation

MP & R. Workman, Phys. Rev. C ***82****, 035202 (2010)*

1. Current SAID photoproduction parametrization form lacks full multichannel unitarity
2. Despite χ^2/datum $\sim$ 2, problems with *eg.* $\gamma p \rightarrow \eta p$
3. Extend SAID Chew-Mandelstam approach used in hadronic sector to electromagnetic
   - Hadronic sector $\pi N \rightarrow \pi N$ & $\pi N \rightarrow \eta N$ (untouched)
     4 channel Chew-Mandelstam approach $\{\pi N, \eta N, \pi\Delta, \rho N\}$
   - Electromagnetic sector $\gamma N \rightarrow \pi N$ & $\gamma N \rightarrow \eta N$
     Introduce 4 channel Chew-Mandelstam approach $\{\pi N, \eta N, \pi\Delta, \rho N\}$ with same hadronic "rescattering" matrix
   - Hadronic subprocess dominant in photoproduction $\rightarrow$ 'backconstrain' hadronic amplitudes
   - Obtain $\eta-$photoproduction amplitude with *resonant* phase – various model approaches [*Green & Wycech; Kaiser et. al.; Aznauryan*] yield wide range of phases
4. Study baryon resonances in ηN channel
5. Study η-sector physics

‘Old’ SAID K-matrix formalism

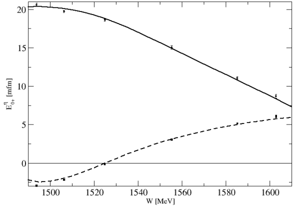
compare to *Green & Wycech PRC**55**(1997); Arndt et. al. PR**C58**(1998); Green & Wycech PR**C60**(1999)*

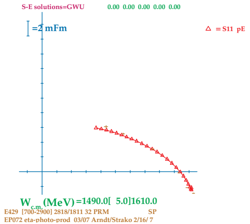
Two-channel formalism (can be generalized to N 2-body channels)

$$T_{\pi\gamma} = (1 + iT_{\pi\pi})K_{\pi\gamma} + iT_{\pi\eta}K_{\eta\gamma} \qquad T_{\eta\gamma} = (1 + iT_{\eta\eta})K_{\eta\gamma} + iT_{\eta\pi}K_{\pi\gamma}$$

Reduction via hadronic matrix to various forms

$$\begin{aligned} T_{\eta\gamma} &= \left(K_{\eta\gamma} - \frac{K_{\pi\gamma}K_{\eta\eta}}{K_{\pi\eta}} \right)(1 + iT_{\eta\eta}) + \frac{K_{\pi\gamma}}{K_{\pi\eta}} T_{\eta\eta} \\ &= A(W)(1 + iT_{\eta\eta}(W)) + B(W)T_{\eta\eta}(W) && \text{Form 1} \\ &= A'(W)(1 + iT_{\pi\pi}(W)) + B'(W)T_{\pi\pi}(W) && \text{Form 2} \end{aligned}$$





Anticipate resonant phase in region 1.49 GeV $\lesssim W \lesssim$ 1.6 GeV

‘New’ SAID Chew-Mandelstam parametrization

$\pi-$ & $\eta-$photoproduction *MP & R. Workman Phys. Rev. C **82**, 035202 (2010)*

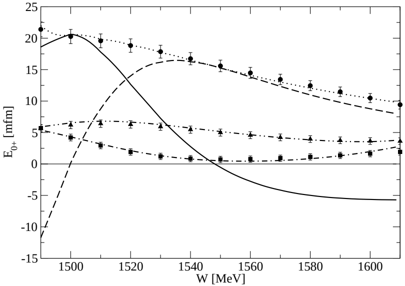
Current hadronic parametrization fits $\pi N \to \pi N, \pi N \to \eta N$, DR, …

$$T_{\alpha\beta} = \sum_\sigma [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\beta} \quad \to \quad \text{CM ‘rescattering’ matrix}$$

Generalized to photoproduction (hadronic matrix fixed by above)

$$T_{\alpha\gamma} = \sum_\sigma [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\gamma}$$

Perform fit at amplitude level to Re E^{π}_{0+}, Im E^{π}_{0+} & $|E^{\eta}_{0+}|$



Amplitudes
Solid curve: Re E^{η}_{0+}
Dashed curve: Im E^{η}_{0+}
Dot-dashed curve: Re E^{π}_{0+}
Double dot-dashed curve: Im E^{π}_{0+}
Dotted curve: $|E^{\eta}_{0+}|$

'New' SAID Chew-Mandelstam parametrization

$\pi-$ & $\eta-$photoproduction

MP & R. Workman Phys. Rev. C ***82****, 035202 (2010)*

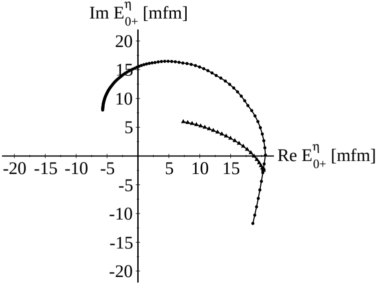
Current hadronic parametrization fits $\pi N \to \pi N, \pi N \to \eta N$, DR, ...

$$T_{\alpha\beta} = \sum_\sigma [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\beta} \;\longrightarrow\; \text{CM 'rescattering' matrix}$$

Generalized to photoproduction (hadronic matrix fixed by above)

$$T_{\alpha\gamma} = \sum_\sigma [1 - \overline{K}C]^{-1}_{\alpha\sigma} \overline{K}_{\sigma\gamma}$$

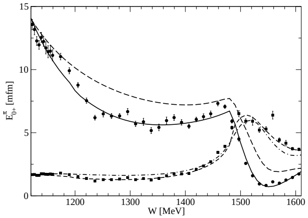
Perform fit at amplitude level to Re E^π_{0+}, Im E^π_{0+} & $|E^\eta_{0+}|$



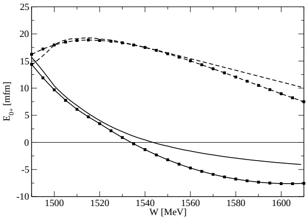
Argand diagram
Triangles – 'Old' Heitler K matrix (non-unitary) formalism
Circles – 'New' Chew-Mandelstam K matrix (non-unitary) formalism

Comparison to MAID

E^{π}_{0+} SAID and MAID solutions



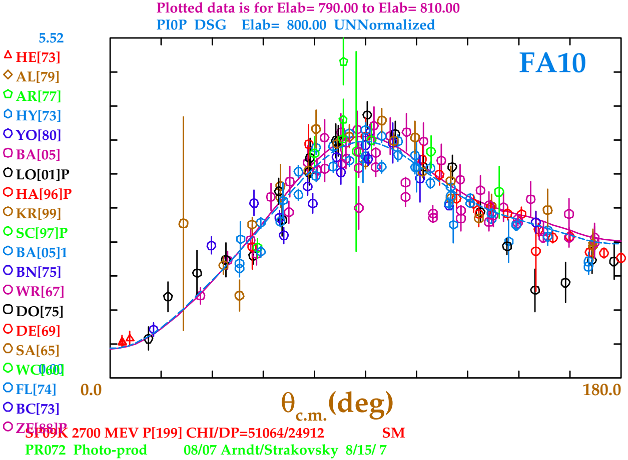
Refitting with MAID E^{π}_{0+} and same $|E^{\eta}_{0+}|$



Outline

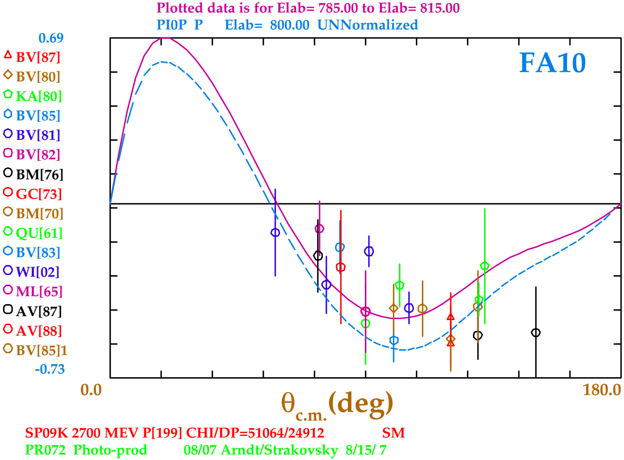
CM form for $\gamma N \rightarrow \pi N$

Fixed CM rescattering matrix



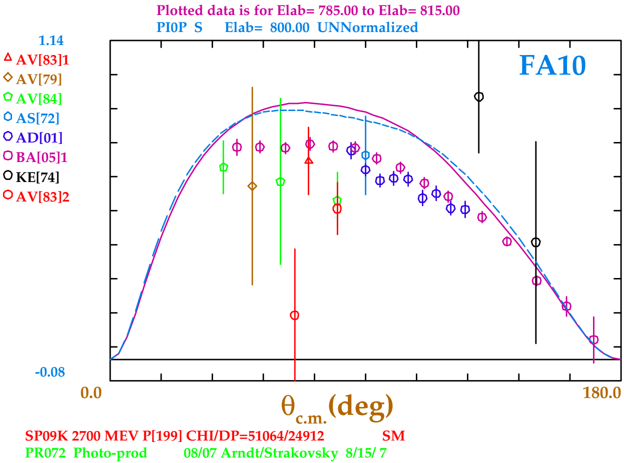
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



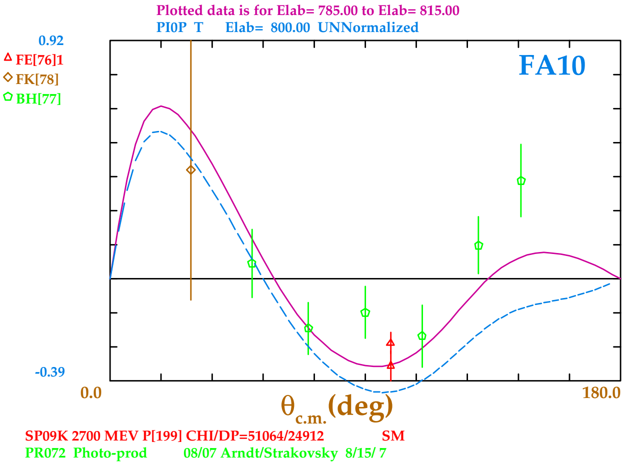
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



