DEVELOPING INFORMATION-GAP MODELS OF UNCERTAINTY FOR TEST-ANALYSIS CORRELATION

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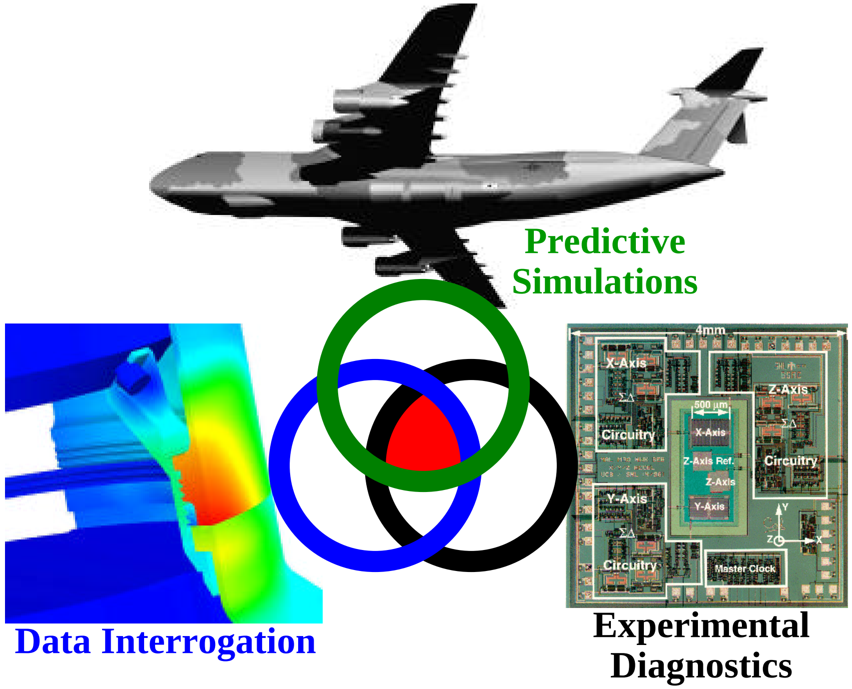

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Abstract

Developing Information-gap Models of Uncertainty for Test-analysis Correlation
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Relying on numerical simulations, as opposed to field measurements, to analyze the structural response of complex systems requires that the predictive accuracy of the models be assessed. This activity is generally known as "model validation". Model validation requires the comparison of model predictions with test measurements at several points of the design / operational space. For example, numerical models of flutter must be validated for various combinations of fluid velocity and wing angle-of-attack. Because validation experiments become expensive when the system investigated is complex, only a few data sets are generally available. This lack of adequate representation of the design / operational space makes it questionable whether statistical models of predictive accuracy can be developed.

In this work, we focus on one aspect of model validation that consists in assessing the robustness of a decision to uncertainty. In this context, "decision" refers to assessing the accuracy of predictions and verifying that the accuracy is adequate for the purpose intended. Likewise, "uncertainty" can represent experimental variability, variability of the model's parameters but also inappropriate modeling rules in regions of the design / operational space where experiments are not available.

An alternative to the theory of probability is applied to the problem of assessing the robustness of model predictions to sources of uncertainty. The analysis technique is based on the theory of information-gap, which models the clustering of uncertain events in embedded convex sets instead of assuming a probability structure. Unlike other theories developed to represent uncertainty, information-gap does not assume probability density functions (which the theory of probability does) or membership functions (which fuzzy logic does). It is therefore appropriate in cases where limited data sets are available. The main disadvantage of information-gap is that the efficiency of sampling techniques cannot be exploited because no probability structure is assumed. Instead, the robustness of a decision with respect to uncertainty is studied by solving a sequence of optimization problems, which becomes computationally expensive as the number of decision and uncertainty variables increases.

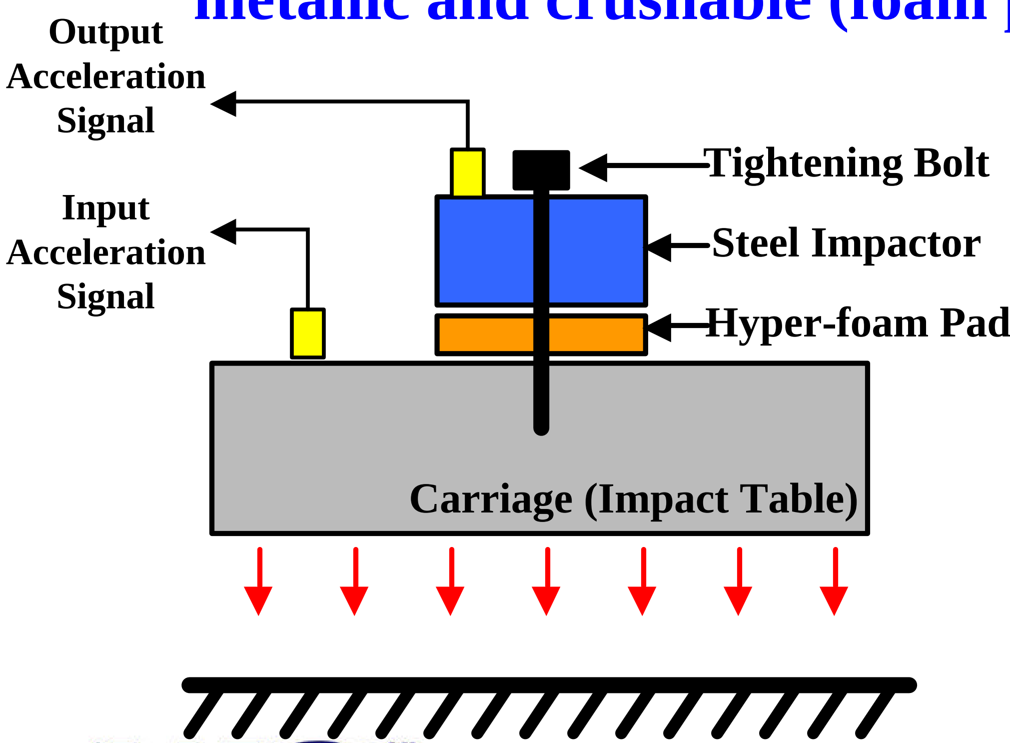
The concepts are illustrated with the propagation of a transient impact through a layer of hyper-elastic material. The numerical model includes a softening of the hyper-elastic material's constitutive law and contact dynamics at the interface between metallic and crushable materials. Although computationally expensive, it is demonstrated that the information-gap reasoning can greatly enhance our understanding of a moderately complex system when the theory of probability cannot be applied.

