Application of Extreme Value Statistics for Structural Health Monitoring

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Four Goals for Structural Health Monitoring

- Step 1: Damage Identification
- Step 2: Damage Localization
- Step 3: Damage Quantification
- Step 4: Damage Prognosis

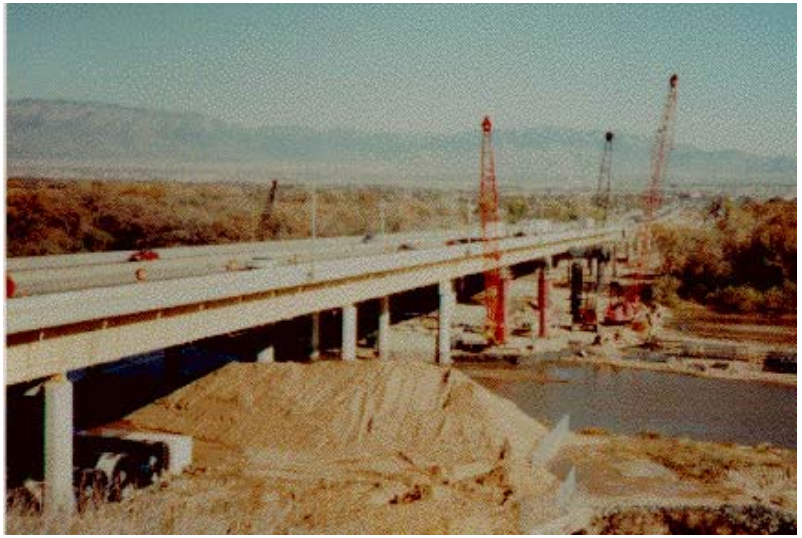
Structural Health Monitoring

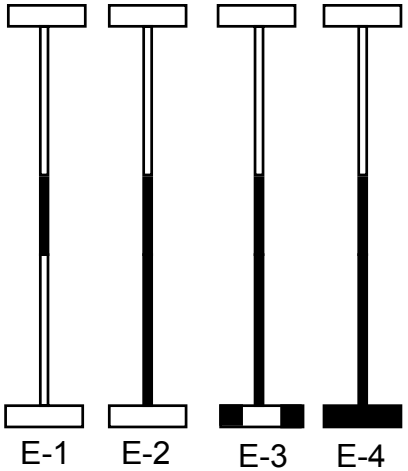
Is this bridge damaged? → **Perform pattern comparison.**

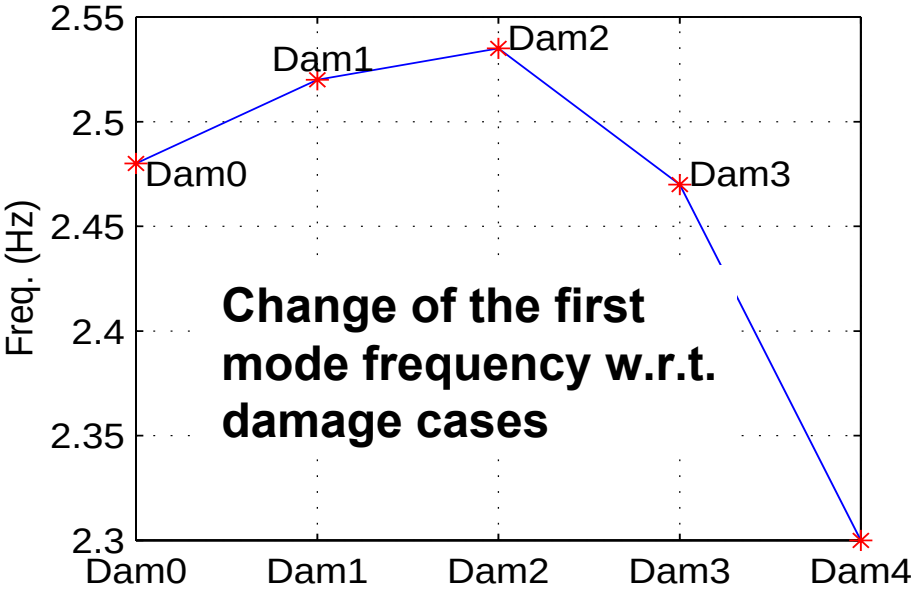
Five Steps for Structural Health Monitoring