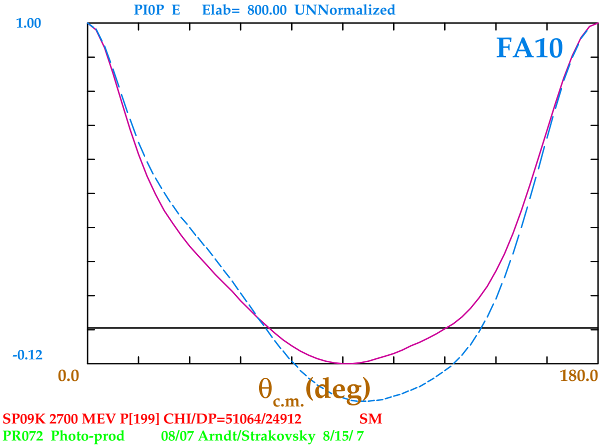
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



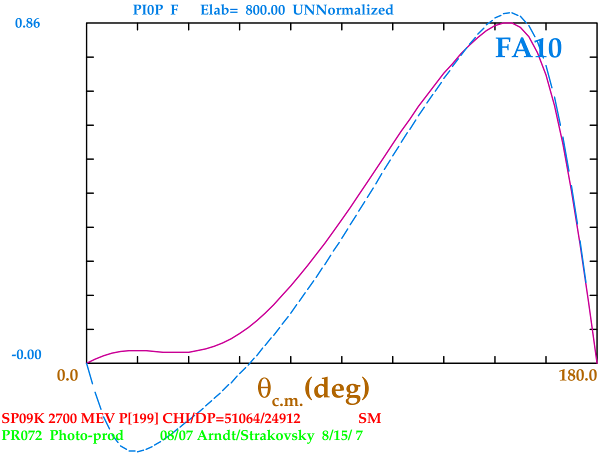
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



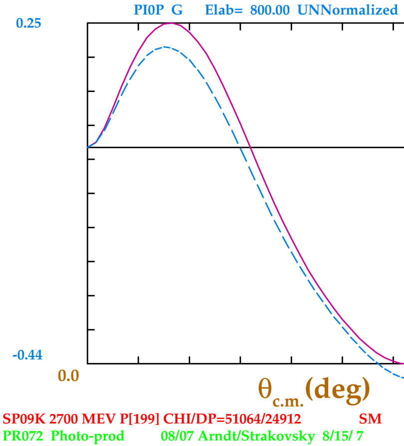
CM form for $\gamma N \rightarrow \pi N$

Fixed CM rescattering matrix



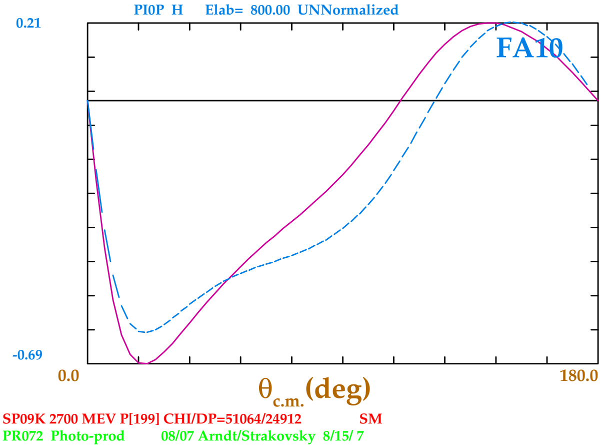
CM form for $\gamma N \rightarrow \pi N$

Fixed CM rescattering matrix



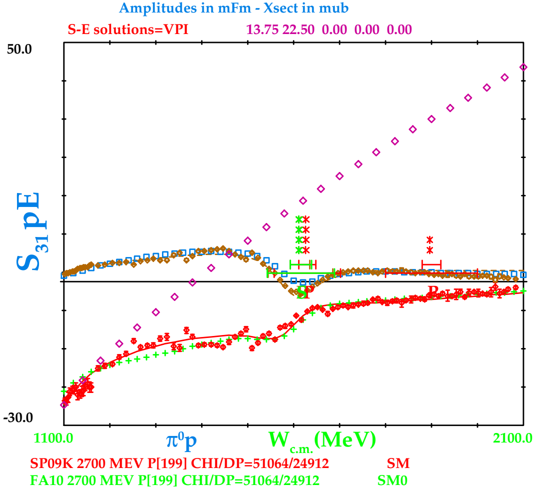
CM form for $\gamma N \rightarrow \pi N$

Fixed CM rescattering matrix



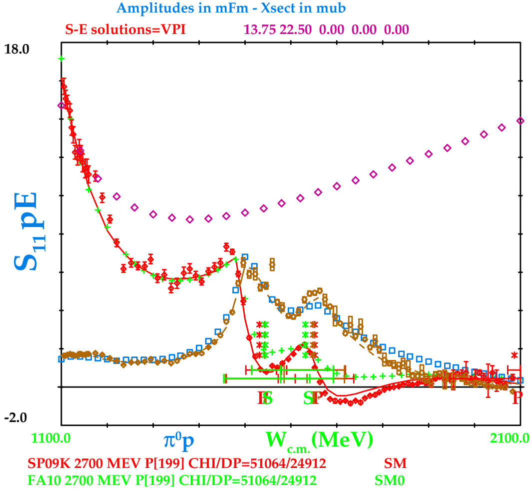
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



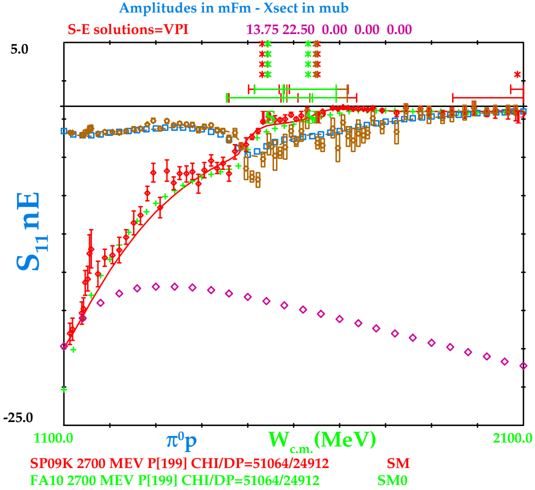
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



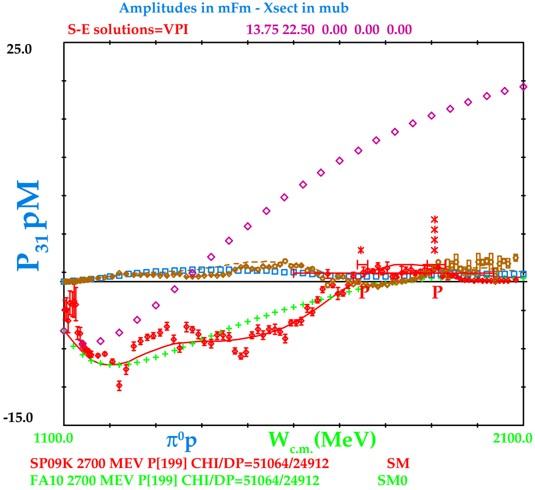
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



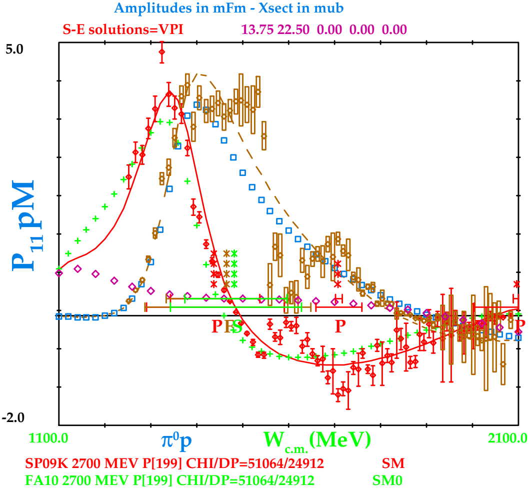
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



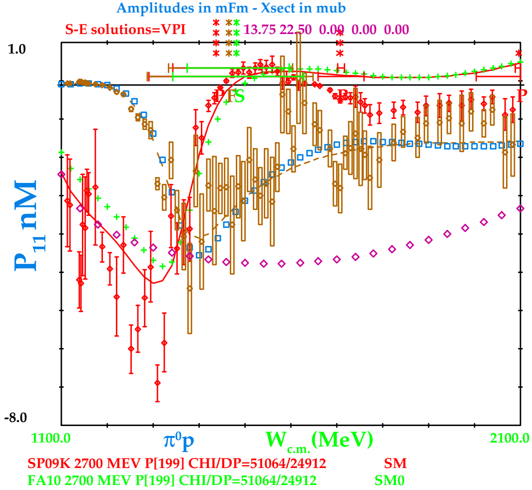
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



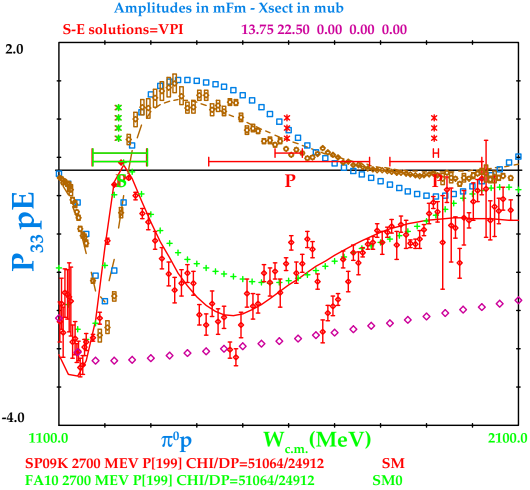
CM form for $\gamma N \rightarrow \pi N$

Fixed CM rescattering matrix



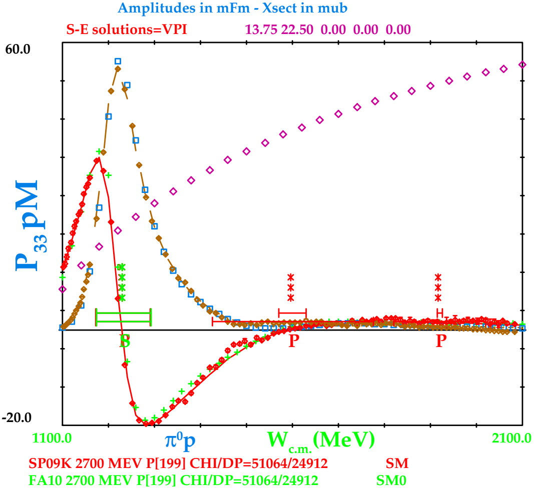
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



