Primer on Quantum Mechanics

E.E. Ford Summer Teachers' Colloquium

Joe Ginocchio & Mark Paris

Los Alamos National Lab Theory Division

T-2 Nuclear & Particle, Astrophysics & Cosmology

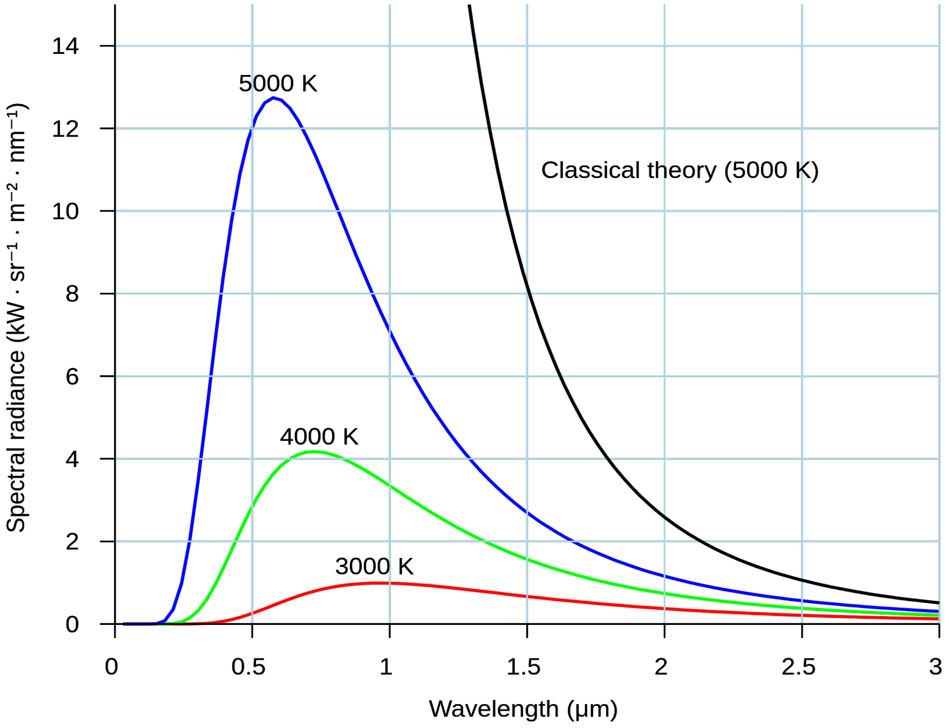
Why Quantum Mechanics?

Blackbody Radiation

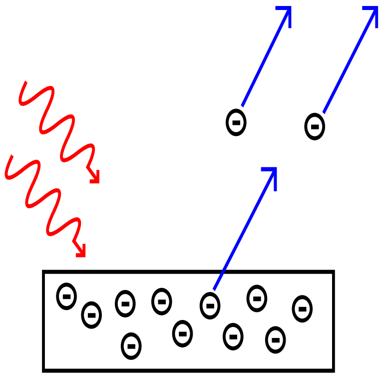
- The energy density of radiation measured from a “black” cavity does not agree with theoretical expectations. Max Planck proposed that light has energy inversely proportional to its wavelength or directly proportional to its frequency (1900).
- Summing over all possible discrete standing waves in the cavity, he derived a simple equation energy density of light versus its wavelength for a given temperature.
- As the temperature increases his result approaches that of the classical result.
- Hence light beams of a given wavelength carry quanta of energy which we call photons.

$$E = h\nu = \frac{hc}{\lambda}$$

h Planck’s constant



Consequence of Planck’s Quanta of Light



Photoelectric Effect

- This gave Einstein the idea that if you shine light on materials electrons will be emitted with energy determined by the frequency of the light, not by its intensity (1905).
- The intensity of the light (number of photons) determines the number of electrons emitted.
- Below a certain threshold of energy no electrons will be emitted which implies that the electrons are quantized in energy also.

Energy Quantization for Particles

- This leads to quantization of energy in atoms and nuclei.
- The quantization of energy in atoms depends on how electrons interact with each other. This is well understood.
- The quantization of energy in nuclei depends on how quarks interact with each other. This is not well understood.