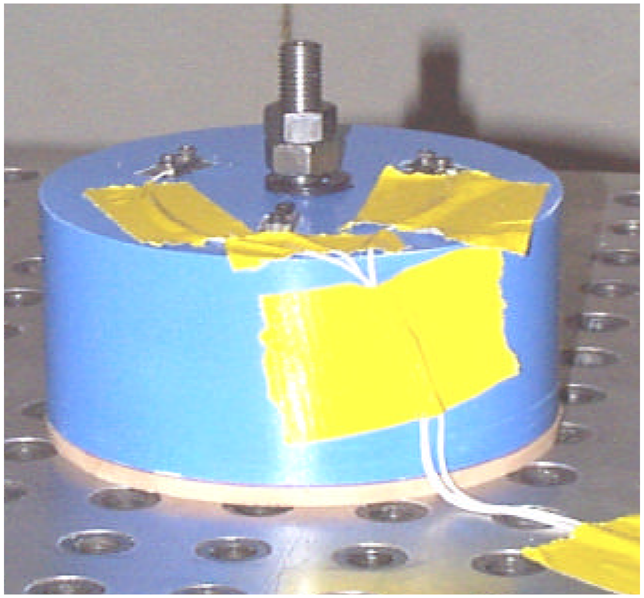
Outline

- **The Foam Impact Experiment**
- **Brief Overview of Information-gap Theory**
- **Implementation and Results of Info-gap Analysis**
- **Perspectives for Decision-making**

Hyper-foam Impact Experiments

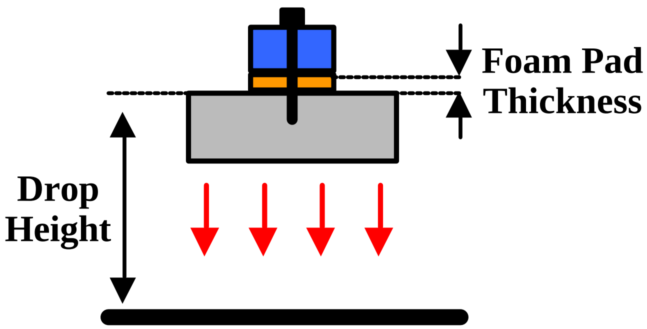
- Physical experiments are performed to study the propagation of an impact through an assembly of metallic and crushable (foam pad) components.

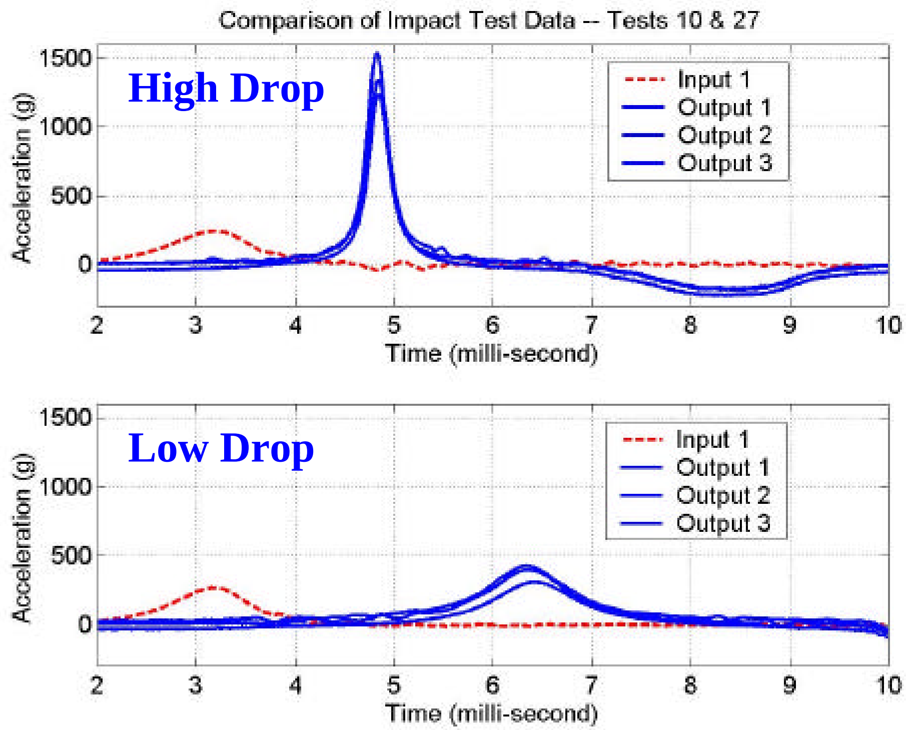




Experimental Data

- **Several configurations of the system are tested by varying the foam pad thickness and drop height.**

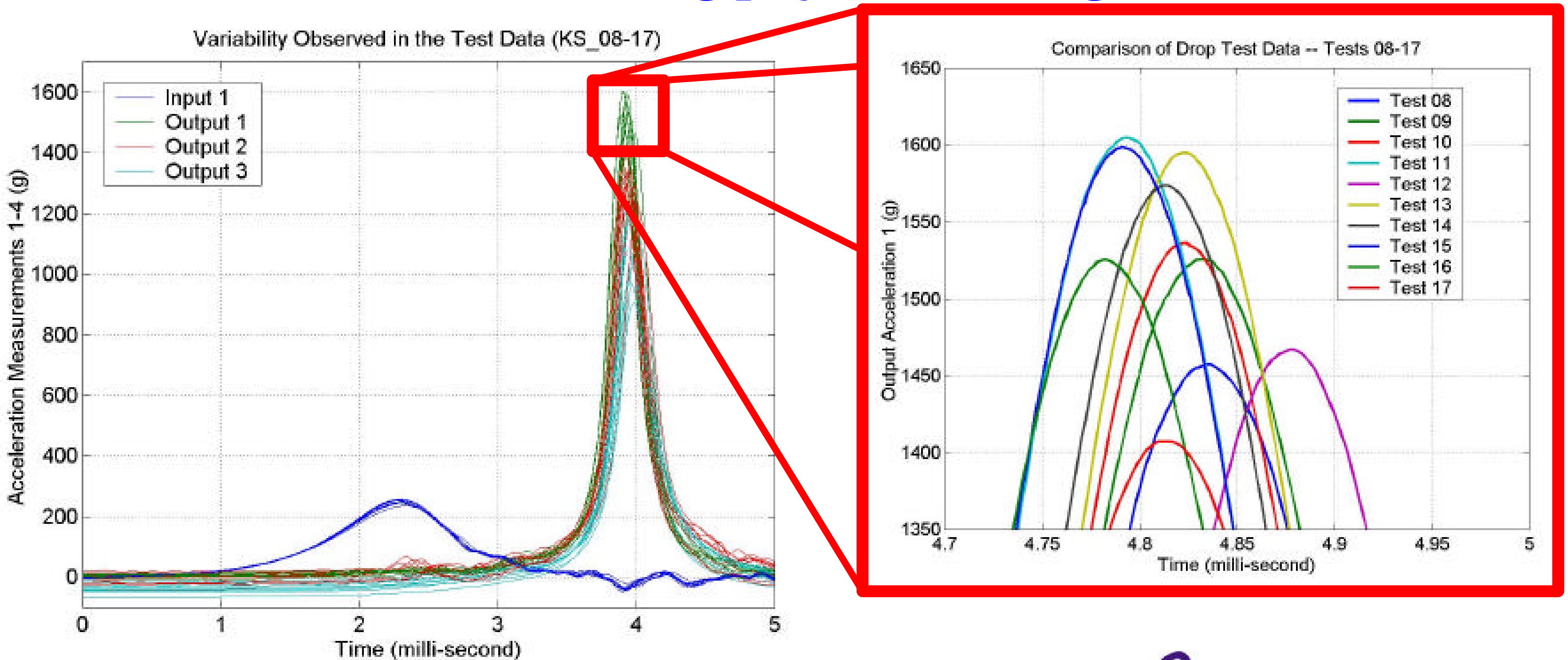




	Low Drop (13 in. / 0.3 m)	High Drop (155 in. / 4.0 m)
Thin Layer (0.25 in. / 6.3 mm)	10 Replicates	5 Replicates
Thick Layer (0.50 in. / 12.6 mm)	10 Replicates	5 Replicates