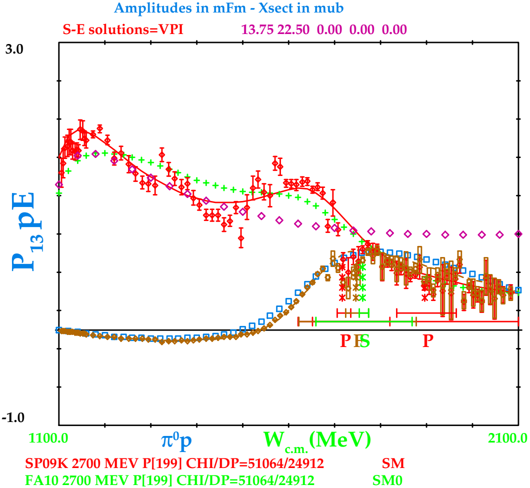
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



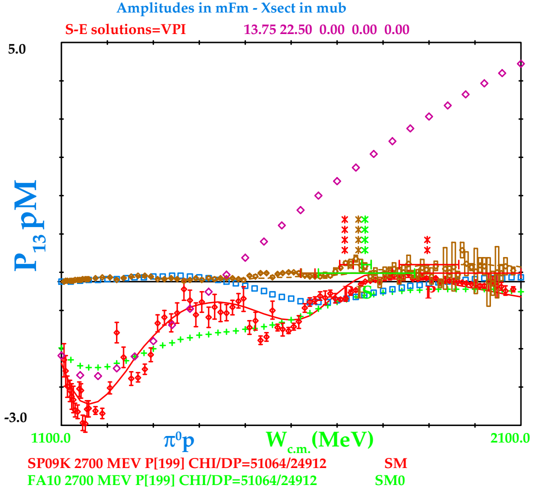
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



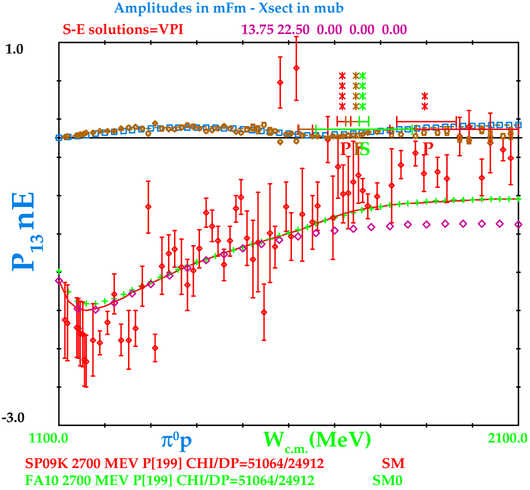
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



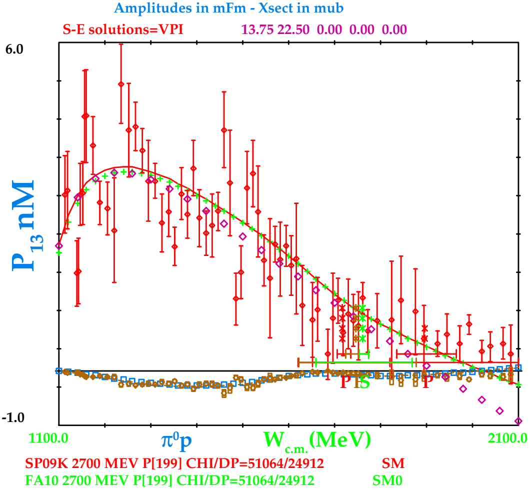
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



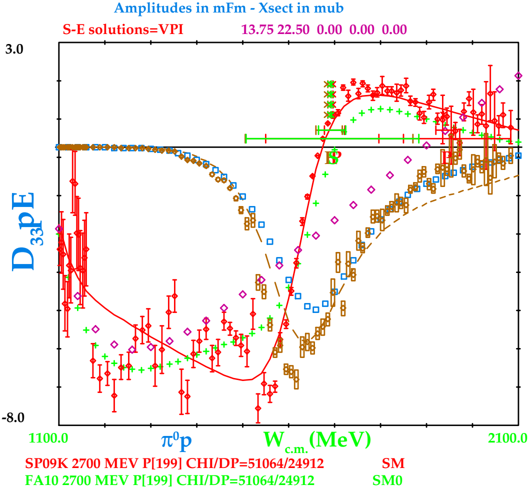
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



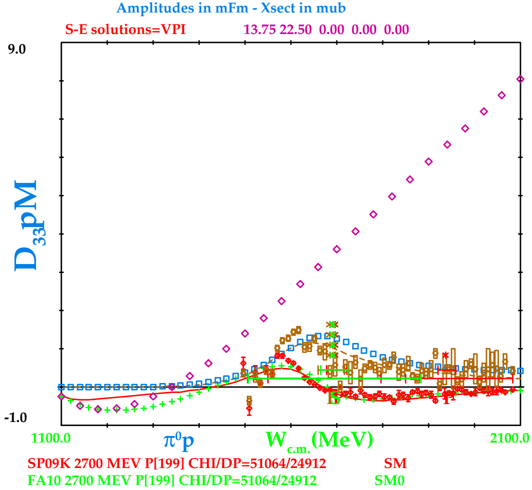
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



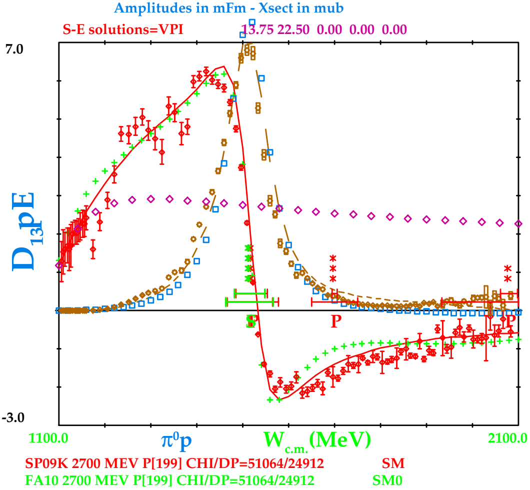
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



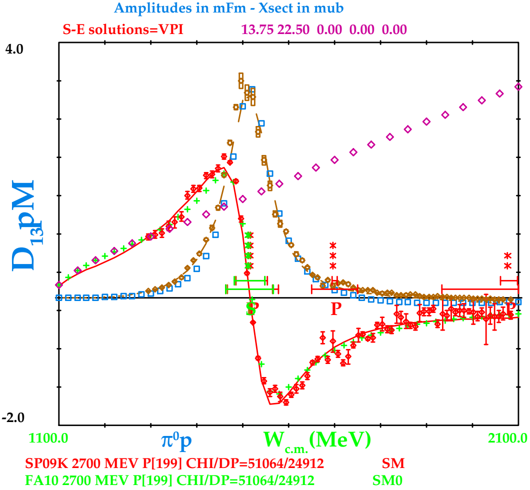
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



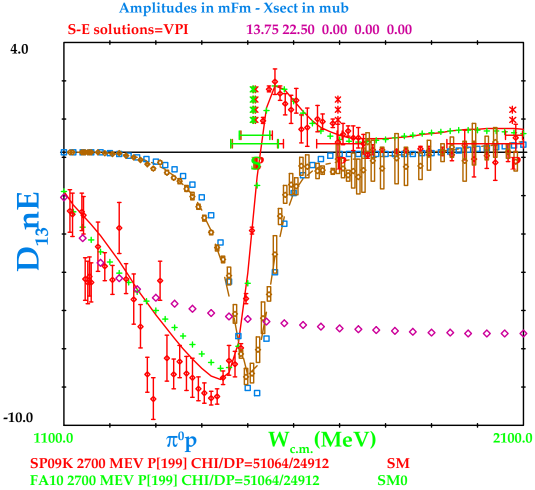
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



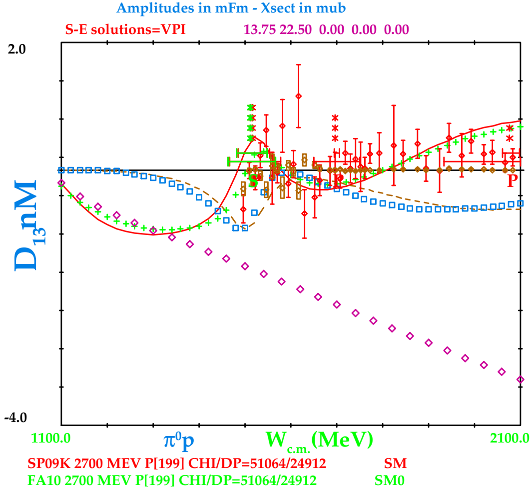
CM form for $\gamma N \rightarrow \pi N$

Fixed CM rescattering matrix



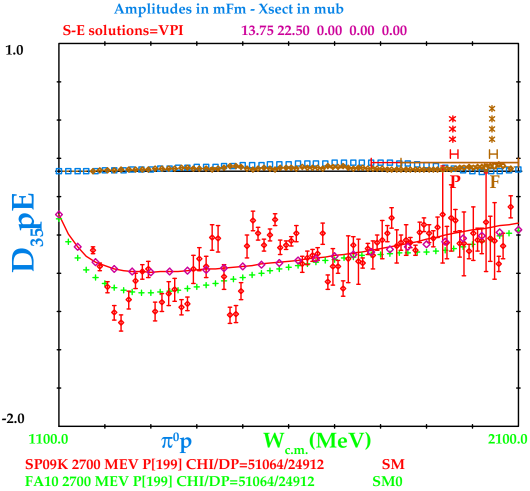
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