- Step 1: Operational Evaluation
- Step 2: Data Acquisition
- Step 3: Data Normalization
- Step 4: Feature Extraction
- Step 5: Statistical Inference

Environmental Variation



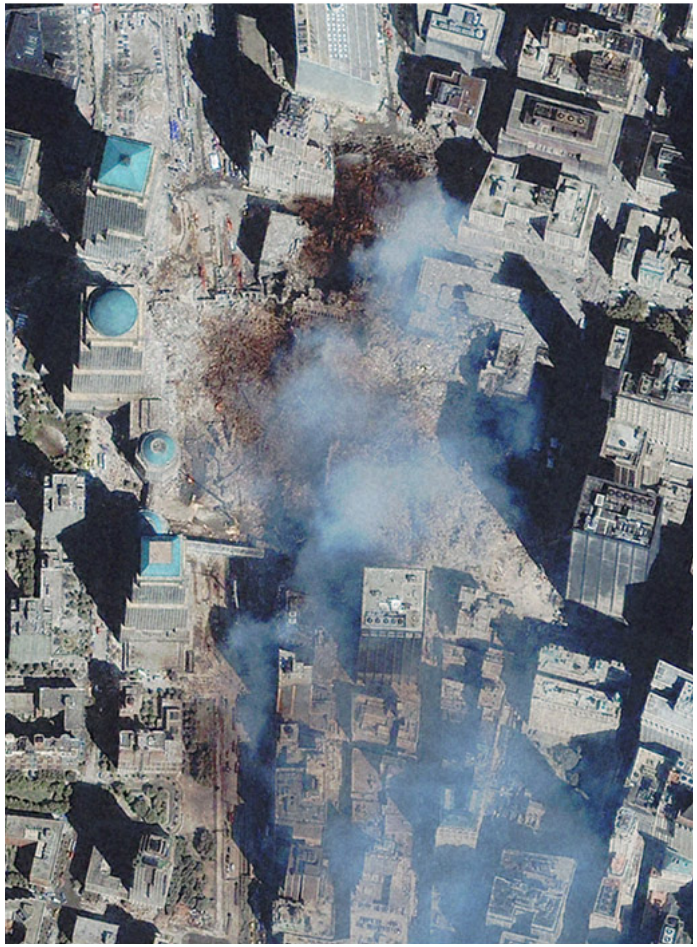




Example of Debris in Expansion Joint of the Alamosa Canyon Bridge

This debris can effect the boundary conditions of the structure and its response to environmental changes

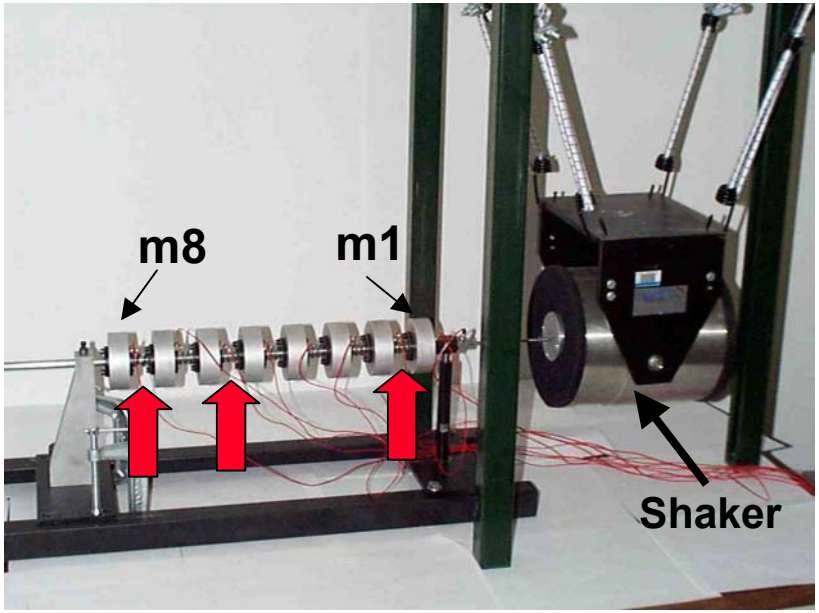
Statistical Pattern Recognition Paradigm for SHM