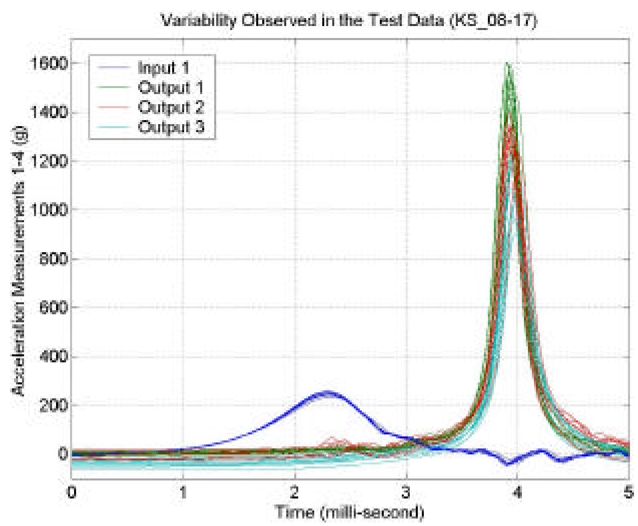
Variability

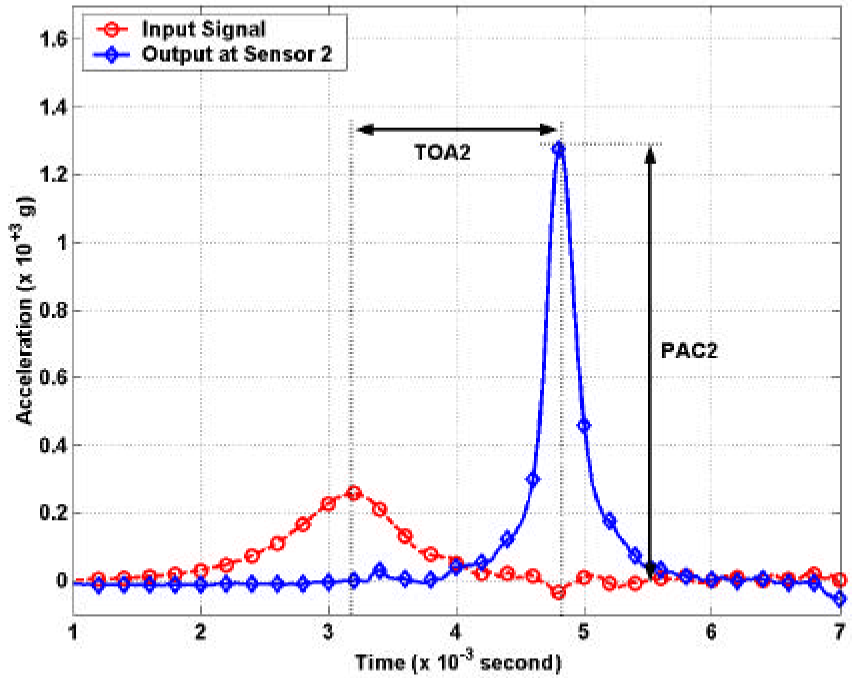
- **Significant variability is observed from the replicate measurements during physical testing.**



Response Features

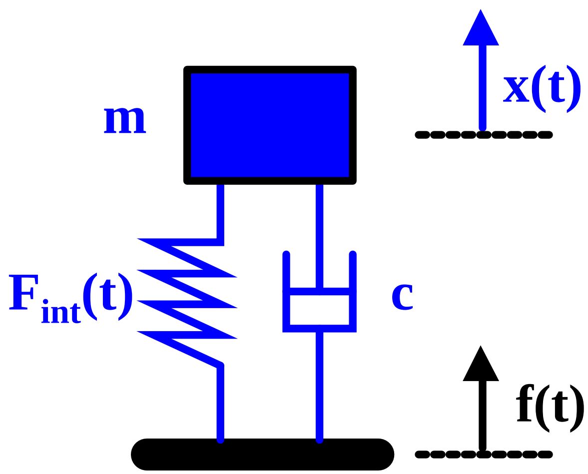
- The response features of interest are the peak acceleration (*PAC*) and the time-of-arrival (*TOA*) at output sensor 2.

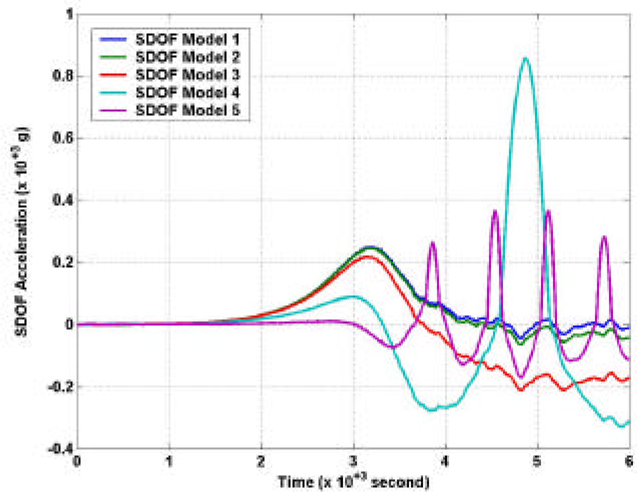




SDOF Modeling

- **A single degree-of-freedom (SDOF) oscillator model is developed to predict the features of interest without describing the dynamics with high-fidelity.**





$$m\ddot{x}(t) ? c\dot{x}(t) ? F_{int}(t) ? m\ddot{x}_{applied}(t)$$

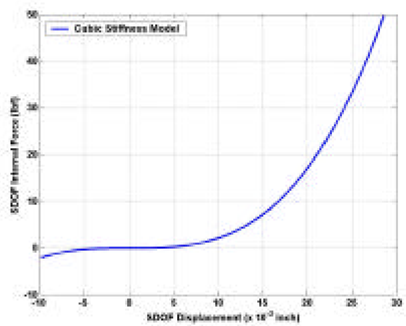
- – **The only source of non-linearity of the SDOF model is defined by the internal force *$F_{int}(t)$*.**

Parameters of the SDOF Model

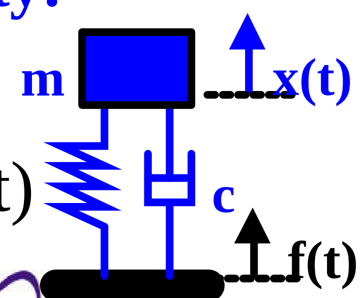
- The input variables that control the SDOF model are:

Variable	Description	Minimum	Maximum	Nominal
1	Foam Thickness (inch)	0.25	0.50	0.25
2	Drop Height (inch)	13.00	155.00	13.00
3	Linear stiffness (lbf/inch)	0.00	?	?
4	Damping (lbf x sec/inch)	0.00	?	?
5	Cubic stiffness (lbf/inch^3)	0.00	?	?