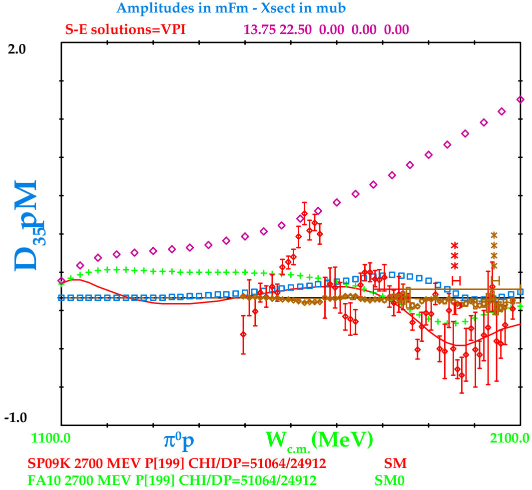
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



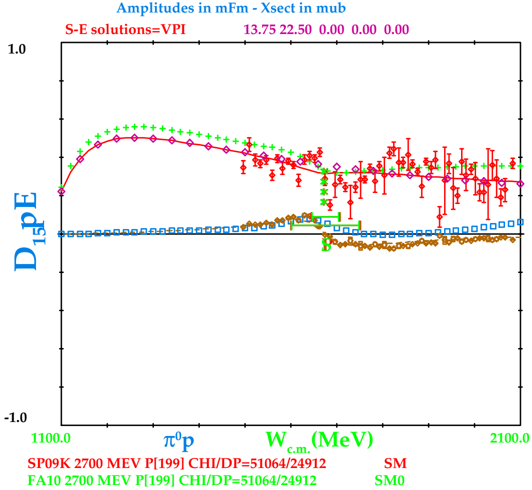
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



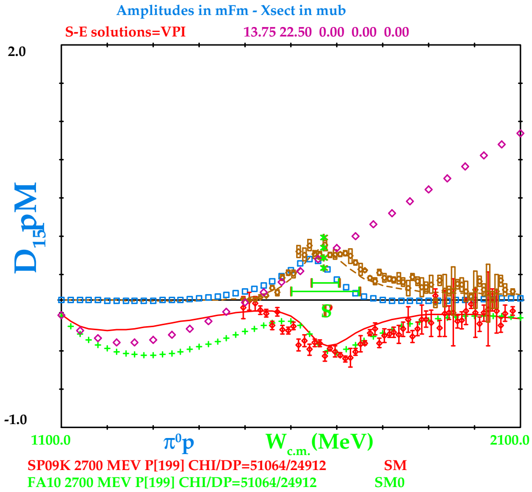
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



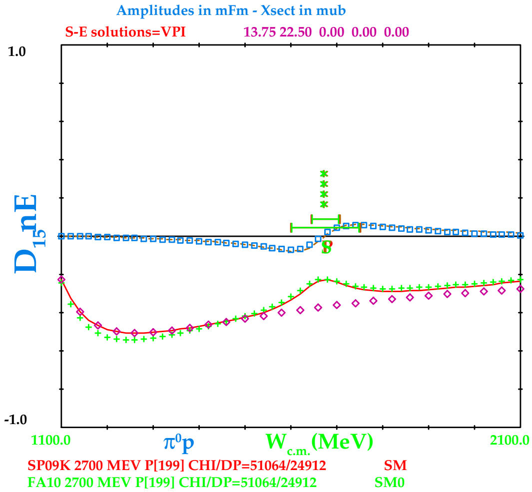
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



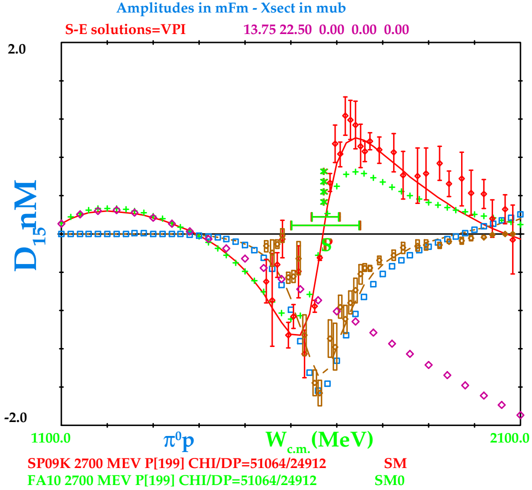
CM form for $\gamma N \rightarrow \pi N$

Fixed CM rescattering matrix



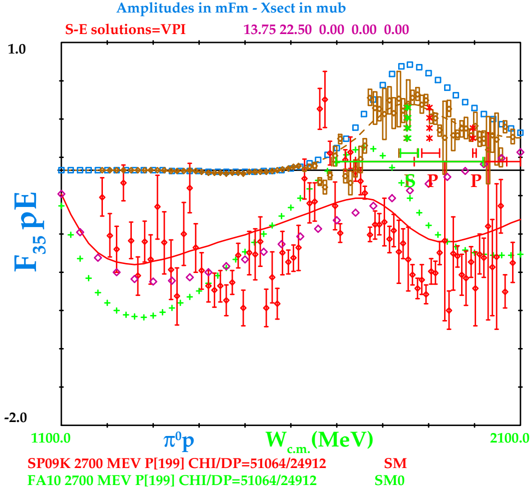
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



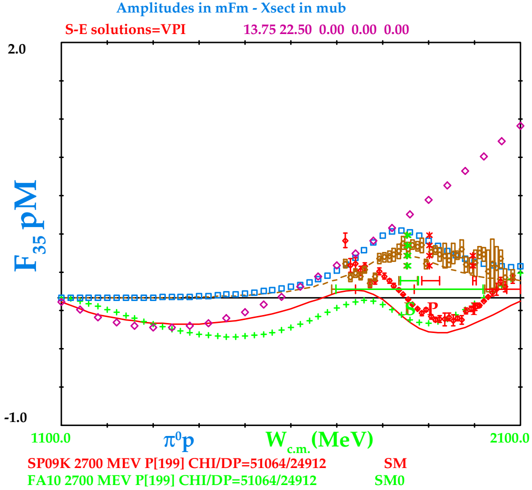
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



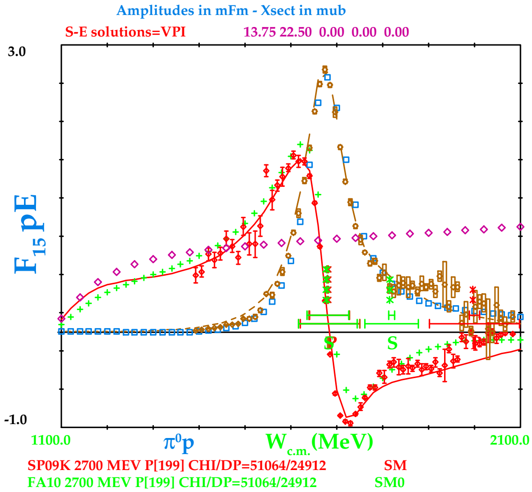
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



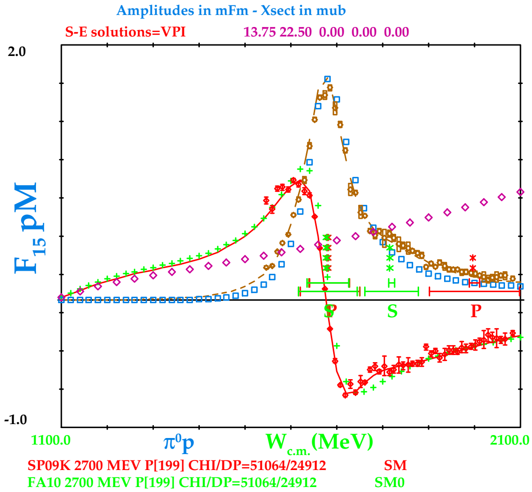
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



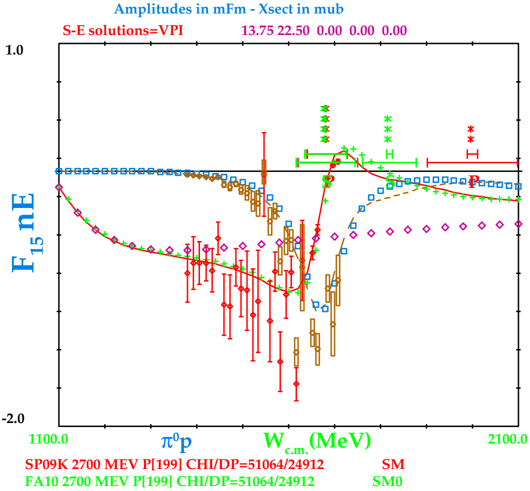
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



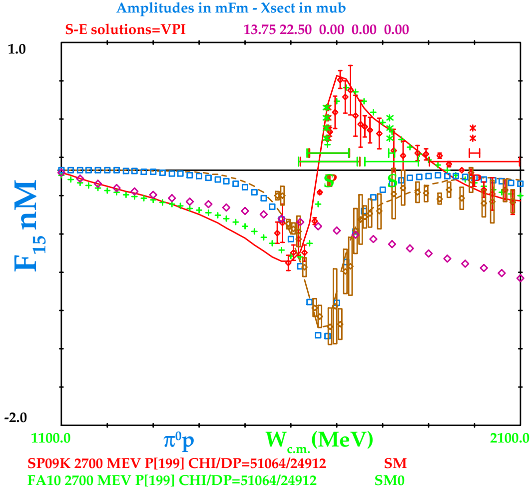
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



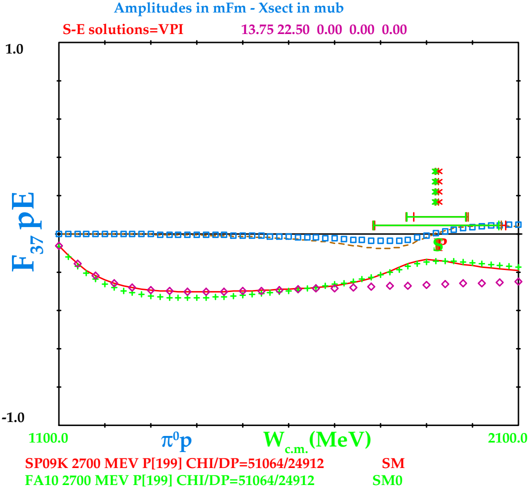
CM form for $\gamma N \rightarrow \pi N$