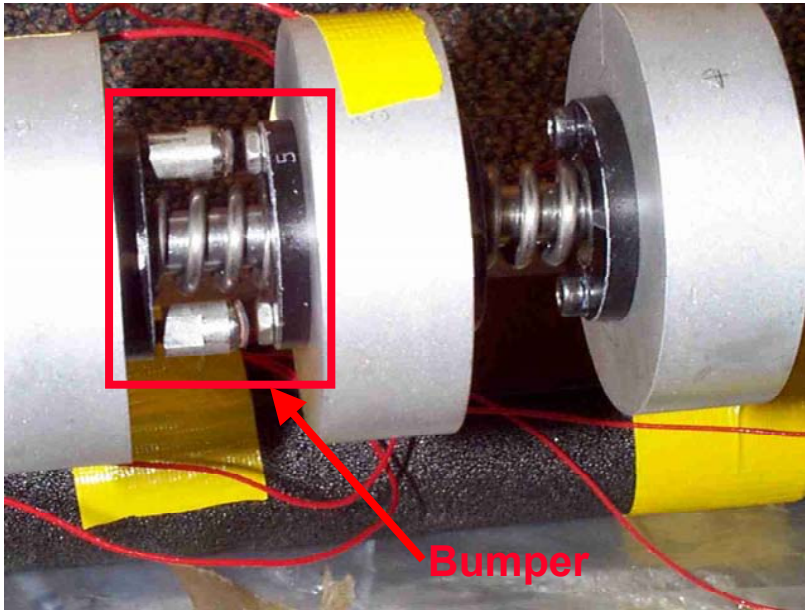


After Sep. 11, 2001

Before Sep. 11, 2001

Data Normalization Example [Sohn et al. 2002]



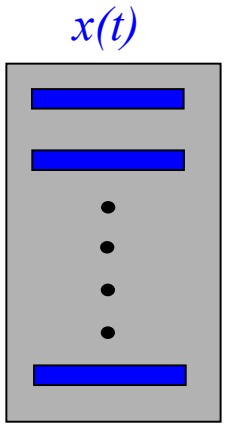


List of time series employed in this study

Case	Description	Input level	Data # per input	Total data #
0	No bumper	3, 4, 5, 6, 7 Volts	15 sets	75 sets
1	Bumper between m1-m2	3, 4, 5, 6, 7 Volts	5 sets	25 sets
2	Bumper between m5-m6	3, 4, 5, 6, 7 Volts	5 sets	25 sets
3	Bumper between m7-m8	4, 5, 6, 7 Volts	5 sets	20 sets

Outline of Damage Diagnosis using AR-ARX, Auto-Associative Network and Hypothesis Test

1. COLLECT BASELINE DATA

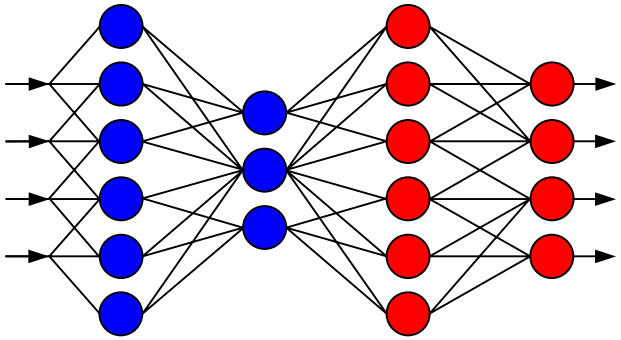


Time series from various operation & environmental conditions

2. FEATURE EXTRACTION

$$\varepsilon_x(t) = x(t) - \sum_{i=1}^{a} \alpha_i \, x(t-i) - \sum_{j=1}^{b} \beta_i \, e_x(t-j)$$