- Example of a cubic stiffness non-linearity:



$$F_{int}(t) ? k_{nl} x^3(t)$$

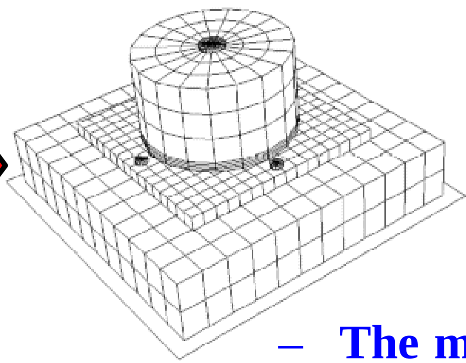


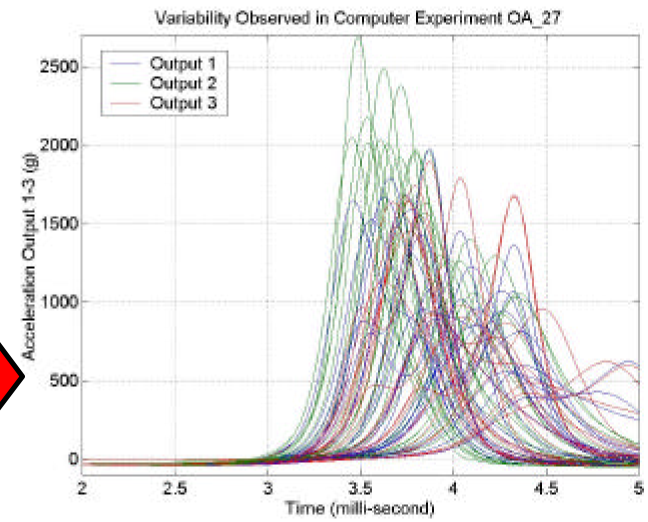
Finite Element Modeling

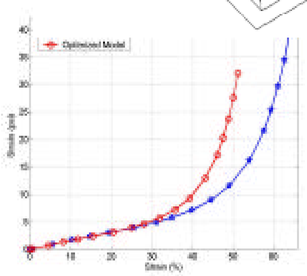
- **A finite element (FE) model is developed to simulate the impact dynamics with high-fidelity.**

Min	Input	Max
¼	p_1	½
13	p_2	155
0	p_3	2
0	p_4	2
0	p_5	500
0.8	p_6	1.2
0.8	p_7	1
0.9	p_8	1.1
0	p_9	1
0	p_{10}	1

$$y = M(p_1;\ldots;p_{10})$$



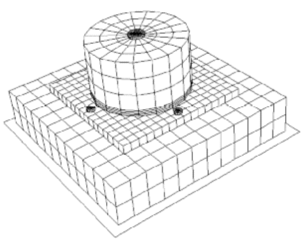




– **The main sources of non-linearity are the hyper-foam constitutive behavior and contact between the crushable and metallic components.**

Parameters of the FE Model

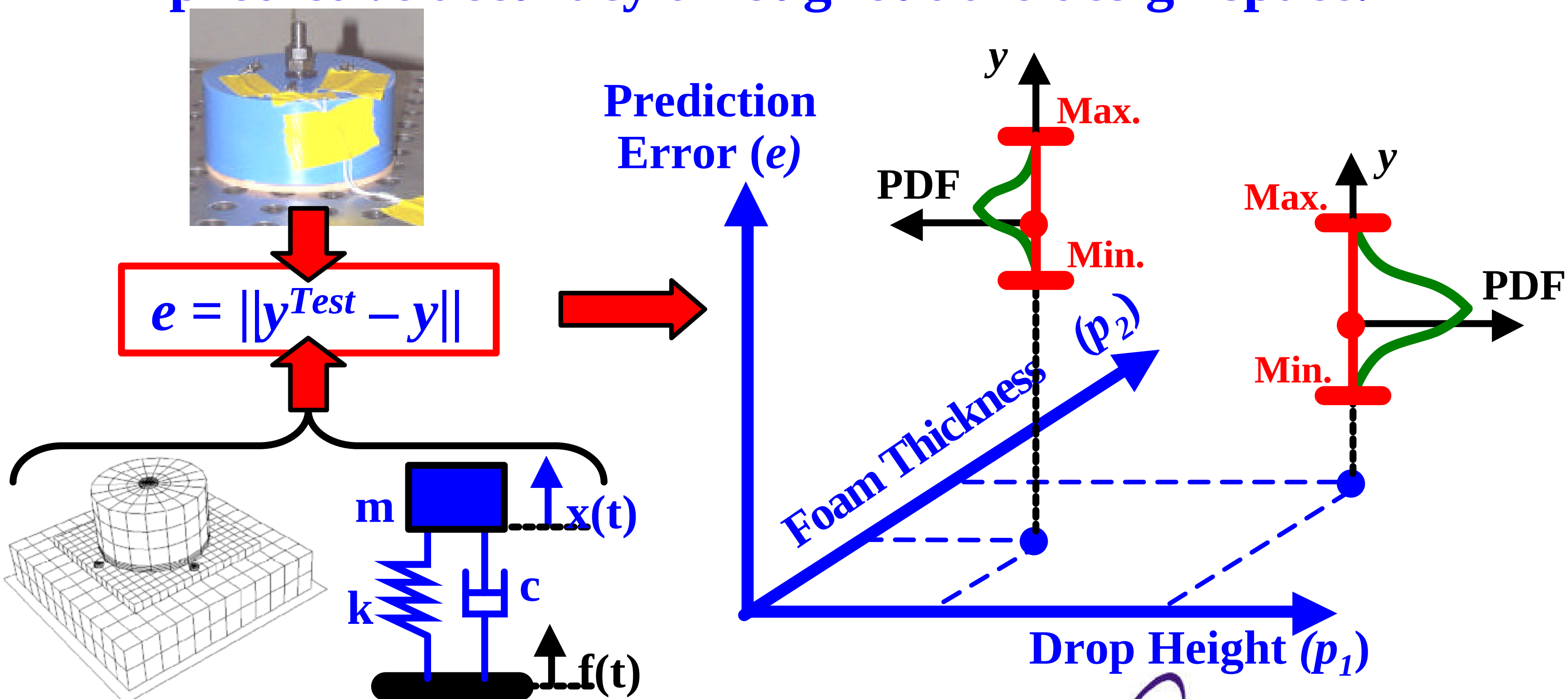
- **The input variables that control the FE model are:**



Variable	Description	Minimum	Maximum	Nominal
1	Foam Thickness (inch)	0.25	0.50	0.25
2	Drop Height (inch)	13.00	155.00	13.00
3	Angle 1 (degree)	0.00	2.00	0.50
4	Angle 2 (degree)	0.00	2.00	0.50
5	Bolt Preload (psi)	0.00	500.00	250.00
6	Stress Scaling (unitless)	0.80	1.20	1.00
7	Strain Scaling (unitless)	0.80	1.00	1.00
8	Input Scaling (unitless)	0.90	1.10	1.00
9	Friction (unitless)	0.00	1.00	0.10
10	Bulk Viscosity (unitless)	0.00	1.00	0.60

Predictive Accuracy Assessment

- The objective of this study is to assess the model's predictive accuracy throughout the design space.