Fixed CM rescattering matrix



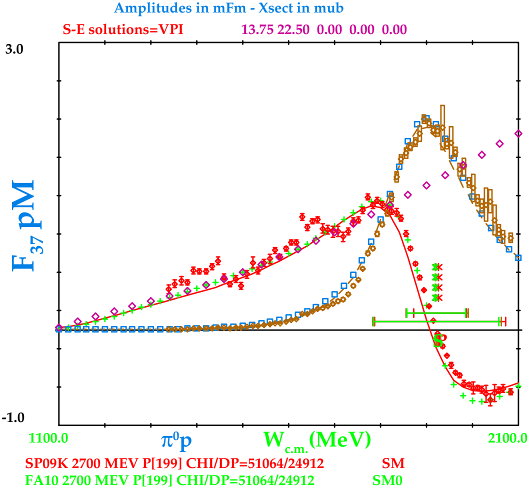
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



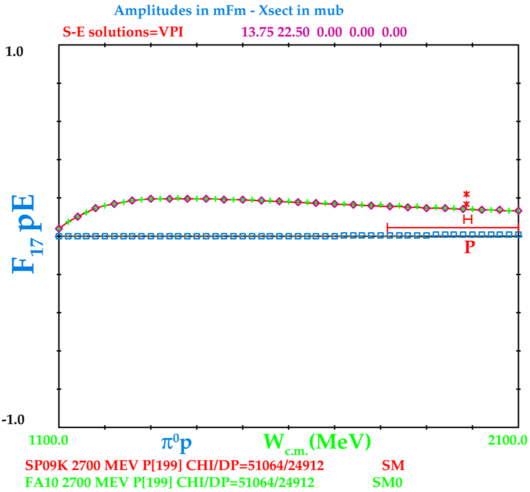
CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



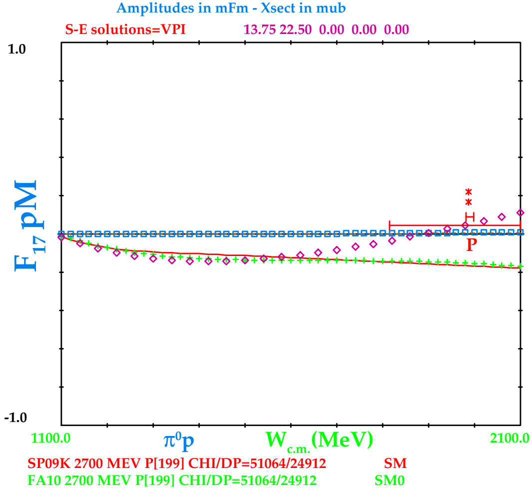
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



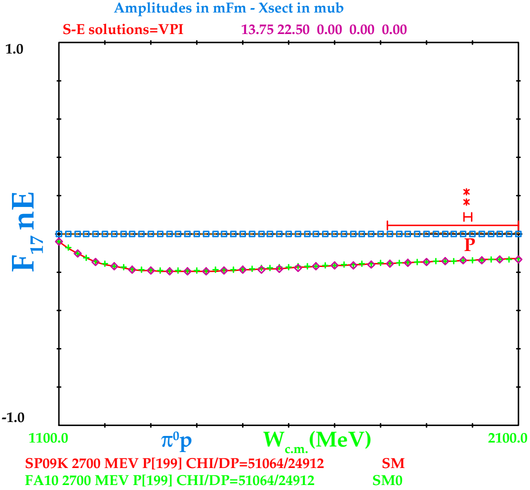
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



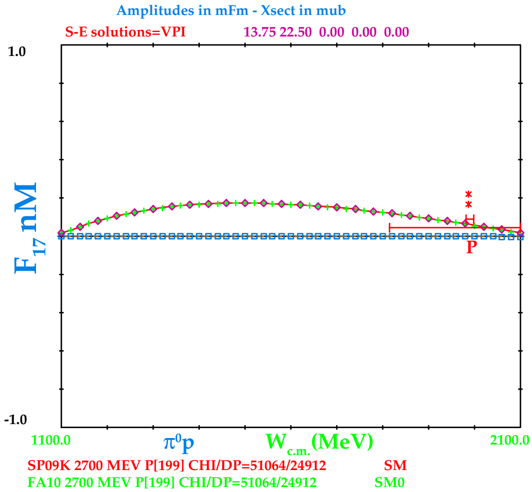
CM form for $\gamma N \longrightarrow \pi N$

Fixed CM rescattering matrix



CM form for $\gamma N \to \pi N$

Fixed CM rescattering matrix



Summary

- Unitarity determines non-analyticities in physical region, $w > m_i + m_t$
- Related Chew-Mandelstam form to N/D approach $\rightarrow$ 'left-hand cut' neglected in C-M
- Realistic description of hadroproduction data requires correct analytic form $\leftrightarrow$ unitarity
- Forthcoming photoproduction data
- Performed simultaneous coupled-channel fit of $\eta-$photoproduction S_{11} multipole modulus, $|E^{\eta}_{0+}|$ and $\pi-$photoproduction amplitude, E^{π}_{0+}
- Current approach yields resonant E^{η}_{0+} phase $\rightarrow$ encourages us to pursue the C-M approach in fits to photoproduction observables (not amplitudes)
- Performed fit to $\pi-$photoproduction *data* using C-M form, yields similar but distinct partial waves with comparable χ-squared
- Outlook
  1. Perform simultaneous fit to $\pi-$ and $\eta-$photoproduction *data* using C-M form
  2. Perform simultaneous, global fit to $\pi N \rightarrow \pi N$, $\pi N \rightarrow \eta N$, $\gamma N \rightarrow \pi N$, $\gamma N \rightarrow \eta N$ using C-M form: **offers opportunity for precision electromagnetic data to 'back-constrain' hadronic amplitudes (some of which are very poorly known)**
  3. 'Left-hand' branch points?

Dedication

To the memory of our friend and colleague, Dick Arndt, GWU Research Professor and Virginia Tech Emeritus Professor, who passed Saturday, April 10, 2010.

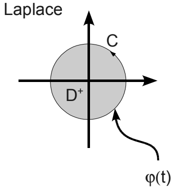
Supplementary material

Follow-on material

The $\mathcal{R}$iemann–$\mathcal{H}$ilbert problem

Properly: 'The scalar $\mathcal{R}$–$\mathcal{H}$ method'

- Reconstruction of a complex function given boundary data
- Riemann [1851]: analytic function find $f(z) = u(z) + iv(z)$ given $\alpha(z(t))u(z(t)) + \beta(z(t))v(z(t)) = \gamma(z(t))$ on a curve C
- Hilbert [1904]: find an analytic function given the **discontinuity data** along a curve $\leftrightarrow$ Disc T, transition matrix
- Applications in math
    - Solve (singular) linear integral equations
    - Solve partial differential equations
    - Hilbert transforms or Kramers-Krönig dispersion relations
    - ...
- ... & physics
    - Elasticity: Laplace boundary value prob. on D^+
    - Hydrodynamics: non-linear Korteweg-deVries (KdV) equation, $u_t + u_{xxx} + uu_x = 0$ shallow water *soliton* waves
    - Electrostatics: find surface density on $C \Rightarrow$ constant potential
    - Hadronic physics: discontinuity data from unitarity
    - Renormalization group: *Connes & Kreimer* showed that renorm. is equivalent to solving an $\mathcal{R}$–$\mathcal{H}$ problem
    - ...



Chew-Mandelstam approach

Recap: solving the $\mathcal{R}$–$\mathcal{H}$ problem

- Have: scattering data on the real axis, *eg.* $\frac{d\sigma}{d\Omega}$
- Want: transition amplitude T in complex W plane $\rightarrow$ resonances
- Know: T is discontinuous from unitarity
- Approach: parametrize T matrix using Chew-Mandelstam technique
- Result: search for poles $\leftrightarrow$ resonances

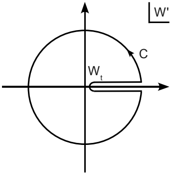
- Discontinuities in Chew-Mandelstam function $C(W)$

$$T^{-1} = K_{CM}^{-1} - C \qquad \mathrm{Im}\, C = -\rho$$

- Parametrize K_{CM} (boundary data isn't analytic)

$$K_{CM}(p; W) = \sum_{n=0}^{3 \text{ or } 4} p_n [W - W_t]^n$$

- Parameters p are fixed by fitting **scattering observables** (unpolarized diff. x-sec., pol. asymmetries, . . .)



SAID: Scattering Analysis Interactive Database amplitudes

πN elastic scattering and inelastic reactions

- Chi-squared per datum compared with model calculations
- Optimize χ–squared w.r.t. $p \to K_{CM}(p)$

$$\chi^2(p) = \frac{1}{N_{data}} \sum_{i=1}^{N_{data}} \left[\frac{\Phi_{n(i)} y_i(p) - Y_i}{\Delta Y_i} \right]^2 + \frac{1}{N_{exp}} \sum_{n=1}^{N_{exp}} \left[\frac{\Phi_n - 1}{\Delta \Phi_n} \right]^2$$

χ^2/Data	SP06		FA02		KA84		EBAC		Gießen
Reaction	Norm		Norm		Norm		Norm		Norm
$\pi^+p \to \pi^+p$	2.0		2.1		5.0		13.1		10.5
$\pi^-p \to \pi^-p$	1.9		2.0		9.1		4.9		12.1
$\pi^-p \to \pi^0n$	2.0		1.9		4.4		3.5		6.3
$\pi^-p \to \eta n$	2.5		2.5						

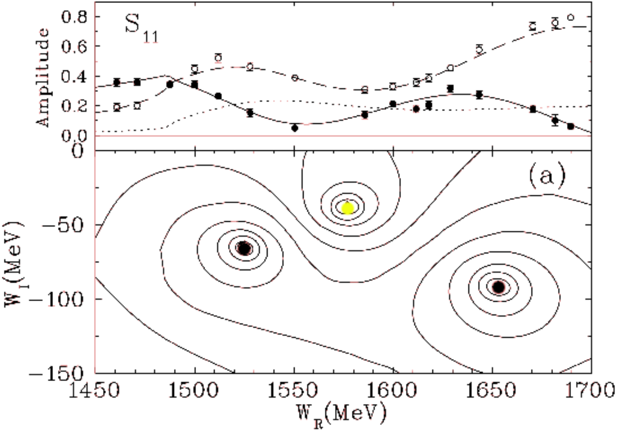
FA02 [R. Arndt *ea* Phys Rev C **69**, 035213 (2004)]
KA84 [R. Koch, Z Phys C **29**, 597 (1985)]
EBAC [B. Julia-Diaz *ea* Phys Rev C **76**, 065201 (2007)]
Gießen [V. Shklyar *ea* Phys Rev C **71**, 055206 (2005)]

- ***Correct analytic behavior ensures realistic description (low–χ^2) of the data***

$\pi N \longrightarrow \pi N$

Analytic continuation

Spectroscopic notation: $L_{2I,2J}$ – L: rel. πN orb. ang. mom.; I: isospin; J: total intrinsic ang. mom.

