# MEASUREMENTS OF ELASTIC ELECTRON-PROTON SCATTERING AT HIGH MOMENTUM TRANSFER BY A COINCIDENCE TECHNIQUE* 

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#### Abstract

We present measurements of elastic electron-proton scattering cross sections in which both the scattered electron and recoil proton have been detected. Cross sections have been measured for a range of four-momentum transfers from 10 to $150 \mathrm{~F}^{-2}$ with an estimated accuracy of between $3 \frac{1}{3}$ and $10 \%$.


Electrons from the external beam of the Cambridge Electron Accelerator were allowed to strike a liquid-hydrogen target and the unscattered beam was stopped in a Faraday cup. A thin secondary emission monitor was placed just before the Faraday cup and served as an additional monitor of the beam intensity.

Scattered electrons were detected in a magnetic spectrometer and recoil protons were detected in a triple scintillation-counter telescope protected from low-energy charged particles by a sweeping magnet [see Fig. (1)]. The spectrometer had a momentum acceptance of $15 \%$ and could focus particles with momenta up to $5 \mathrm{BeV} / c$. The maximum solid angle subtended was 1.8 msr and the momentum resolution was approximately $1.7 \%$ (full width at half-maximum).


FIG. 1. Schematic view of the electron spectrometer and proton telescope. The external beam-transport system and Faraday cup are not shown.

Electronic information from the outputs of discriminators, coincidence units, pulse height analyzers, and other equipment was transmitted to an on-line computer where it was recorded on magnetic tape for subsequent analysis. The criteria for triggering the computer were deliberately chosen to be rather nonselective. In particular, they did not include the requirement of a proton coincidence. The criteria for accepting an event as an elastic scattering were (1) that the shower and Cherenkov pulses be above certain bias levels, (2) that the electron trajectory cross the focusing plane of the spectrometer within a momentum band usually chosen to be $7 \%$ wide and centered on the elastic peak, and (3) that a coincident proton be detected.

The solid angle was defined by the electron spectrometer. For momentum transfers up to $45 \mathrm{~F}^{-2}$, an $0.82-\mathrm{msr}$ tungsten-edged aperture was placed in front of the quadrupole; for the higher momentum transfers a $1.8-\mathrm{msr}$ aperture behind the magnet was used. The two apertures were intercalibrated and the measurements agreed with the calculated ratio of their solid angles. We assign a $1 \%$ uncertainty to the solid angle subtended by the front aperture and $2 \%$ to that subtended by the back aperture.

The average energy of the incident electrons was monitored with a relative precision of $\pm 0.2 \%$ and with a possible additional $\pm 0.2 \%$ systematic error. These errors are magnified by up to a factor of 7 in the cross section. Fluctuations in the incident beam direction give rise to uncertainties in the scattered angle. The beam position was monitored by a tuned rf cavity mounted on a moving table, and the resul-
tant uncertainty in scattered angle is estimated to be less than $\pm 1 \mathrm{mrad}$; this leads to approximately $\pm \frac{1}{2} \%$ error in the cross section.
Corrections totaling about $14 \%$ were applied for the effect of radiator between the point of scattering and the magnet, for the effect of the fringe fields of the sweeping magnet on the trajectories of scattered and incident electrons, for the efficiencies of the proton, shower, and Cherenkov counters, and for the computer dead time. Preliminary estimates have been made of the inelastic contamination and of the number of events thrown out of the acceptance by the tails of the resolution function. These effects contribute no more than a $3 \%$ correction in the worst case. The radiative corrections of Meister and Yennie ${ }^{1}$ for electron detection only have been applied. An estimate of the additional radiative correction due to the detection of protons in coincidence has been made. ${ }^{2}$ It was found to be less than $0.2 \%$.

The target was used as its own vapor-pressure thermometer and the temperature used to predict the density. A value of $0.0708 \mathrm{~g} / \mathrm{cm}^{3}$ at atmospheric pressure was used. ${ }^{3}$ No correction has been made for bubbling in the target. Intensity-dependent studies and calculation suggest that this effect is small. Emptycup runs were taken to subtract out the contributions from the end walls of the target which were typically between 1 and $4 \%$. The Faraday cup was taken to be ( $100 \pm 0.35$ ) \% efficient on the basis of the variation of response with bias voltage. A recent measurement on this cup ${ }^{4}$ confirms that this estimate can not be more than $1 \%$ in error.
We feel that the coincident detection of pro-
tons provides an important overdetermination of the elastic kinematics as well as assisting in the rejection of inelastic backgrounds. At $q^{2}=115,130$, and $150 \mathrm{~F}^{-2}$ the removal of the requirement of the proton coincidence would raise our estimates of the cross section by 4,6 , and $2 \%$, respectively. At the low momentum transfers there is no significant change. Although this change in cross section is not understood, it is felt to be encouragingly small.
The results are summarized in Table I. The cross sections are quoted for nominal momentum transfers and angles or energies. The factor applied to the measurements to bring them to the nominal values introduces less than $0.2 \%$ error. The errors quoted represent the combination of both experimental uncertainties and uncertainties in the present analysis. The latter are expected to be reduced in the near future. For convenience, we also give the value of $\left(G_{M} / \mu\right)^{2}$ based on the assumption $G_{E}=G_{M} / \mu$.