Requirements

- To generate a numerical simulation that we can trust to predict the dynamics of interest, we need to …
  - Quantify the experimental uncertainty.
  - Quantify the modeling uncertainty.
  - Understand where the uncertainty comes from and what its effects are.
  - Make decisions: Is the model good enough?

What happens when uncertainty cannot be represented probabilistically?

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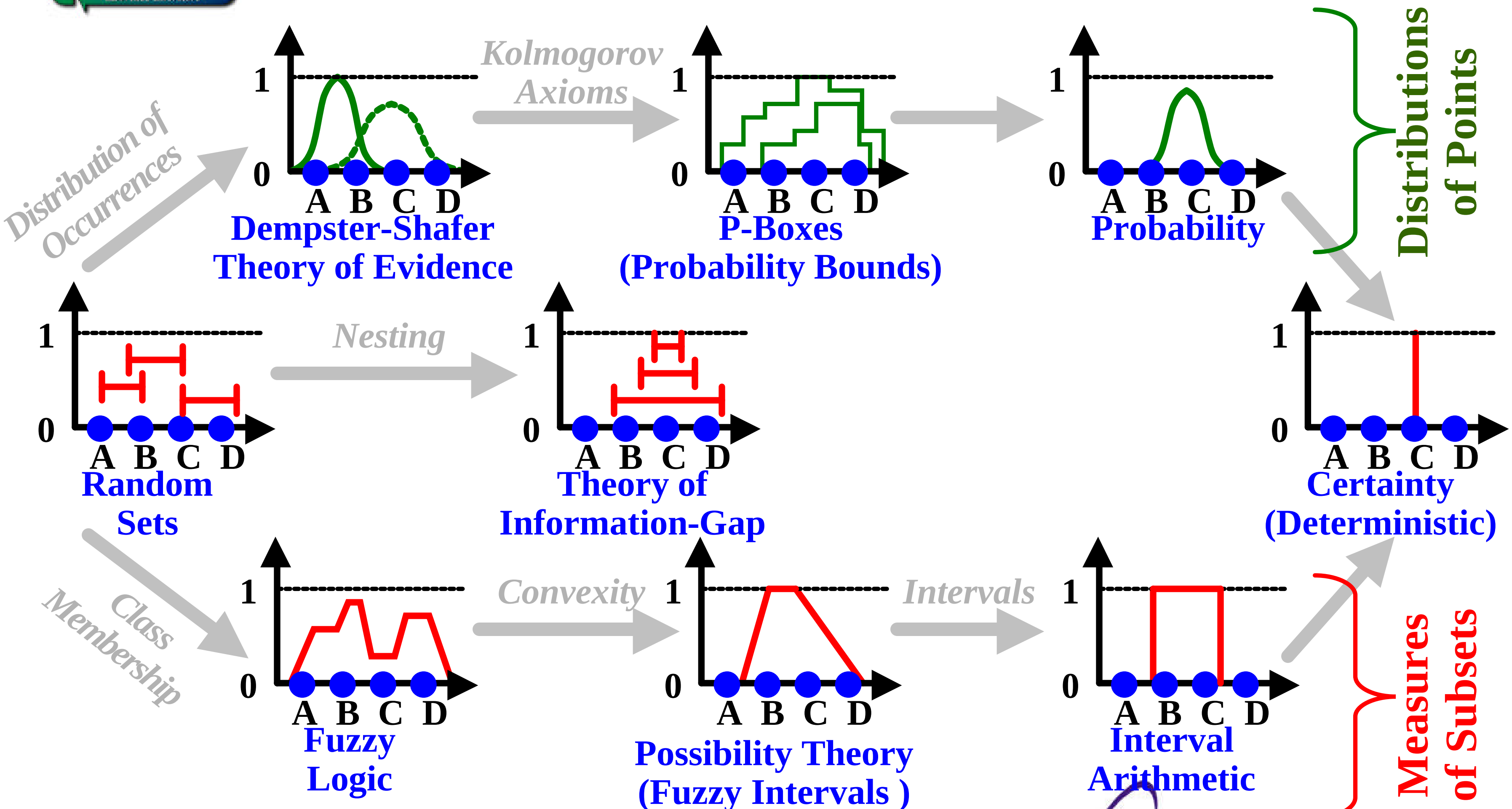
Motivations

- How to describe uncertainty when *evidence* is not available that probability theory is adequate?
- How to describe expert judgment, scarce data sets, rare events or epistemic uncertainty (i.e., lack-of-knowledge)?
- How to interface other theories with probabilities?
- How to propagation alternate models of uncertainty through our "black-box" computational codes?

General Information Theory



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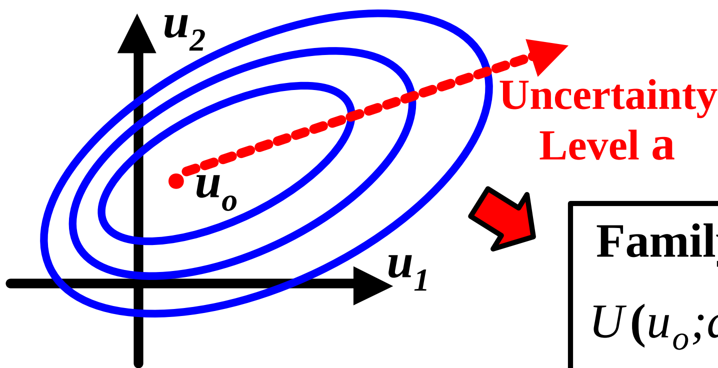
Theory of Information-gap

- **Information-gap seeks to represent the *gap* between what is currently known and what is needed to make a decision.**



Family of nested sets:

$$U(u_o;a) = \left\{ u \;\middle|\; (u-u_o)^T W^{-1} (u-u_o) \le a \right\}, \quad a \ge 0$$

- **The basic principle of information-gap is to model the *clustering* of uncertain events in families of nested sets instead of assuming a probability structure.**

Components of Info-gap

- **The three components of info-gap analysis are the *decision model*, the *info-gap model* of uncertainty and the *performance criterion*.**

1 **Decision model**
$y = M(q;u)$

Decision variables q
Uncertainty variables u

2 **Info-gap model**
u ? $U(u_o;a)$, a ? 0

Nominal settings u_o
Uncertainty parameter a

3 **Performance criterion**
$R(q;u)$? R_C (*)

Performance criterion
Critical level or target performance

(*) Or $R(q;u)$?R_C or any other criterion.