Uncertainty Principle

- **Quantum Measurement**
  - In the process of measuring a particle the observer disturbs the particle.
  - This uncertainty is expressed as (1928)

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

    which relates uncertainty in position with uncertainty in momentum.
  - The effects are very small because Planck's constant is small. For example an uncertainty of one mile per hour in the speed of your car leads to about 10^(-39) inches uncertainty in its position.

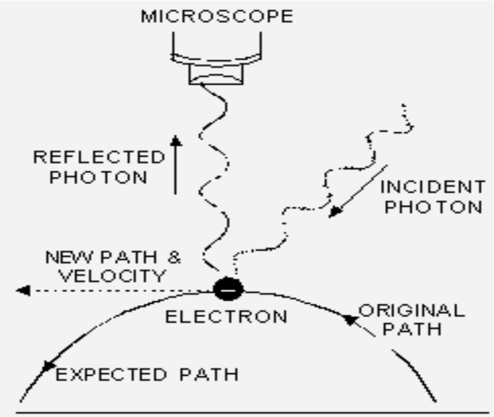
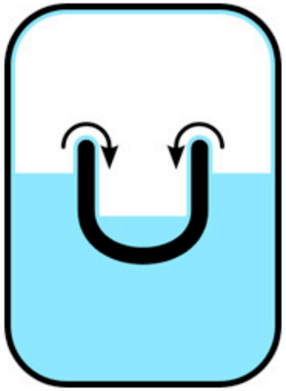


Figure 4. Change of momentum and position of electron on impact with a photon

Interlude: quantum mechanics (QM) is very "real"

- **Visual proof**
  - Superfluid liquid Helium-4
    - Kapitsa, Allen, & Misener 1937
  - Cooled below the "lambda point", T_λ=2.172 K
  - The fluid flows up (against gravity) the inner wall of the vessel and down the outer wall
  - Why? ➔Superfluid component has zero viscosity
  - Capillarity drives flow; no resistance
  - Lower image shows "inverse" process
- **Superfluid is a consequence of a quantum description of nature**
  - It's quite "real."
  - And macroscopic!



http://alfredleitner.com/superfluid.html

Back to the Heisenberg uncertainty principle (HUP)

- **Precision in position (x) & momentum (p=mv) is *correlated***
  - x & p are examples of conjugate variables; all conjugate variables have this prop.
  - Call Δx & Δp precisions; HUP says they're correlated:

$$\Delta x \cdot \Delta p \geq \hbar$$

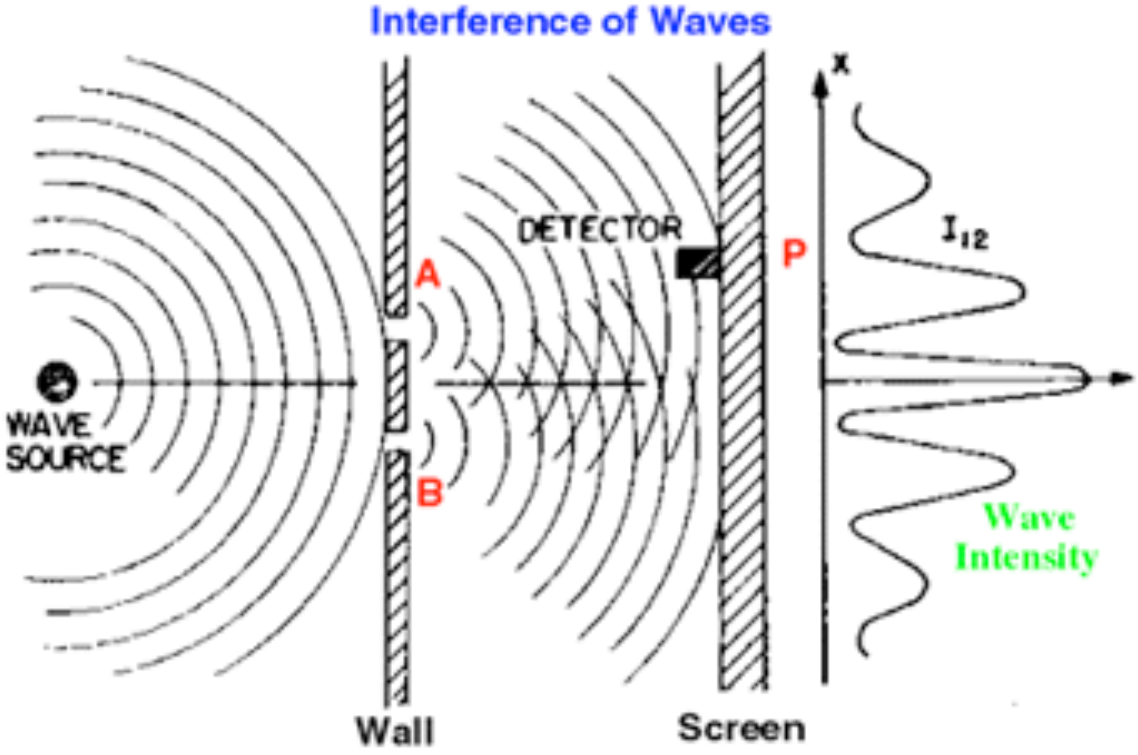
  - $\hbar$ is Planck's constant – a *nearly infinitesimal* number (6.626 × 10^{-34} J s)
  - Example: suppose we know an electron's position to 1 Fermi = 10^{-15} m