In order to compare our results with recently reported data ${ }^{5,6}$ we show, in Fig. 2, the ratio of cross-section values (obtained both by us and by other laboratories) to the predictions of the one-parameter fit:

$$
G_{E}\left(q^{2}\right)=\frac{G_{M}\left(q^{2}\right)}{2.792}=\left[1+\frac{q^{2}}{0.71(\mathrm{BeV} / c)^{2}}\right]^{-2} .
$$

It is important to emphasize that any other reasonably good fit to the data would also serve for the purpose of making these comparisons. The low $-q^{2}$ data fit the relationship $G_{E}=G_{M} / \mu$ very well and the high $-q^{2}$ data have very little contribution from $G_{E}$, so that data taken at different angles are well compared by such a fit.

Table I. Summary of data. The total error is obtained by adding the constituent errors in quadrature.

| $\begin{gathered} q^{2} \\ \left(\mathrm{~F}^{-2}\right) \end{gathered}$ | Electron angle (deg) | Incident electron energy (BeV) | $\begin{gathered} \text { Cross } \\ \text { section } \\ \left(10^{-32} \mathrm{~cm}^{2} / \mathrm{sr}\right) \end{gathered}$ | Errors (in \%) due to |  |  | Total error (\%) | $\begin{gathered} \left(\boldsymbol{G}_{\boldsymbol{M}} / \mu\right)^{2} \\ \text { assuming } \\ \boldsymbol{G}_{\boldsymbol{E}}=\boldsymbol{G}_{M} / \mu \end{gathered}$ | Ratio of cross section to oneparameter fit |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | Counting statistics | Other random errors | Systematic errors |  |  |  |
| 10 | 20 |  | 39.1 | 0.8 | 2 | 2.5 | 3.3 | 0.166 | 0.956 |
| 30 | 20 |  | 2.27 | 1.5 | 2 | 2.5 | 3.5 | 0.0208 | 1.017 |
| 45 | 20 |  | 0.600 | 1.7 | 3 | 2.5 | 4.3 | 0.00736 | 1.064 |
| 70 |  | 5.5 | 0.113 | 3.4 | 3 | 2.5 | 5.2 | 0.00199 | 1.093 |
| 75 |  | 6 | 0.110 | 5.4 | 3.5 | 2.5 | 7 | 0.00179 | 1.222 |
| 100 |  | 5.5 | 0.0134 | 4.1 | 3 | 2.5 | 5.6 | 0.000642 | 1.135 |
| 115 |  | 6 | 0.00700 | 3.8 | 3 | 2.5 | 5.5 | 0.000386 | 1.099 |
| 130 |  | 6 | 0.00289 | 5.6 | 3 | 2.5 | 6.8 | 0.000243 | 1.061 |
| 150 |  | 6 | 0.000962 | 7 | 3 | 2.5 | 8 | 0.000141 | 1.024 |



FIG. 2. Ratio of experimental data to the one-parameter fit. The value of 1 at $q^{2}=0 \mathrm{~F}^{-2}$ is imposed by the static values of the form factors.

The agreement between the present data and those of other laboratories is excellent below $100 \mathrm{~F}^{-2}$; above this value it is adequate, but there is an indication of a systematic discrepancy of approximately $10 \%$. We feel that these data represent an improvement over previous forward-angle measurements from this laboratory ${ }^{7}$ and should supersede them.

A discussion of the comparison of these data with the available theoretical predictions
appears in an accompanying Letter. ${ }^{8}$
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# COMPARISON OF ELASTIC ELECTRON-PROTON SCATTERING CROSS SECTIONS WITH SOME THEORETICAL PREDICTIONS* 

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New data on elastic electron-proton scattering have recently become available. ${ }^{1-3}$ We wish to point out that the best present theoretical predictions are not adequate to describe the detailed functional dependence of the cross sections on the four-momentum transfer.

We will directly compare the theoretically predicted cross sections with the experimental data. It is customary to remove the trivial but rapidly varying dependence of cross sections on energy and angle by presenting them as ratios to the point (Mott) cross sections.

We remove the major remaining dependence by using instead an approximate fit to the form factors in conjunction with the Rosenbluth formula. We use the form-factor fit

$$
\begin{equation*}
G_{E}=\frac{G_{M}}{\mu}=\left[1+\frac{q^{2}}{0.71(\mathrm{BeV} / c)^{2}}\right]^{-2} \tag{1}
\end{equation*}
$$

and refer to cross sections evaluated with this fit as the "Hofstadter-Wilson cross sections." This fit has no theoretical basis. However, this simple formula describes data to within