Remarks

- An information-gap model includes *all possible* representations of uncertainty within the nested sets.
- Information-gap focuses on *decision making* instead of attempting to represent the uncertainty.
- Sampling cannot be taken advantage of to propagate uncertainty because no probability structure is assumed.
  - Optimization is used to propagate uncertainty, which may be less efficient & rigorous (convergence?) than sampling.

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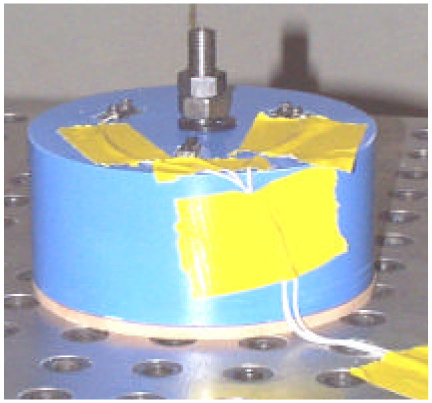
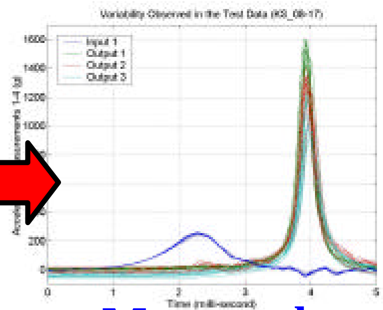
Engineering Application

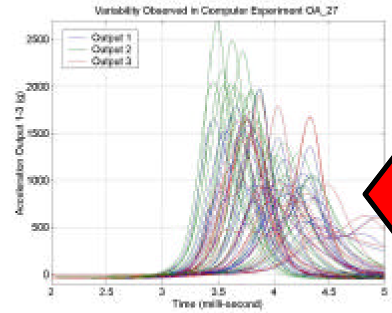
- **The objective is to identify the numerical models that best reproduce the physical measurements.**



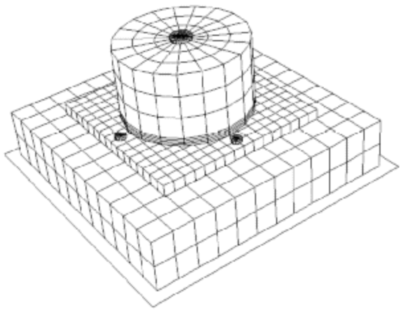
Physical Experiments



Measured Responses



Simulated Responses



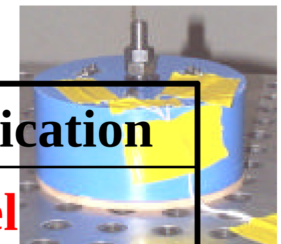
Finite Element Modeling

Agreement?

- **Experimental and modeling sources of uncertainty are accounted for in a non-probabilistic framework.**

Analogy

- **The *performance* of a numerical model is deemed acceptable if the model provides less than R_C=20% test-analysis correlation error.**

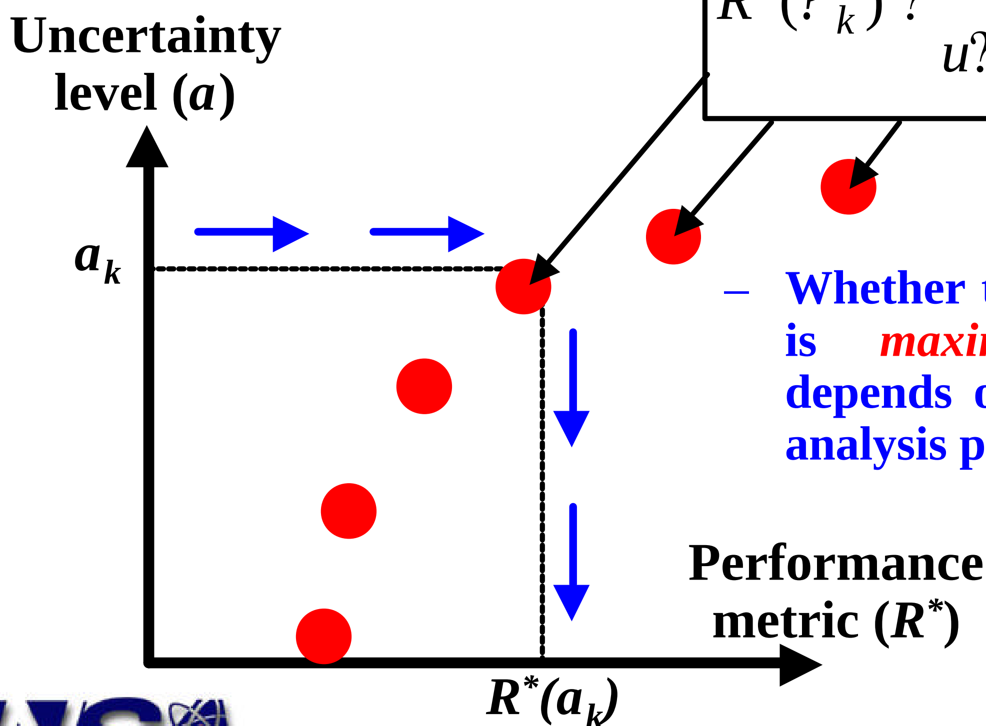


Information-gap Analysis	Symbol	Foam Impact Application
Decision model	$y=M(q;u)$	**Finite element model**
Output	y	**Features *PAC, TOA***
Decision variables	q	**Input parameters, p_1, p_2, …**
Uncertainty variables	u	**Input parameters, p_1, p_2, …**
Horizon-of-uncertainty	a	**Range of an interval**
Performance criterion	$R(q;u)$	**Prediction error, $e=\|\|y^{Test}-y\|\|$**
Acceptance criterion	$R(q;u)<R_C$	**"No more than 20% error"**

Information-gap Analysis — Step 1

- **In an info-gap analysis, uncertainty is propagated by optimizing the performance of the system at any given uncertainty level.**

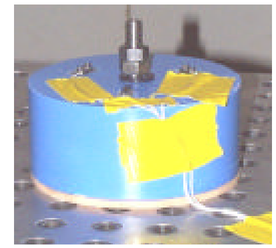
$$R^*(?_k)\ ?\ \max_{u?U(u_0;?_k)} R(q;u)$$



  - **Whether the performance *R(q;u)* is *maximized* or *minimized* depends on the type of info-gap analysis performed.**