$$\begin{aligned} p &= mv \\ \Rightarrow \Delta p &= m\Delta v \\ \text{HUP} \Rightarrow \Delta v &\geq \frac{\hbar}{m\Delta x} \approx 390\,\frac{m}{s} \end{aligned}$$

  - An uncertainty of 390 m/s in the velocity is ***the best*** we can do.
  - And this is non-classical:
    - Classically, measurements of one variable can always be done without affecting any other

Classical double-slit experiment

- **Waves of light or water or sound or …**

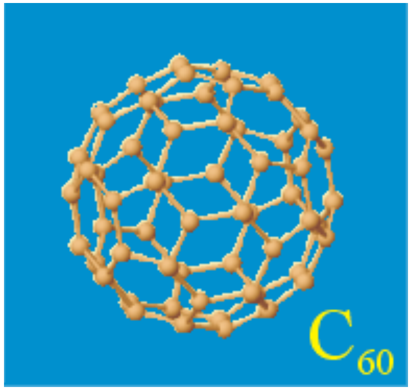


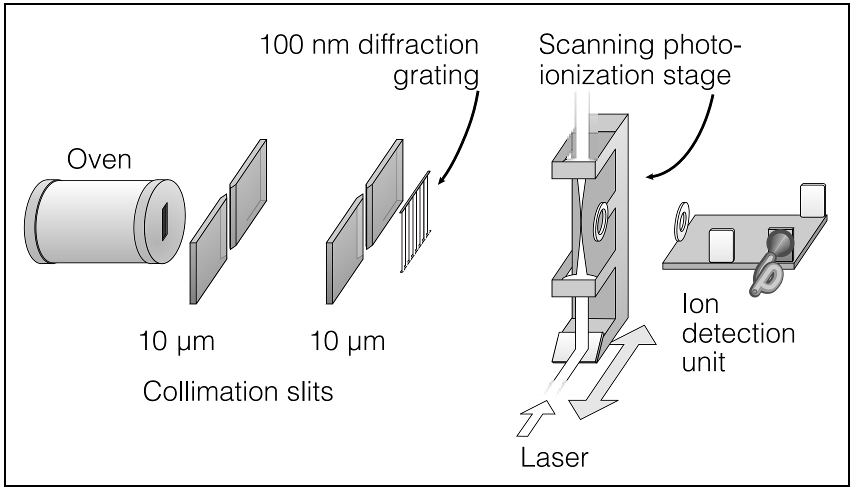
  - Kirchoff diffraction pattern

Another "real," nearly classical example

- **Wave-particle duality of C_{60} molecules**
  - Zeilinger et.al., Nature 401 (1999)
  - Classic double-slit type experiment except it uses fullerenes ("buckyballs") rather than light
  - Fullerenes are "nearly classical":

    C_{60} size ~ 400 x (de Broglie wavelength)
  - ***de Broglie wave interference pattern is seen***
  - Compared to standard Kirchoff diffraction pattern
  - Is it a "wave" or is it a "particle"?