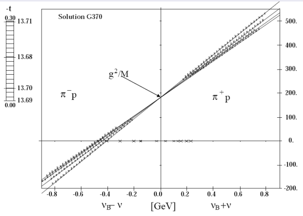


$\pi N \longrightarrow \pi N$ dispersion relations

πNN coupling

- Old debate with Karlsruhe-Helsinki: 'Dispersion relations aren't satisfied'
- Correct analytic behavior ensures DR's satisfied
- Use the DR's to compute the πNN coupling constant

Fixed-t DR



$$(\nu_B \pm \nu)\left\{\mp \mathrm{Re}\, B_\pm(\nu, t)\right.$$

$$\left.\pm \frac{\nu}{\pi}\int_{\nu_{\mathrm{th}}}^{\infty}\frac{d\nu'}{\nu'}\left[\frac{\mathrm{Im}\, B_+}{\nu' \mp \nu} + \frac{\mathrm{Im}\, B_-}{\nu' \pm \nu}\right]\right\}$$

$$= \frac{g^2}{M} + \tilde{B}(0, t)(\nu_B \pm \nu)$$

$g = 13.69 \pm 0.07$ $\qquad$ $f = 0.0757 \pm 0.0004$

Effective field theory

Hadronic interactions:

π, η, N, Δ:

$$L_{\pi NN} = -\frac{f_{\pi NN}}{m_\pi}\,\bar{\psi}_N\gamma_\mu\gamma_5\vec{\tau}\psi_N\cdot\partial^\mu\vec{\phi}_\pi,$$

$$L_{\pi N\Delta} = -\frac{f_{\pi N\Delta}}{m_\pi}\,\bar{\psi}^\mu_\Delta\vec{T}\psi_N\cdot\partial_\mu\vec{\phi}_\pi,$$

$$L_{\pi\Delta\Delta} = \frac{f_{\Delta\Delta\pi}}{m_\pi}\,\bar{\psi}_{\Delta\mu}\gamma^\nu\gamma_5\vec{T}_\Delta\psi^\mu_\Delta\cdot\partial_\nu\vec{\phi}_\pi,$$

$$L_{\eta NN} = -\frac{f_{\eta NN}}{m_\eta}\,\bar{\psi}_N\gamma_\mu\gamma_5\psi_N\partial^\mu\phi_\eta.$$

ρ:

$$L_{\rho NN} = g_{\rho NN}\bar{\psi}_N\left[\gamma_\mu - \frac{\kappa_\rho}{2m_N}\sigma_{\mu\nu}\partial^\nu\right]\vec{\rho}^\mu\cdot\frac{\vec{\tau}}{2}\psi_N,$$

$$L_{\rho N\Delta} = -i\,\frac{f_{\rho N\Delta}}{m_\rho}\,\bar{\psi}^\mu_\Delta\gamma^\nu\gamma_5\vec{T}\cdot[\partial_\mu\vec{\rho}_\nu - \partial_\nu\vec{\rho}_\mu]\psi_N + [h.c.],$$

$$L_{\rho\Delta\Delta} = g_{\rho\Delta\Delta}\bar{\psi}_{\Delta\alpha}\left[\gamma^\mu - \frac{\kappa_{\Delta\Delta\rho}}{2m_\Delta}\,\sigma^{\mu\nu}\partial_\nu\right]\vec{\rho}_\mu\cdot\vec{T}_\Delta\psi^\alpha_\Delta,$$

$$L_{\rho\pi\pi} = g_{\rho\pi\pi}[\vec{\phi}_\pi\times\partial_\mu\vec{\phi}_\pi]\cdot\vec{\rho}^\mu,$$

$$L_{NN\rho\pi} = \frac{f_{\pi NN}}{m_\pi}g_{\rho NN}\,\bar{\psi}_N\gamma_\mu\gamma_5\vec{\tau}\psi_N\cdot\vec{\rho}^\mu\times\vec{\phi}_\pi,$$

$$L_{NN\rho\rho} = -\frac{\kappa_\rho g^2_{\rho NN}}{8m_N}\,\bar{\psi}_N\sigma^{\mu\nu}\vec{\tau}\psi_N\cdot\vec{\rho}_\mu\times\vec{\rho}_\nu.$$

ω:

$$L_{\omega NN} = g_{\omega NN}\bar{\psi}_N\left[\gamma_\mu - \frac{\kappa_\omega}{2m_N}\sigma_{\mu\nu}\partial^\nu\right]\omega^\mu\psi_N,$$

$$L_{\omega\pi\rho} = -\frac{g_{\omega\pi\rho}}{m_\omega}\,\epsilon_{\mu\alpha\lambda\nu}\partial^\alpha\vec{\rho}^\mu\partial^\lambda\vec{\phi}_\pi\omega^\nu.$$

σ:

$$L_{\sigma NN} = g_{\sigma NN}\bar{\psi}_N\psi_N\phi_\sigma,$$

$$L_{\sigma\pi\pi} = -\frac{g_{\sigma\pi\pi}}{2m_\pi}\,\partial^\mu\vec{\phi}_\pi\partial_\mu\vec{\phi}_\pi\phi_\sigma.$$

Electromagnetic ints:

$\pi, \eta, N, \omega, \Delta, \rho, \sigma$:

$$L_{\gamma NN} = \bar{\psi}_N\left[\hat{e}_N\gamma^\mu - \frac{\hat{\kappa}_N}{2m_N}\,\sigma^{\mu\nu}\partial_\nu\right]\psi_N A_\mu,$$

$$L_{\gamma\pi\pi} = [\vec{\phi}_\pi\times\partial^\mu\vec{\phi}_\pi]_3 A_\mu,$$

$$L_{\gamma N\pi N} = \frac{f_{\pi NN}}{m_\pi}\,[\bar{\psi}_N\gamma^\mu\gamma_5\vec{\tau}\psi_N)\times\vec{\phi}_\pi]_3 A_\mu,$$

$$L_{\gamma\rho\rho} = [(\partial^\mu\vec{\rho}^\nu - \partial^\nu\vec{\rho}^\mu)\times\vec{\rho}_\nu]_3 A_\mu,$$

$$L_{\gamma\rho\pi\pi} = -g_{\rho\pi\pi}[(\vec{\rho}^\mu\times\vec{\phi}_\pi)\times\vec{\phi}_\pi]_3 A_\mu,$$

$$L_{\gamma N\pi\Delta} = \frac{f_{\pi N\Delta}}{m_\pi}\,[(\bar{\psi}^\mu_\Delta\vec{T}\psi_N)\times\vec{\phi}_\pi]_3 A_\mu,$$

$$L_{\gamma N\rho N} = g_{\rho NN}\left[\frac{\kappa_\rho}{2m_N}\left(\bar{\psi}_N\frac{\vec{\tau}}{2}\,\sigma^{\nu\mu}\psi_N\right)\times\vec{\rho}_\nu\right]_3 A_\mu,$$

$$L_{\gamma N\Delta} = -i\bar{\psi}^\mu_\Delta\Gamma^{em,\Delta}_{\mu\nu}T_3\psi_N A^\nu + (h.c.),$$

$$L_{\gamma\rho\pi} = \frac{g_{\rho\pi\gamma}}{m_\pi}\,\epsilon_{\alpha\beta\gamma\delta}\vec{\phi}_\pi\cdot(\partial^\gamma\vec{\rho}^\delta)(\partial^\alpha A^\beta),$$

$$L_{\gamma\omega\pi} = \frac{g_{\omega\pi\gamma}}{m_\pi}\,\epsilon_{\alpha\beta\gamma\delta}(\partial^\alpha A^\beta)\phi^3_\pi(\partial^\gamma\omega^\delta),$$

$$L_{\gamma\rho\eta} = \frac{g_{\rho\eta\gamma}}{m_\rho}\,\epsilon^{\mu\nu\alpha\beta}\partial_\mu\rho^3_\nu\partial_\alpha A_\beta\phi_\eta,$$

$$L_{\gamma\rho\sigma} = -\frac{g_{\rho\sigma\gamma}}{m_\rho}\,(\partial_\mu\rho^3_\nu)(\partial^\mu A^\nu - \partial^\nu A^\mu)\sigma,$$

$$L_{\gamma\Delta\Delta} = \bar{\psi}^\eta_\Delta\left(T^3_\Delta + \frac{1}{2}\right)\left[-\gamma^\mu g_{\eta\nu} + (g^\mu_\eta\gamma^\nu + g^\mu_\nu\gamma^\eta) + \frac{1}{3}\gamma_\eta\gamma^\mu\gamma_\nu\right]\psi^\nu_\Delta A_\mu.$$

CNS Center for Nuclear Studies

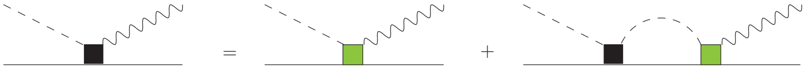
Dynamical model

Lagrangian density of preceeding page→Hamiltonian density

$$H = \int d^3x\, \mathcal{H}(\mathbf{x}) = H_0 + H_{\rm int} \qquad H_{\rm int} = \sum_{M,B,B'} \Gamma_{MB,B'} + \sum_{M.M',M''} \Gamma_{MM',M''}$$