Information-gap Models

- **Examples of info-gap models used in the analysis:**



  - **Uncorrelated intervals:**

    $$U(u_o;a) ? \{u \mid -? ? \{u ? u_o\}? ?a\},\quad a ? 0$$

  - **Correlated intervals:**

    $$U(u_o;a) ? \{u \mid (u ? u_o)^T W^{?1}(u ? u_o)? a\},\quad a ? 0$$

  - **Hybrid probabilistic/info-gap models:**

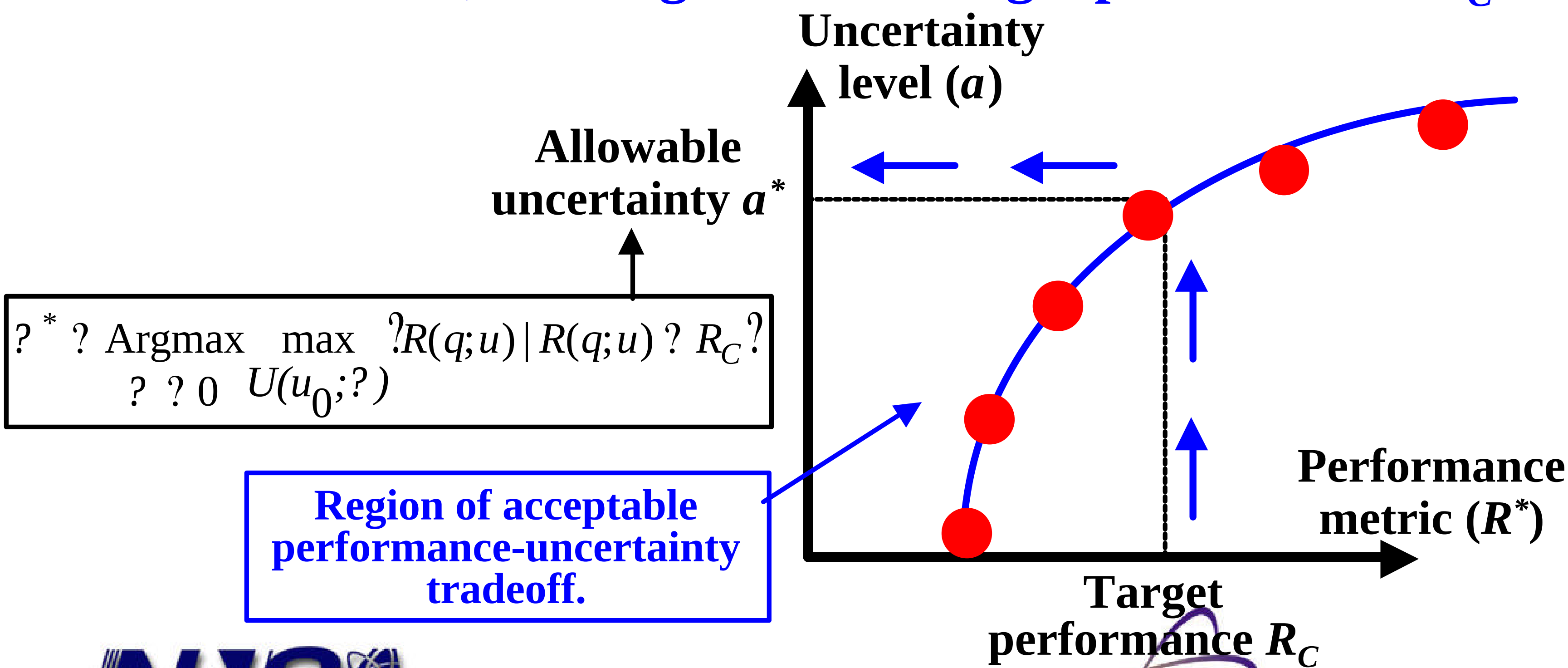
    $$u ? N(?_u;?_{uu}),$$

    $$U(?_o;?_o;a;b) ? \{(?_u;?_{uu}) \mid |?_u ? ?_o| ? a \text{ and } \|?_{uu} ? ?_o\| ? b\},$$

    $$a ? 0,\quad b ? 0$$

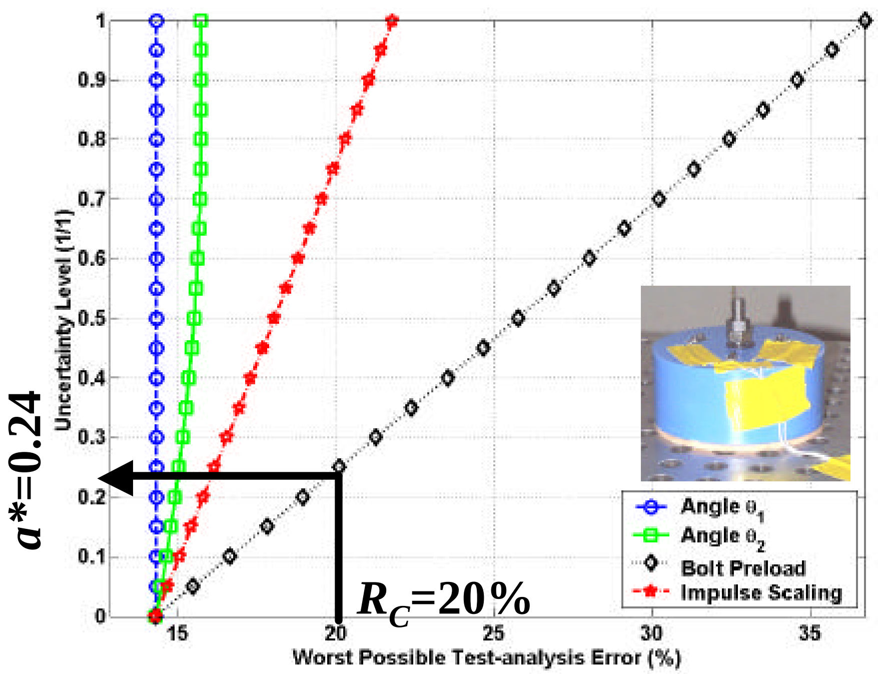
Information-gap Analysis — Step 2

- **The allowable uncertainty *a** is obtained by reading the curve of performance (*R**) versus uncertainty (*a*) backwards, starting from the target performance R_C.**



Immunity to Uncertainty

- **Question of *immunity*: What is the *largest* level of uncertainty α^* that the system can sustain without sacrificing the performance requirement, $R \leq R_C$?**