100 nm diffraction grating

Scanning photo-ionization stage

Oven

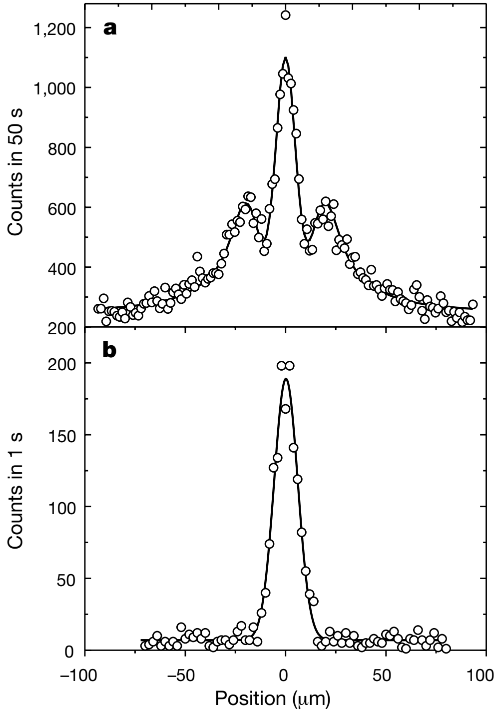
Ion detection unit

10 µm

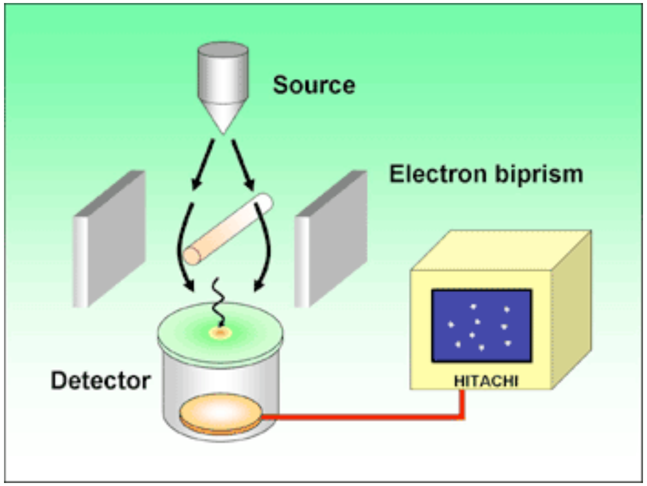
10 µm

Collimation slits

Laser



Quantum particles interfere with themselves



- Standard double-slit but with electrons
- Let electrons pass through slits *one at a time*
- Take four time-lapsed photographs
- Interference pattern builds-up over time!

- Classical waves → lots of particles
- Quantum waves → single particle

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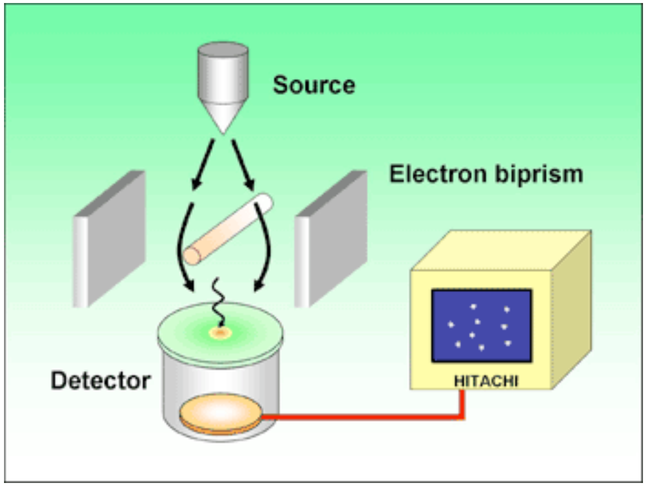

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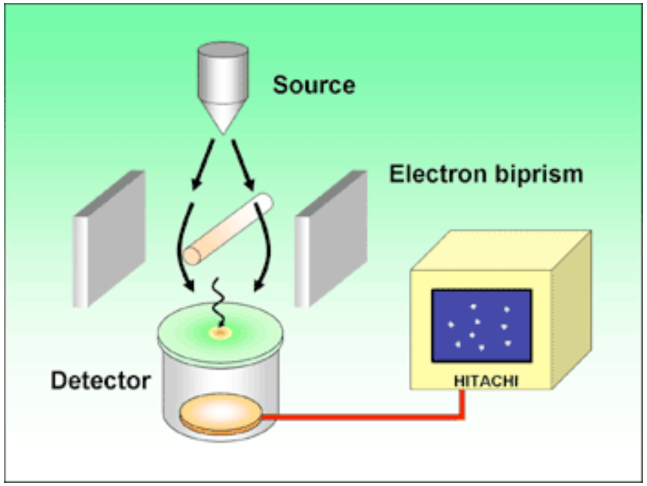