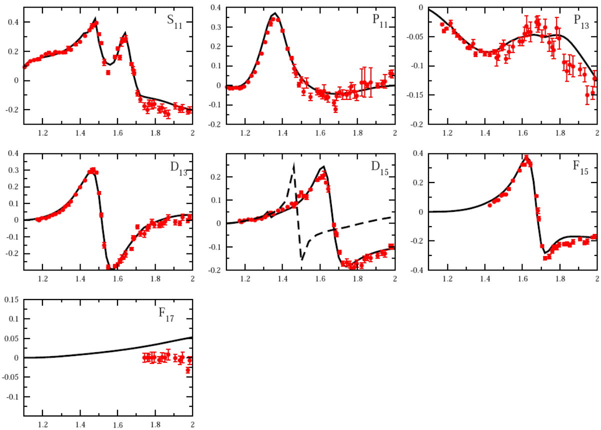
Dynamical Lippmann-Schwinger equation

$$T = V + TG_0 V$$



Hadronic π and ω production

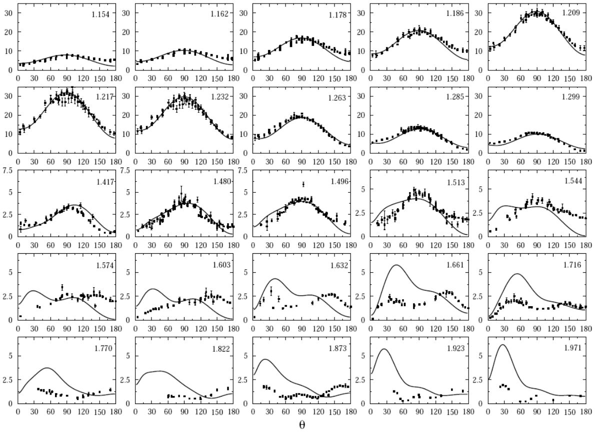
$\pi N \longrightarrow \pi N, \omega N$



Real part, isospin $1/2$

Photoproduction of π and ω mesons

$\gamma N \longrightarrow \pi N, \omega N$



$\frac{d\sigma}{d\Omega}$: $\gamma p \longrightarrow \pi^0 p$

Conclusion/Outlook

Conclusions

- Non-perturbative QCD
  - Resonance spectrum probes confinement regime of QCD
- Analytics
  - Resonances correspond to non-trivial analytic structures–poles–in the complex energy plane
- Model independence
  - Correct analytic behavior is required to obtain good agreement with data
- Modeling
  - Probes one's understanding of theories at the hadronic level

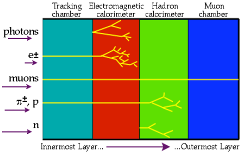
Outlook

- Improve π–photoproduction amplitudes via the Chew-Mandelstam approach
- Simultaneous fitting of π and η photoproduction reactions
- Global fitting of hadronic and electromagnetic reaction data
- Include $\pi\pi N$ channel and its discontinuities

Experimental quantities to ‘observables’

The part SAID doesn’t do

- Process: ionization, bremsstrahlung, TOF, Cerenkov, showers
- Detectors: bubble, PC, MWPC, DC, TPC, spark, streamer, scintillators, emulsions, semiconductors, . . .



- Detector simulation: GEANT, GSIM, SIM12, . . .
- Event-based fitting

A Topical Collaboration in the Analysis of the Light Hadron Spectrum
submitted to the
Department of Energy, Division of Nuclear Physics

- Applicant/Institution:

- Particle state: species, four-momenta, polarization
- Scattering observables *e.g.* $\gamma N \longrightarrow \pi N$ [after Barker et. al., NPB **96**, 347, 1975]
  - unpolarized: $d\sigma/d\Omega$
  - single polarization: Σ, T, P
  - beam-target: E, F, G, H
  - beam-recoil: O_x, O_z, C_x, C_z

Amplitudes→Observables

Elastic πN scattering

- Spin dependence $f(\theta) = g(\cos\theta) + i\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}h(\cos\theta)$
- Mixed state $\langle\overline{O}\rangle = \frac{Tr\rho O}{Tr\rho}$ with $\rho = \sum_s |s\rangle p_s\langle s|$
- $\rho_f = f\rho_i f^\dagger$
- Unpolarized nucleon $\rho = \frac{1}{2}$
  - $\frac{d\sigma}{d\Omega} = \frac{Tr\rho_f}{Tr\rho_i} = |g(\theta)|^2 + |h(\theta)|^2$
  - $\mathbf{P}_f = \frac{Tr\boldsymbol{\sigma}\rho_f}{Tr\rho_f} = -\frac{2Im(g^* h)}{|g|^2+|h|^2}\hat{\mathbf{n}} = P(\theta)\hat{\mathbf{n}}$
- Polarized nucleon $\mathbf{P}_i = P_i\hat{\mathbf{n}}$
  - $\frac{d\sigma}{d\Omega}\Big|_{pol} = \frac{d\sigma}{d\Omega}(1 + P_i P(\theta))$
- Longitudinally polarized target nucleon
  - R ‘spin rotation’: final pol. in scatt. plane along nucleon momentum
  - A ‘spin rotation’: final pol. in scatt. plane $\perp$ nuc. mom.
- Partial wave representation
  eg. $g(\cos\theta) = \frac{1}{q}\sum_{\ell=0}^{\infty}\{(\ell+1)f_{\ell+} + \ell f_{\ell-}\}\, P_\ell(\cos\theta)$

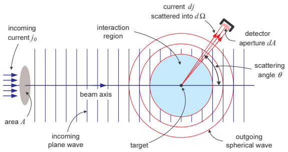
Observables → Amplitudes

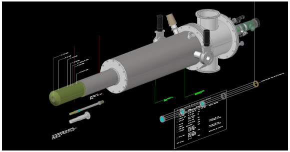
Differential cross section 2 → 2 reactions:
$\pi N \to \pi N, \gamma N \to \pi N, \ldots$ at center-of-mass energy, W

$$\frac{d\sigma}{d\Omega} = \frac{\#\text{ particles scattered into } (\theta, \phi)}{\text{unit time} \cdot \text{incident flux}}$$
$$= \frac{(4\pi)^2}{k^2} \rho_{1'2'}(k')\rho_{12}(k) \left| T_{\lambda_{1'},\lambda_{2'},\lambda_1\lambda_2}(k', k; W) \right|^2$$

- T transition matrix: complex function of W
- New polarized experiments (Bonn, JLab, Lund, Mainz)
- Unitarity requires *multi-channel* data, *eg.* $\gamma N \to \pi N, \gamma N \to \pi\pi N, \gamma N \to \eta N, \ldots$
- Provide 'boundary data' on the real energy axis → allows analytic continuation into complex plane

Figure courtesy H. Haberzettl





FROzen Spin Target

Ambiguities

Elastic πN scattering

Consider $2 \rightarrow n-$body scattering

$$\mathcal{O}_{\alpha\beta}(\mathbf{k}_\alpha, W) = \frac{1}{k_\beta^2} \left| \sum_{P(\alpha)} Z_{P(\alpha)}(\mathbf{k}_\alpha) T_{\alpha\beta} Z_{P(\beta)}(\mathbf{k}_\beta) \right|^2$$

$Z_{P(\alpha)}(\mathbf{k})$ contains angular information for the partial wave, $P(\alpha)$

Specific example: $MB \longrightarrow M'B'$

$$\frac{d\sigma}{d\Omega} = \frac{1}{k_{MB}^2} \left| T_{M_{M'} M_{B'}, M_M M_B}(\theta, W) \right|^2$$

$$\begin{aligned} T_{M_{M'} M_{B'}, M_M M_B}(\theta, W) = & \sum_{JTLSL'S'} \\ & \times \langle S_{M'} S_{B'} M_{M'} M_{B'} | S_{M'} S_{B'}; S' M' \rangle \langle S' [S_{M'} S_{B'}] L' M_{S'} M_{L'} | L' S'; JM \rangle Y_{L'}^{M_{L'}}(\hat{\mathbf{k}}') \\ & \times T^{JT}_{L'S'M'B', LSMB}(W) \\ & \times \sqrt{\frac{2L+1}{4\pi}} \langle LS; JM | S[S_M S_B] L M_S M_L \rangle \langle S_M S_B; SM | S_M S_B M_M M_B \rangle, \end{aligned}$$

Resonance production

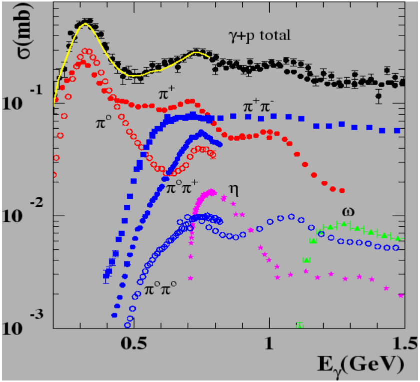
- Photoproduction exhibits strong resonance signature (bumps) in all channels
- Single meson production falls-off $E_\gamma \sim 750$ MeV, $W \sim 1500$ MeV
- Coupled-channel treatment absolute necessity
- Aside: energies
  E_γ photon lab energy [experiment]
  W total COM energy [calculations]

$$W = [m_N^2 + 2m_N E_\gamma]^{1/2}$$

$$\approx m_N + E_\gamma - \frac{E_\gamma^2}{4m_N}$$

$$E_\gamma = \frac{W^2 - m_N^2}{2m_N}$$

$$= \tfrac{1}{2}(1 + \frac{W}{m_N})[W - m_N]$$



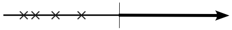
Analytic structure of S

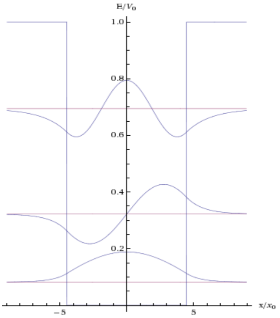
Bound states, resonances, & poles

Bound states: $W < 0$

$$\psi_1(x) = e^{\kappa x} \qquad \psi_2(x) = A \begin{pmatrix} \cos \\ \sin \end{pmatrix} \overline{p}x \qquad \psi_3(x) = \pm e^{-\kappa x}$$

$$x < -a/2 \qquad -a/2 \leq x \leq a/2 \qquad a/2 < x$$





$$\kappa = \sqrt{-2mW} > 0, W \leq 0$$

$$S(E)e^{ipa} = \frac{1}{\cos \overline{p}a - \frac{i}{2}\left[\frac{p}{\overline{p}} + \frac{\overline{p}}{p}\right] \sin \overline{p}a}$$

Denominator zeros→bound states when $W < 0$

$$\tan \frac{\overline{p}a}{2} = \frac{\kappa}{\overline{p}}$$

$$\tan \frac{\overline{p}a}{2} = -\frac{\overline{p}}{\kappa}$$

$$p = i\kappa.$$

Analytic structure of S

Bound states, resonances, & poles

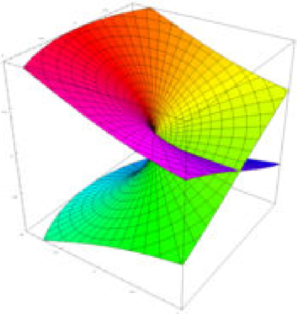
Riemann surface

$$p = \sqrt{2mW} \qquad\qquad W \in \mathbb{C}$$

$$\sqrt{W} = |W|^{\frac{1}{2}}\, e^{i\theta/2}$$

$$\theta = \begin{cases} 0 \le \theta < 2\pi & \text{‘upper’ sheet} \\ 2\pi \le \theta < 4\pi & \text{‘lower’ sheet} \end{cases}$$

$$\text{Disc } p \equiv p(W + i\epsilon) - p(W - i\epsilon)$$
$$= \sqrt{2m|W|}[e^{i\cdot 0/2} - e^{i\cdot 2\pi/2}] = 2\sqrt{2m|W|}$$



Riemann surface representation of the function $\sqrt{W}$. The complex–W plane is horizontal. The vertical axis gives the imaginary part of the function.

