Graphical Models: Bayesian modeling in the large

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Section in the tutorial at *Maximum Entropy and Bayesian Methods,* Sante Fe, New Mexico, June 31st, 1995.

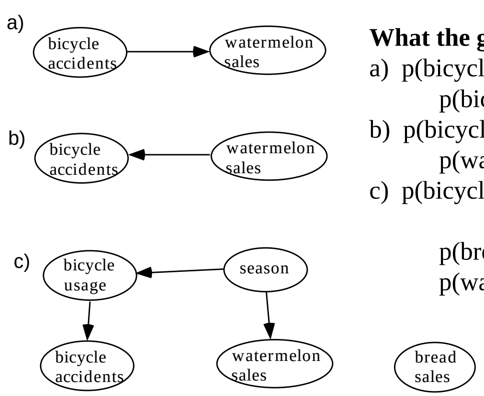
Outline

- Example graphical models and their analysis
  - a causal and non-causal model
  - causal vs. inference model
  - repeated trials
  - a random walk versus a Markov chain
- Using graphical models
  - applications of Bayesian inference
  - knowledge acquisition and inference in diagnosis
  - compiling a data analysis algorithm from specification
  - a framework for understanding Bayesian inference
- For tutorial references see

  http://www.Heuristicrat.com/wray/uaiconnections.html

Two non-causal models and a causal one

(from Shachter, 199?)



What the graphs mean:

a) p(bicycle-acc,waterm-sales) = p(bicycle-acc)p(waterm-sales|bicycle-acc)

b) p(bicycle-acc,waterm-sales) = p(waterm-sales)p(bicycle-acc|waterm-sales)

c) p(bicycle-acc,waterm-sales,bicycle-use, season,bread-sales) = p(bread-sales)p(season)p(bicycle-use|season) p(waterm-sales|season)p(bicycle-acc|bicycle-use)

Causality and graphs:

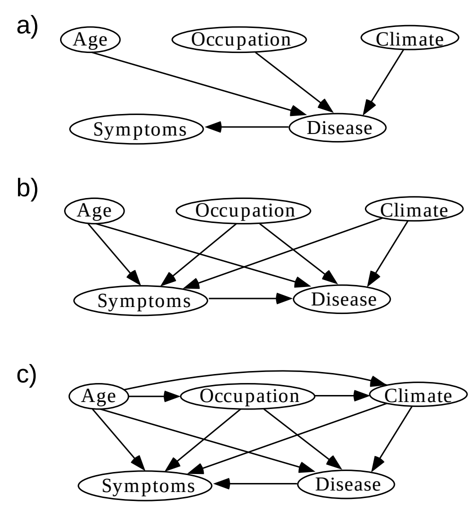
c) could be said to be "causal models", but a) and b) merely represent probabilities

We can also conclude independence statements:

e.g. from c), bicycle-acc *independent of* waterm-sales *given* season
bicycle-acc *not necessarily independent of* waterm-sales

Causal vs. Inference models

(See Shachter and Heckerman, 87)



a) a causal model

b) an inference model

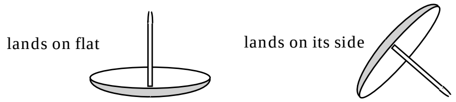
c) a model with NO assumptions

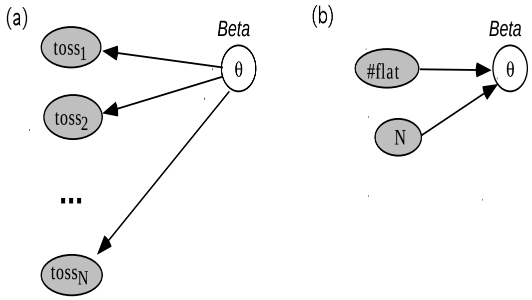
- Inference can work in any direction, i.e., marginalizing and conditioning allows arbitrary probabilities to be inferred.
- Probability tables for the inference model can be derived automatically using Bayes theorem from the causal model (see Shachter, Andersen and Szolovits, 94).