3. DATA NORMALIZATION



4. FEATURE EXTRACTION

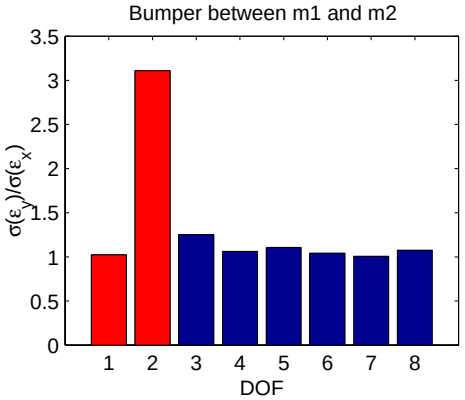
$$\varepsilon_y(t) = x(t) - \sum_{i=1}^{a} \hat{\alpha}_i \, x(t-i) - \sum_{j=1}^{b} \hat{\beta}_i \, e_x(t-j)$$

5. STATISTICAL INFERENCE

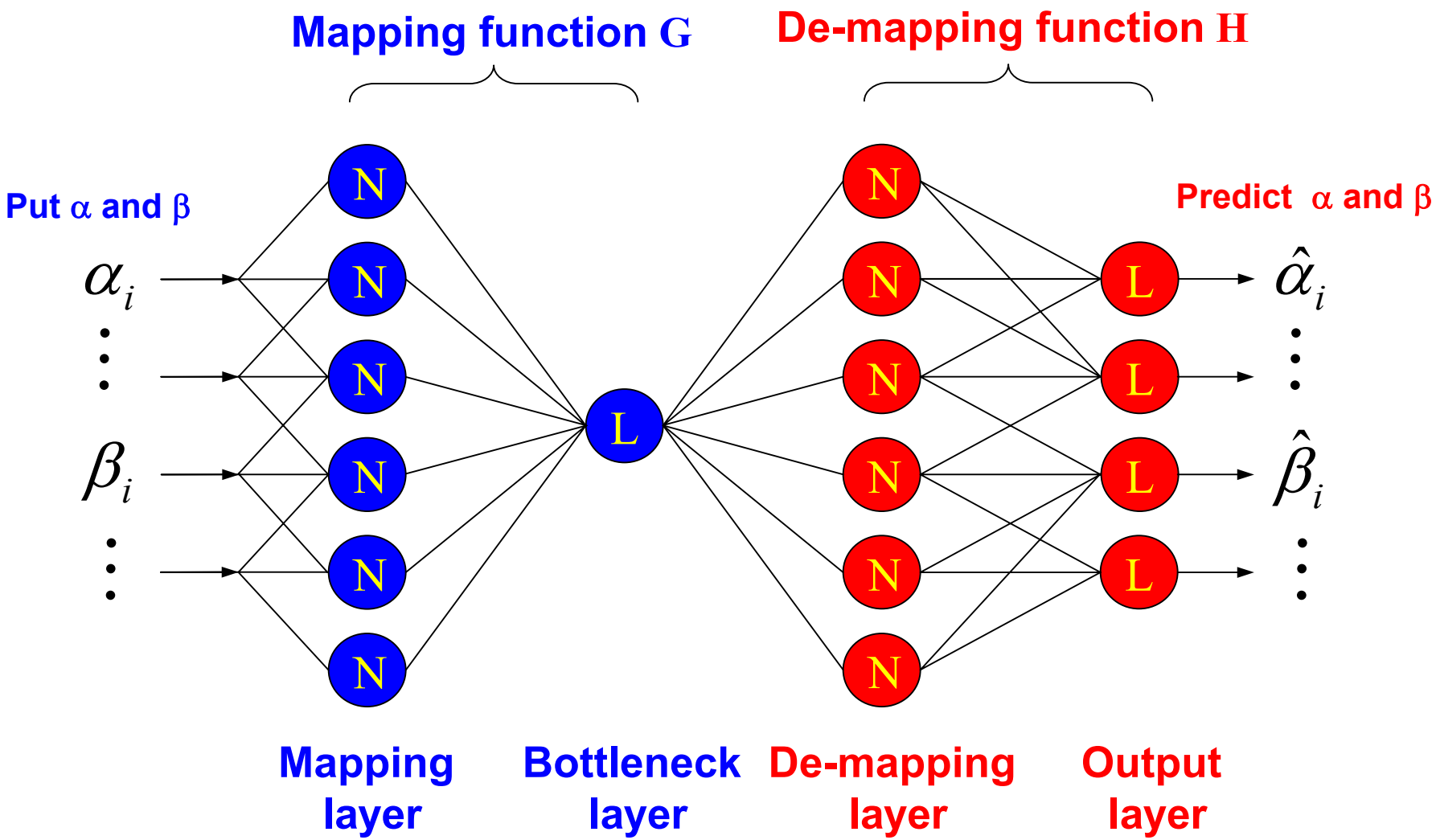
$$H_o : \sigma(\varepsilon_y) \leq \sigma(\varepsilon_x)$$