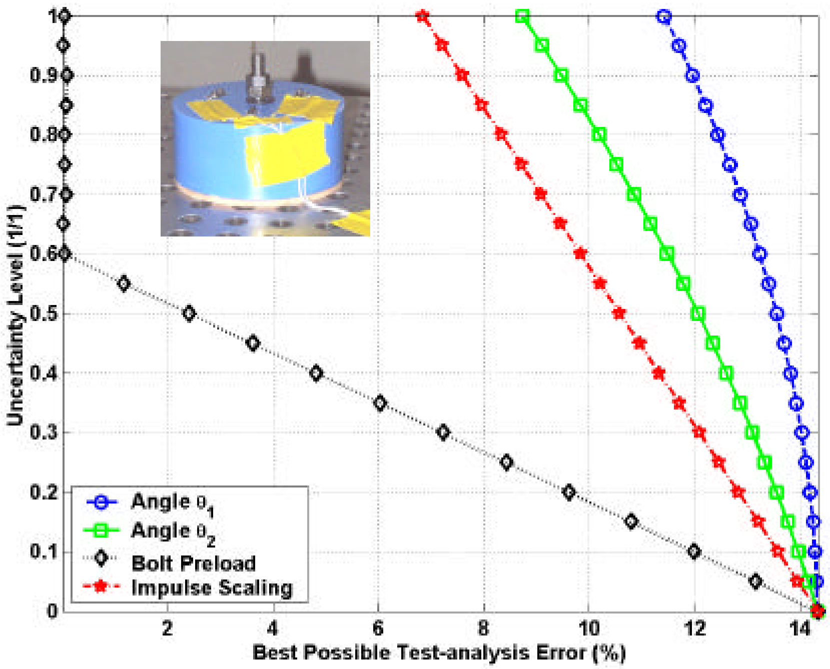


– **The immunity α^* quantifies the *adverse* effect of uncertainty on the system's performance $R(q;u)$.**

$$\alpha^* = \underset{\alpha \geq 0}{\text{Argmax}} \; \max_{u \in U(u_0;\alpha)} \{R(q;u) \mid R(q;u) \leq R_C\}$$

Opportunity Arising From Uncertainty

- **Question of *opportunity*: What is the *smallest* level of uncertainty b^* that could potentially improve the performance while satisfying the requirement, $R?R_C$?**



  - **The opportunity b^* quantifies the *beneficial* effect of uncertainty on the performance *R(q;u)*.**

$$b^* ? \text{Argmin} \min_{? \; ? \; 0 \;\; U(u_0;?)} ?R(q;u) \,|\, R(q;u) ? R_W ?$$

Decision-making

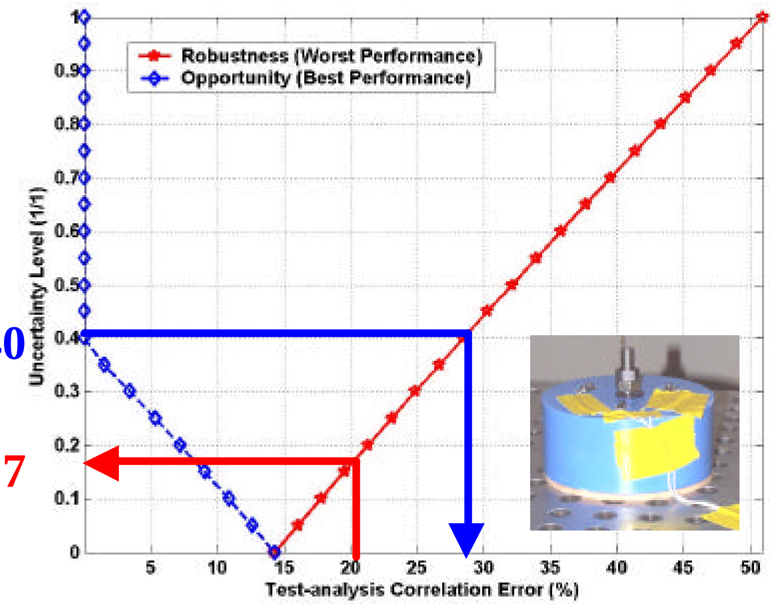
- **When the sources of uncertainty are combined, which performance can be expected and how much uncertainty can be tolerated?**

  – **To guarantee 20% prediction error at most, no more than 17% uncertainty can be tolerated.**

  – **If 40% uncertainty could be tolerated, it might be possible to find a model that yields perfect predictions. In this case, however, no less than 28% error can be guaranteed.**

b*=0.40

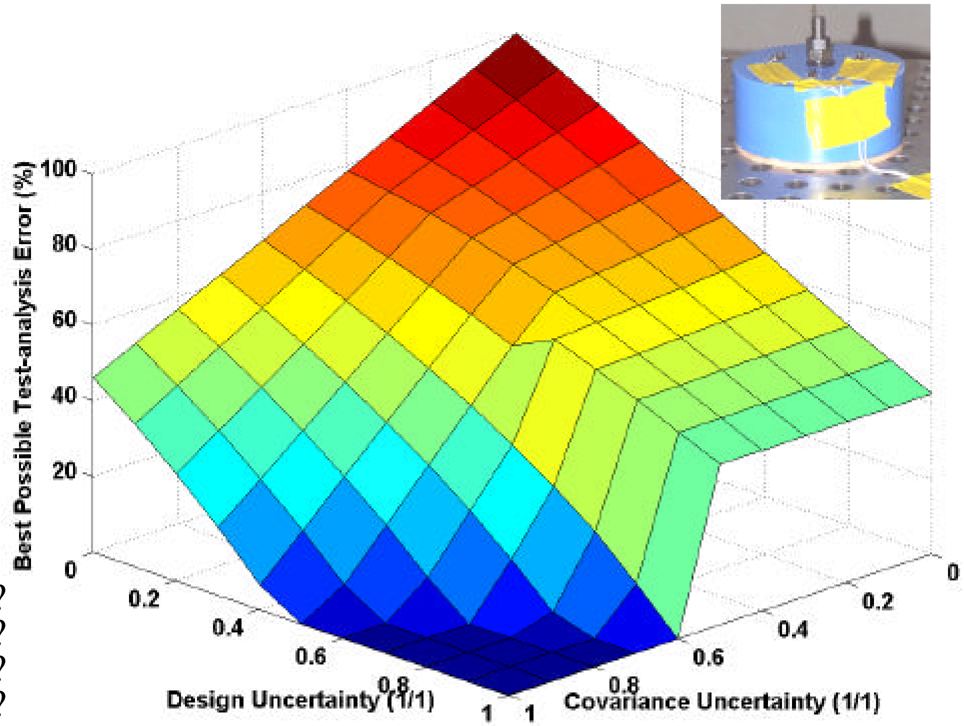
a*=0.17



R_C=20% R_C=28%

Hybrid Models of Uncertainty

- **Can probability and info-gap models of uncertainty be embedded?**



– **The uncertainty is represented by a probability model whose parameters are not precisely known. This lack-of-knowledge is represented by an info-gap model of uncertainty.**

$$u\ ?\ N(?_u;?_{uu})$$

$$?_u\ ?\ \begin{bmatrix} t_1 \\ t_2 \\ P_B \\ s_I \end{bmatrix} \qquad W_{uu}\ ?\ \begin{bmatrix} v_1 & v_5 & 0 & 0 \\ v_5 & v_2 & 0 & 0 \\ 0 & 0 & v_3 & 0 \\ 0 & 0 & 0 & v_4 \end{bmatrix}$$

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