Repeated trials

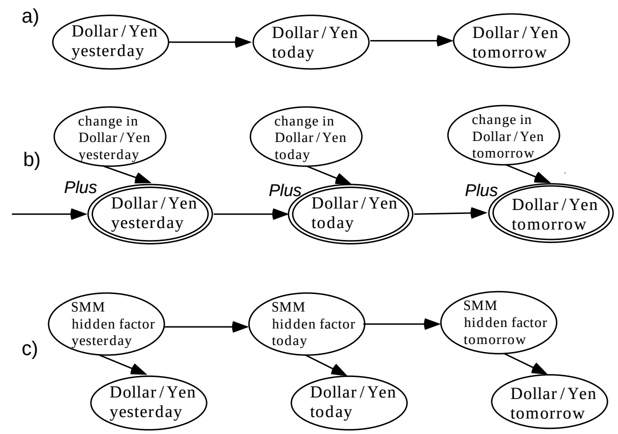
(see Howard, 70)

- we toss a thumbtack N times with unknown probability θ of it landing on its flat
- a) gives the evidence (the outcome of each toss), and its probabilistic relationship with θ
  - tosses are independent given θ
  - also called i.i.d. sampling
- b) gives the result of simplifying a) where the tosses are summarized in *sufficient statistics* N and #flat





Random walk versus a Markov chain



a) today's price is effected by yesterday's price only

b) the random walk: daily changes are independent of one another

c) Sincere Trading Co.s secret hidden Markov model that lets them predict the exchange rate accurately

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Potential applications of Bayesian inference

- million/billion dollar applications in industry waiting for the right software:
  - image understanding, e.g., face or signature recognition
  - document processing and retrieval
  - natural language understanding and intelligent user I-O
  - industrial diagnosis, process control, and instrumentation
  - scientific, medical and sociological data analysis,

    e.g., estimating the number of heroin addicts from indirect sources
  - help desk and intelligent console management
- too many of these to have a CS/NN/Stats/Phys/Eng PhD. work on each one

Knowledge acquisition and inference in diagnosis

- application areas:

  diagnosis of large complex machinery (e.g., turbines), medical diagnosis, computer peripherals (printers), help desk

- need to acquire knowledge:
  - key “hidden” variables and “causal” structure
  - 1000’s of probabilities (often magnitudes are all that matter)
  - probabilities are often the expert’s subjective opinions
- inference is non-directed:
  - each instance may have different observations
  - may involve hundreds of variables
  - system may be required to recommend which of several (expensive) observations to make next
  - hypothetical (what-if) reasoning, and explanation

BUGS: compiling a data analysis algorithm

(See `ftp://ftp.mrc-bsu.cam.ac.uk/pub/methodology/bugs`)

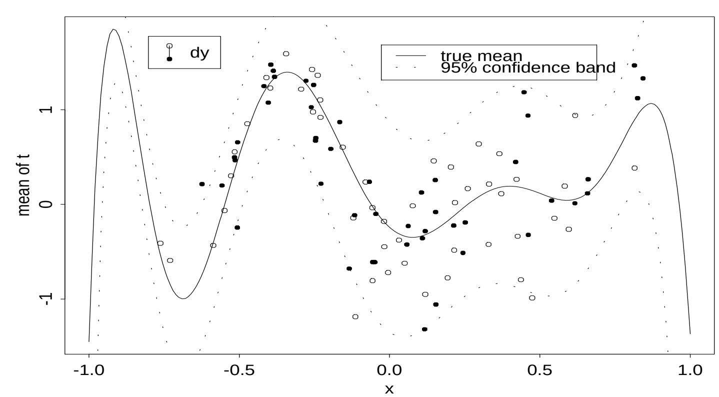
- **B**ayesian analysis **U**sing **G**ibbs **S**ampling,
  - from Gilks, Spiegelhalter, and Thomas (MRC, Cambridge, UK)
  - takes a data analysis problem represented as a Bayesian Network with Plates and compiles a Gibbs sampler for the problem.
- Amazing variety of problems addressed:
  - logistic regression
  - dose response studies
  - normal mixture models
  - non-linear regression with heterogeneous variance
  - discrete variable latent class models
  - spacial smoothing
- Interfaces to S-Plus, both input and output.
- Gibbs sampling is inherently slow, so usefull for smaller samples (e.g., 200 cases), and no multivariate Gaussians in BUGS (!!)