$$H_1 : \sigma(\varepsilon_y) \geq \sigma(\varepsilon_x)$$

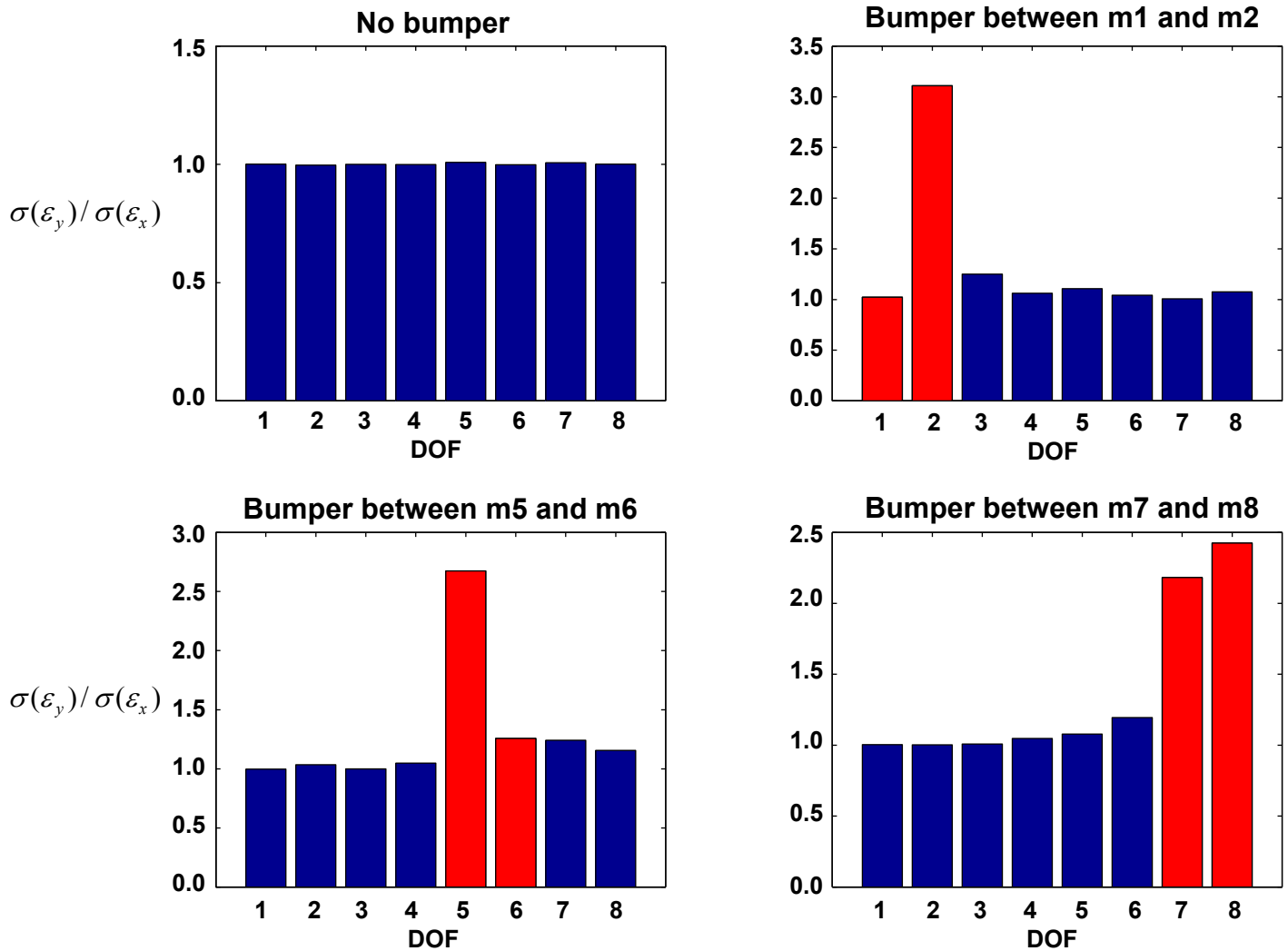
6. DECISION MAKING