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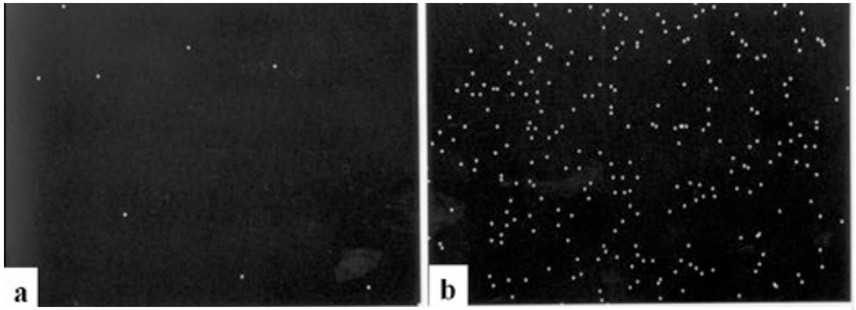

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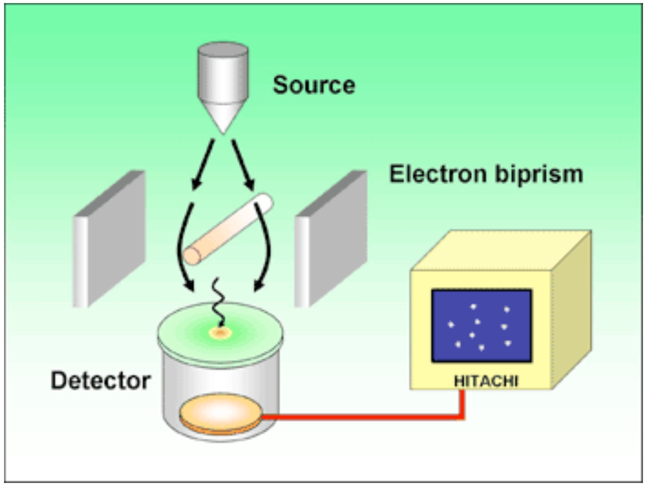

- **Standard double-slit but with electrons**
- **Let electrons pass through slits *one at a time***
- **Take four time-lapsed photographs**
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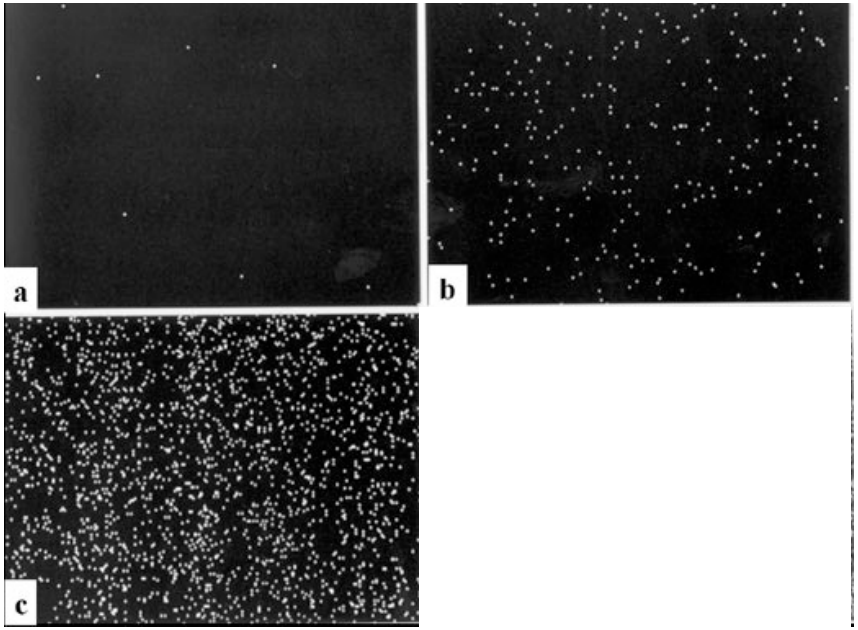
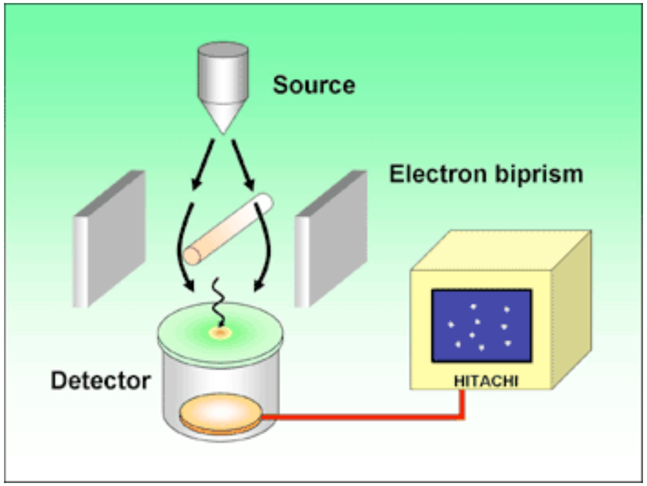
- **Classical waves → lots of particles**
- **Quantum waves → single particle**