Inferring the distance to galaxies

(from D. Mackay, `http://131.111.48.24/mackay/bugs/astro.ps.Z`)

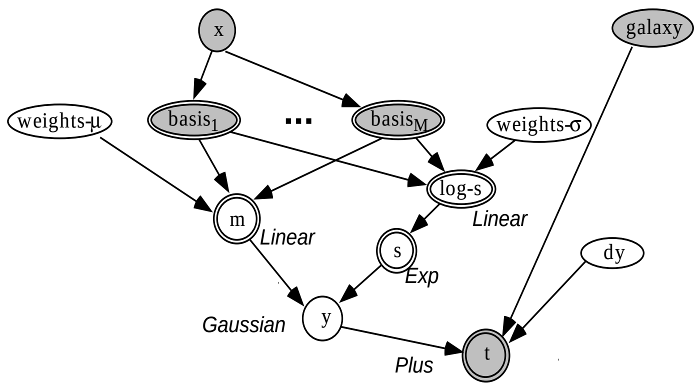
- Measurements of Cephoids (a class of supergiant variable stars) in a galaxy vary by a constant offset depending on distance.
- To infer distance between galaxies, we can jointly estimate the regression line for the measurements from two galaxies, and the constant offset between them.
- See toy data over page.
- Denote **dy** as the constant distance between the regression line for two galaxies.
- Here we:
  - Model the problem as a Bayesian network.
  - Use the BUGS compiler to automatically generate a learning algorithm from the Bayesian network model.
  - Plot the results in S-Plus.

Toy data and Truth for “Distance between Cephoids”



A “Model” for the problem

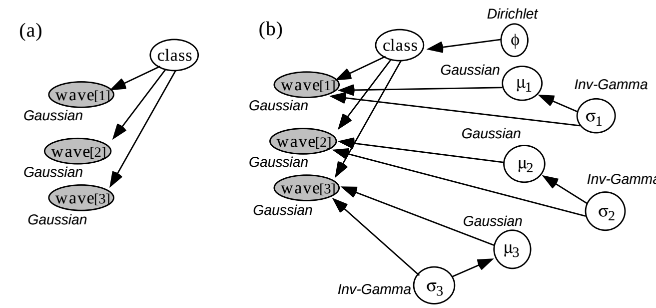
- Data set is list of records of (x,t,galaxy) where galaxy is a boolean indicating whether the offset dy should be added to the measurement t.
- Model parameters are weights-μ, weights-σ, dy and their priors.



Compiling Autoclass

(by Scott Roy, Heuristicrats Research, Inc., hsr@Heuristicrat.COM)

- Simple 3-variable Autoclass III model in (a)
- Fully parameterised model with priors in (b)
- Roy takes the network in (b), combines it with an EM optimizer and compiles out code as efficient as Autoclass in C.
- System also handles: Bayesian Nearest Neighbor, Bayesian version of LVQ, Autoclass with correlation, Jordan and Jacob’s mixture networks, etc.



Understanding Bayesian inference

- many fields use graphs to represent the *structure* of a problem, **e.g.**

  data flow diagrams for visual programming, neural networks, influence diagrams and decision trees in management science, constraint graphs in OR/CS

  and analysis methods transfer across these fields
- with graphs, elicitation, analysis and composition/decomposition of the components of the problem are made simpler, *i.e.*, **they help the user and the application developer**
- probabilistic graphical models provide a natural language for object-oriented specification and construction of software, *i.e.*, **they can support prototyping of probabilistic software**