Auto-Associative Neural Network for Data Normalization



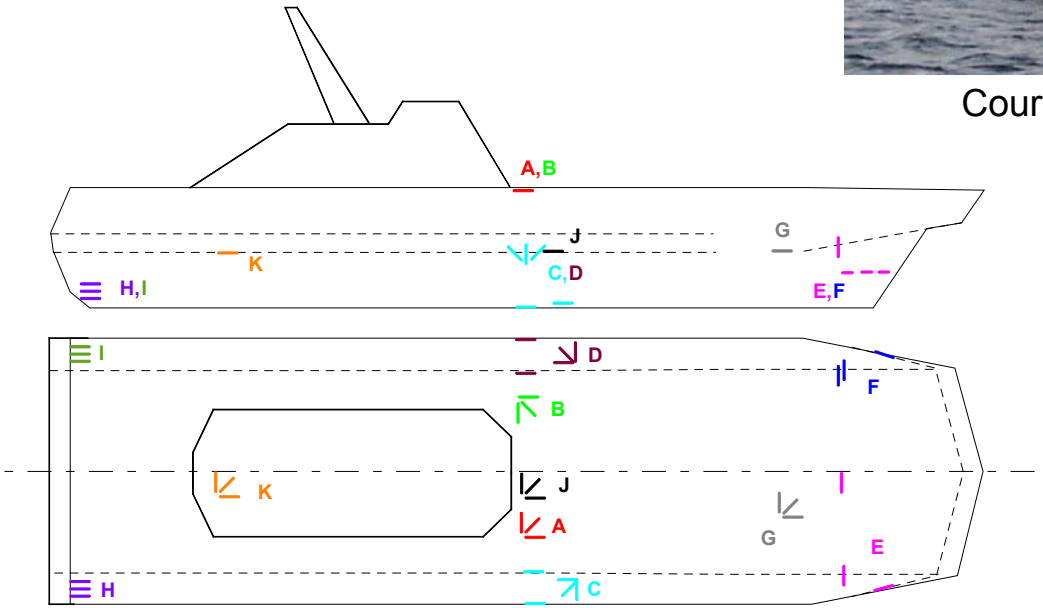
Damage Localization



Real World Application [Sohn et al. 2001]