UNCLASSIFIED

Quantum particles interfere with themselves



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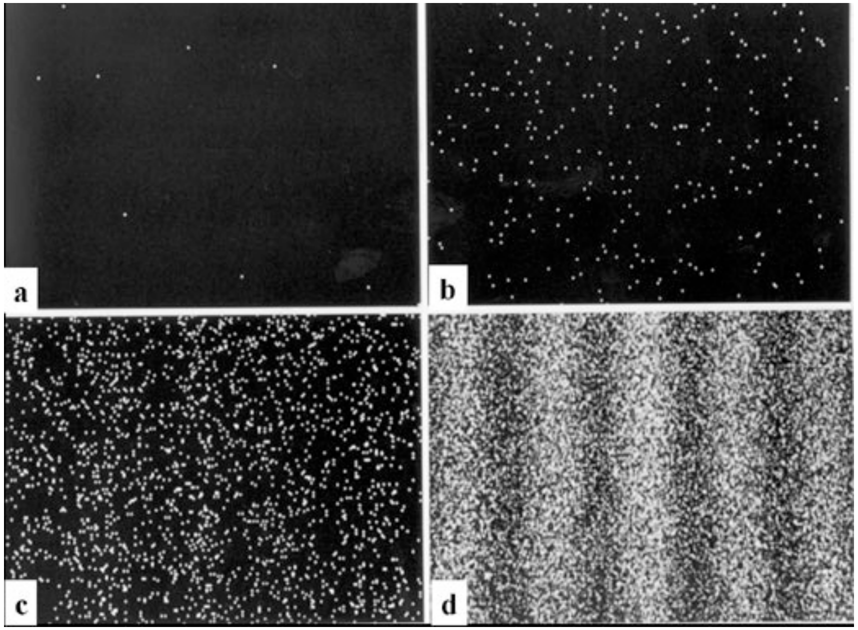

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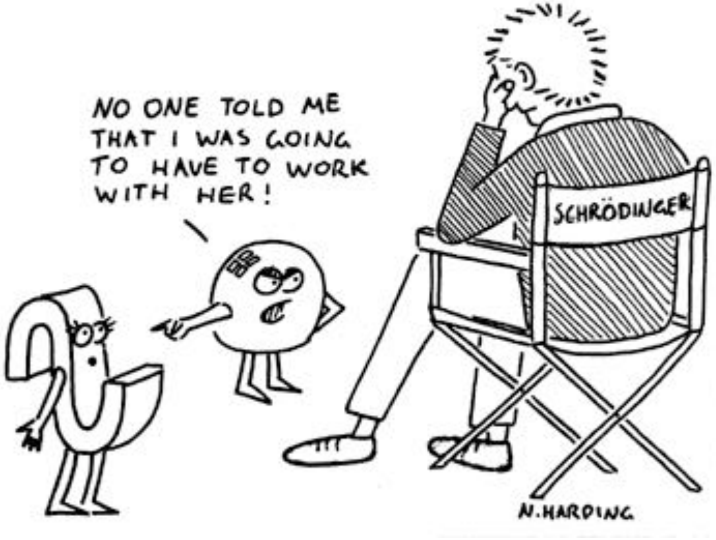
On the wave-particle duality and consistency