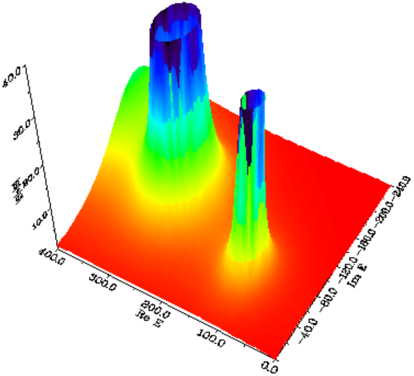
Analytic structure of S

Bound states, resonances, & poles

Given $T(W) = \mathrm{Re}\, T(W) + i \mathrm{Im}\, T(W)$, for $W > 0$ consider AC in $z = W + i\mathrm{Im}\, z$

$$S(E)e^{ipa} = \frac{1}{\cos \bar{p}a - \frac{i}{2}\left[\frac{p}{\bar{p}} + \frac{\bar{p}}{p}\right]\sin \bar{p}a}$$

- Denominator zeros on the second sheet→resonances



CNS Center for Nuclear Studies

Analytic structure of S

Bound states, resonances, & poles

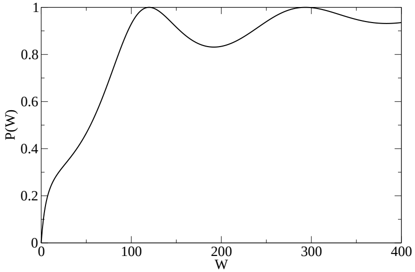
Given $T(W) = \mathrm{Re}\, T(W) + i\mathrm{Im}\, T(W)$, for $W > 0$ consider AC in $z = W + i\mathrm{Im}\, z$

$$S(E)e^{ipa} = \frac{1}{\cos \overline{p}a - \frac{i}{2}\left[\frac{p}{\overline{p}} + \frac{\overline{p}}{p}\right] \sin \overline{p}a}$$

$$T(E) = |S(W)|^2 = \frac{1}{1 + \frac{V_0^2}{4E(E+V_0)} \sin^2 \overline{p}a}$$

$$\overline{p}a = n\pi \rightarrow E_n = n^2 \frac{\pi^2}{2ma^2} - V_0$$

- Denominator zeros on the second sheet→resonances



Analytic structure of S

Bound states, resonances, & poles

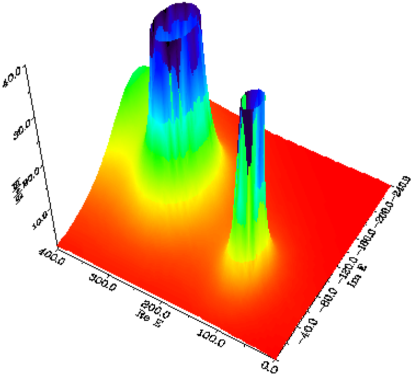
Given $T(W) = \mathrm{Re}\, T(W) + i\mathrm{Im}\, T(W)$, for $W > 0$ consider AC in $z = W + i\mathrm{Im}\, z$

$$S(E)e^{ipa} = \frac{1}{\cos \overline{p}a - \frac{i}{2}\left[\frac{p}{\overline{p}} + \frac{\overline{p}}{p}\right] \sin \overline{p}a}$$

$$T(E) = |S(W)|^2 = \frac{1}{1 + \frac{V_0^2}{4E(E+V_0)} \sin^2 \overline{p}a}$$

$$\overline{p}a = n\pi \to E_n = n^2 \frac{\pi^2}{2ma^2} - V_0$$

- Denominator zeros on the second sheet→resonances



Unitarity $\leftrightarrow$ analytic structure

$$\langle\alpha|\left\{T^+ - T^- = 2iT^+\rho T^-\right\}|\beta\rangle \to T^+_{\alpha\beta} - T^-_{\alpha\beta} = 2i\sum_\sigma T^+_{\alpha\sigma}\rho_\sigma(W)T^-_{\sigma\beta}$$

$$\to \text{Im } T(W) = 2i\sum_\sigma T^+_{\alpha\sigma}(W)\rho_\sigma(W)T^-_{\sigma\beta}(W)$$

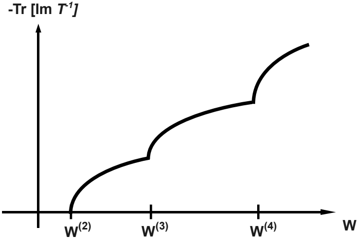
$$\rho^{(2)}_\sigma = \theta(W - (m_{\sigma,1} + m_{\sigma,2}))\mathcal{K}_2$$

$$\rho^{(3)}_\sigma = \theta(W - (m_{\sigma,1} + m_{\sigma,2} + m_{\sigma,3}))\mathcal{K}_3$$

$$\vdots$$

$$\rho^{(n)}_\sigma = \cdots$$

- 'Kinks' due to Heaviside-θ function, due to $\delta(E-H)$
- Non-analytic function? [*Eden (1952)*]
- Violation of Cauchy-Riemann conditions → **branch points**



Unitarity $\leftrightarrow$ analytic structure

$$\langle\alpha|\left\{T^+ - T^- = 2iT^+\rho T^-\right\}|\beta\rangle \to T^+_{\alpha\beta} - T^-_{\alpha\beta} = 2i\sum_\sigma T^+_{\alpha\sigma}\rho_\sigma(W)T^-_{\sigma\beta}$$

$$\to \mathrm{Im}\; T(W) = 2i\sum_\sigma T^+_{\alpha\sigma}(W)\rho_\sigma(W)T^-_{\sigma\beta}(W)$$

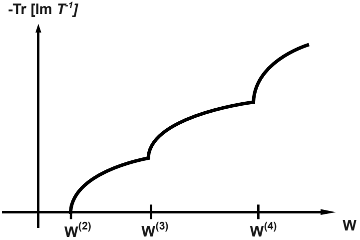
$$\rho^{(2)}_\sigma = \theta(W - (m_{\sigma,1} + m_{\sigma,2}))\mathcal{K}_2$$

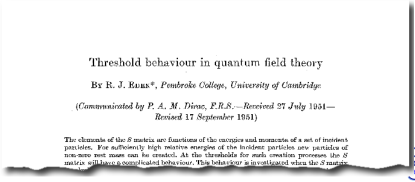
$$\rho^{(3)}_\sigma = \theta(W - (m_{\sigma,1} + m_{\sigma,2} + m_{\sigma,3}))\mathcal{K}_3$$

$$\vdots$$

$$\rho^{(n)}_\sigma = \cdots$$

- ‘Kinks’ due to Heaviside-θ function, due to $\delta(E-H)$
- Non-analytic function? [*Eden (1952)*]
- Violation of Cauchy-Riemann conditions $\to$ **branch points**





Threshold behaviour in quantum field theory

By R. J. Eden*, *Pembroke College, University of Cambridge*

(*Communicated by P. A. M. Dirac, F.R.S.—Received* 27 *July* 1951—*Revised* 17 *September* 1951)

The elements of the S matrix are functions of the energies and moments of a set of incident particles. For sufficiently high relative energies of the incident particles new particles of non-zero rest mass can be created. At the thresholds for such creation processes the S matrix will have a complicated behaviour. This behaviour is investigated when the S matrix

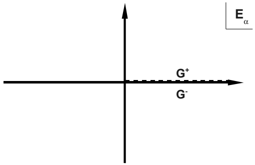
$\mathcal{R}$iemann-$\mathcal{H}$ilbert

First blush

$$([G^- - G^+]V\Phi_\alpha, \Psi^+_\beta) = (\Phi_\alpha, V^\dagger[G^+ - G^-]\Psi^+_\beta) \qquad H\Psi^+_\beta = E_\alpha \Psi^+_\beta$$

Plemelj Formula:

$$\begin{aligned} G^\pm &= \frac{1}{E_\alpha - H \pm i\epsilon} \\ &= \frac{1}{E_\alpha - H} \mp i \lim_{\epsilon \to 0^+} \frac{\epsilon}{(E_\alpha - H)^2 + \epsilon^2} \\ &= \frac{1}{E_\alpha - H} \mp i\pi\delta(E_\alpha - H) \end{aligned}$$



$$[G^+ - G^-]\Psi^+_\beta = -2\pi i \delta(E_\alpha - E_\beta)\Psi^+_\beta$$

$$\to S_{\alpha\beta} = \delta_{\alpha\beta} + 2\pi i \delta(E_\alpha - E_\beta) T^+_{\alpha\beta} \qquad T^+_{\alpha\beta} = -(\Phi_\alpha, V\Psi^+_\beta)$$

- The scattering amplitude is proportional to the discontinuity in G across the real energy axis E_α: Disc $G = G^+ - G^- = 2\pi i \delta(E_\alpha - H)$
- Plemelj formula $\Rightarrow$ imaginary part gives *coupling to the continuum*
- *Sectionally holomorphic* function