(a) Surface-Effect Fast Patrol Boat

Courtesy of Naval Research Laboratory

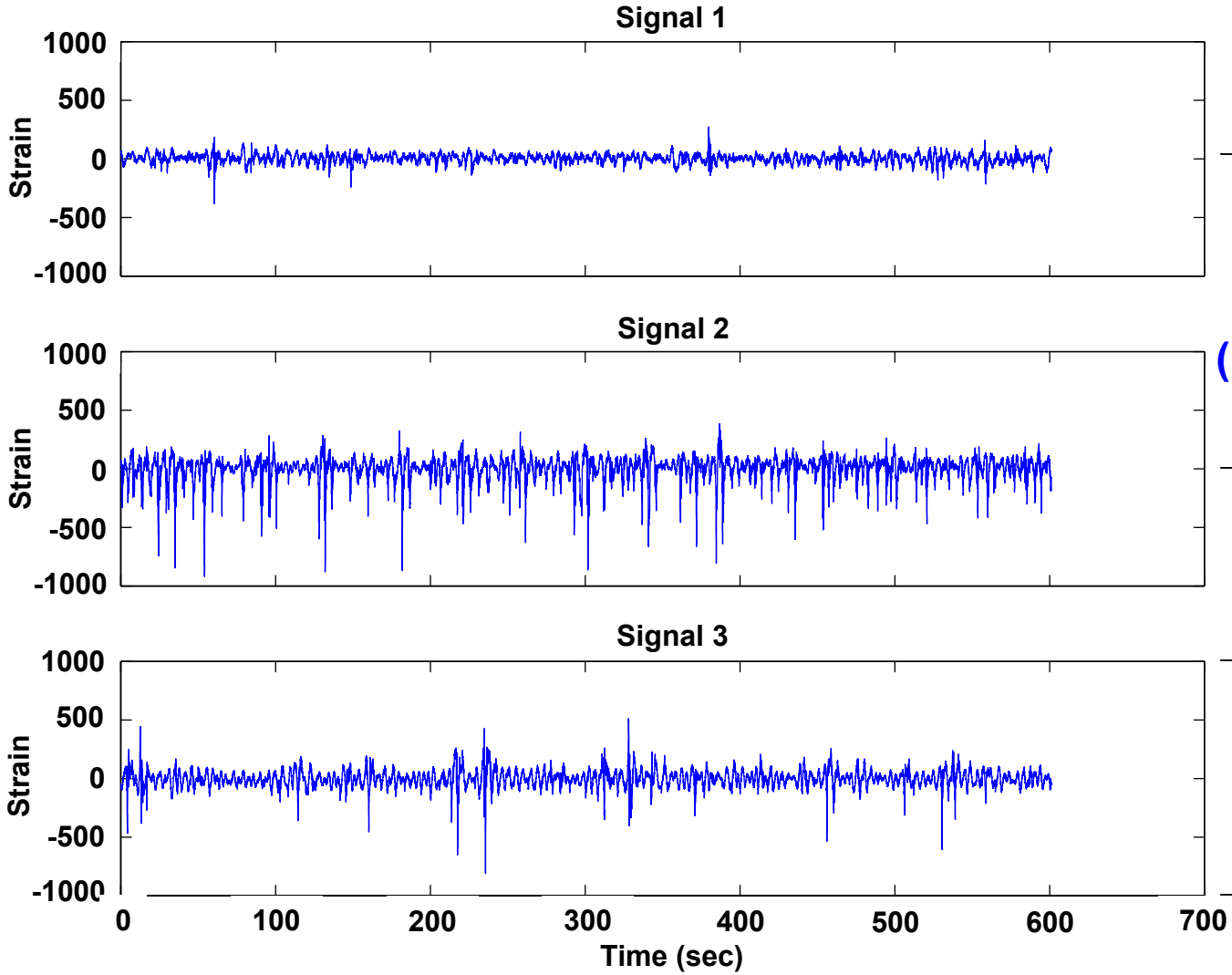