The usual story

- “Sometime it acts as a particle, sometimes it acts as a wave”
- The oft-neglected point: *these “sometimes” never coincide!*
- Ref: Tipler, *Physics for Scientists & Engineers*

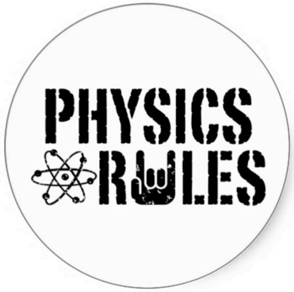


Wavelike

- During propagation (*ie*. going from A to B without interacting with anything along the way)

Pointlike

- During interaction (*ie*. when the electron exchanges energy/momentum)



There are rules!

- Full description of processes requires both ***but not at the same time***.

QM & causality: spin

- **Spin (intrinsic spin)**
  - An electron has intrinsic spin of ½$\hbar$; you might imagine it as spinning*
  - Spin (angular momentum) in QM is weird
    - If you measure spin in one direction, then you affect the spin value in the other two directions
  - Another weird thing
    - Measurement of spin, along any direction gives only two values: ±½$\hbar$
  - And yet, even more weird:
    - Spin can be in a ***superposition*** of +½$\hbar$ and -½$\hbar$
      - Wave function $|\psi\rangle = \alpha|+\hat{\mathbf{z}}\rangle + \beta|-\hat{\mathbf{z}}\rangle$
    - These ***superpositions*** have definite spin in some other direction but the spin measured in the z-direction is random, weighted by

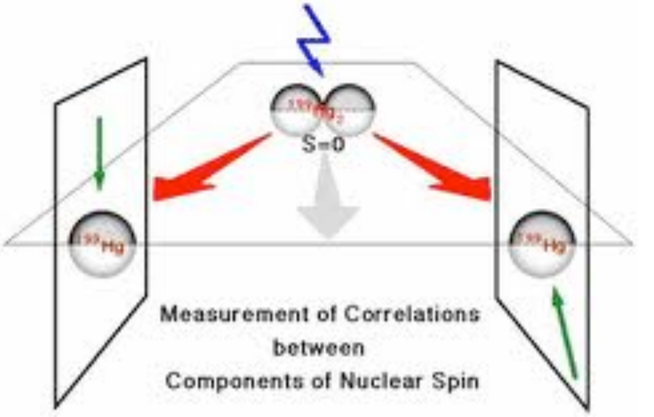
*but this isn't quite right

$$|\alpha|^2 \Rightarrow \text{spin in } +\hat{\mathbf{z}} \text{ direction OR } |\beta|^2 \Rightarrow \text{spin in } -\hat{\mathbf{z}} \text{ direction}$$

Learn more about spin and QM: `http://public.lanl.gov/mparis/qmp.pdf`

QM & causality:

- **Entanglement**
  - A spin-zero particle decays into two spin-½ particles
  - If left-side particle (LSP) is measured along a chosen direction as +½, then right-side particle (RSP) is measured as -½ along this same chosen direction (because angular momentum conserved)
  - In this state of affairs the spins are "entangled"; that is, they're correlated
  - Since the state of the LSP is, in general, a superposition of +½ & -½, it's spin is ***unknown*** until measured
  - Then the state of the RSP is fixed (along the chosen direction) ***seemingly instantaneously*** ?!
    - And this is weird.
    - Or, at least, appears to conflict with special relativity (Einstein)
    - But it doesn't conflict: there's no way to transfer information using these entangled states



Einstein, Podolsky, & Rosen were upset by this state of affairs. They were right to be upset. But QM has proven itself.

Usefulness of Quantum Mechanics

- **Modern Technology**
  - Solar panels depend on the photoelectric effect to convert sunlight to electrical energy.
  - Quantum physics determine the special properties of semi-conductors and superconductors which have spawned the electronics that we depend on daily.
- **Future**
  - Classical computer use bits, on or off, or zero and one. Quantum computers use qubits which are a combination of zero and one. For example a qubit can be linear combination of spin up and spin down. Qubits allow parallel computing with one processor.
  - Quantum Cryptography
- **Conclusion**
  - An abstract physics theory understood by only a few people 100 years ago has led to a new world of technology. Even so, to this day quantum physics is not fully understood. For, example we do not have a complete understanding of quantum gravity.