(b) Fiber optic strain gauges

Raw Dynamic Strain Time Series Data

Low Speed

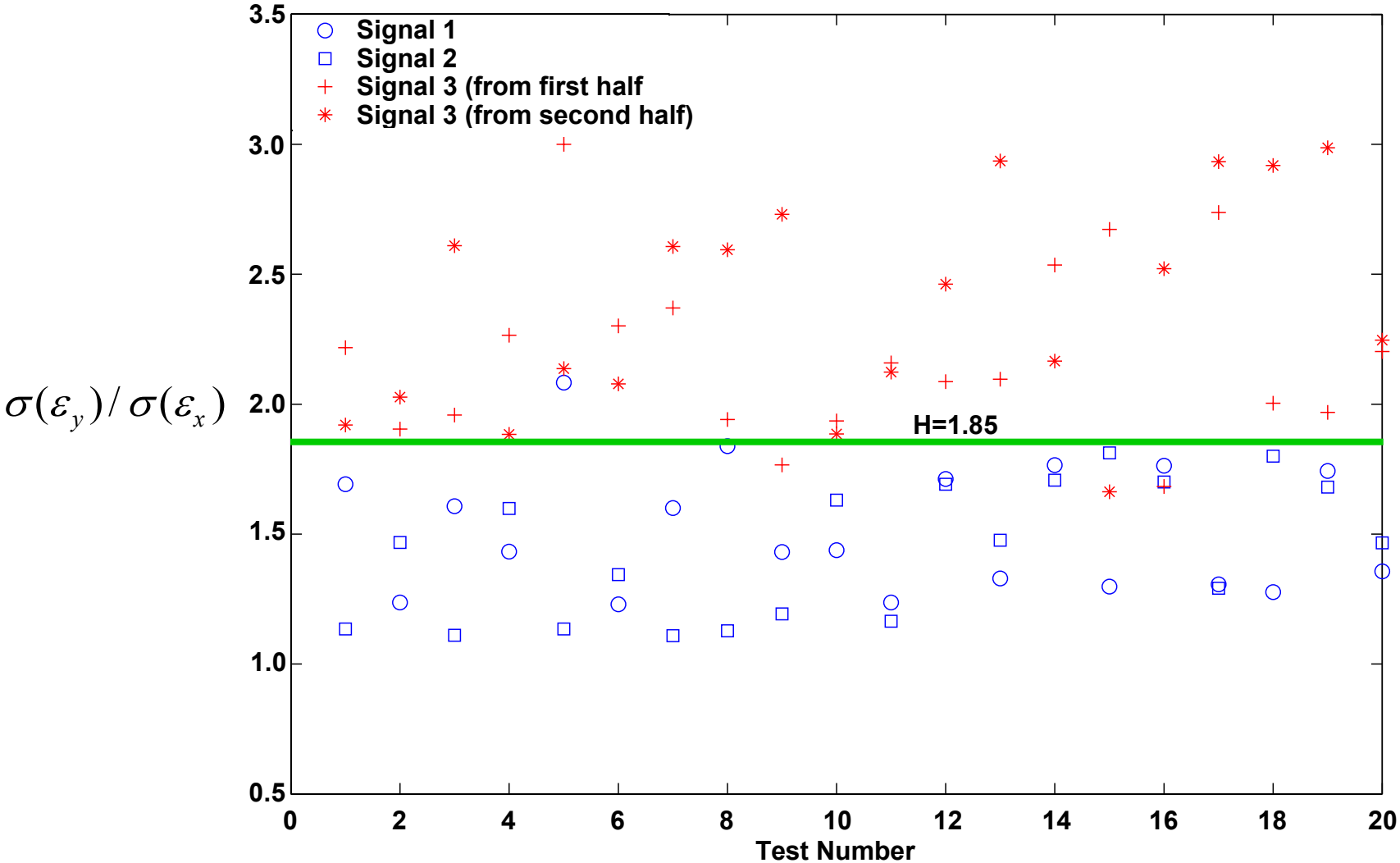
High Speed



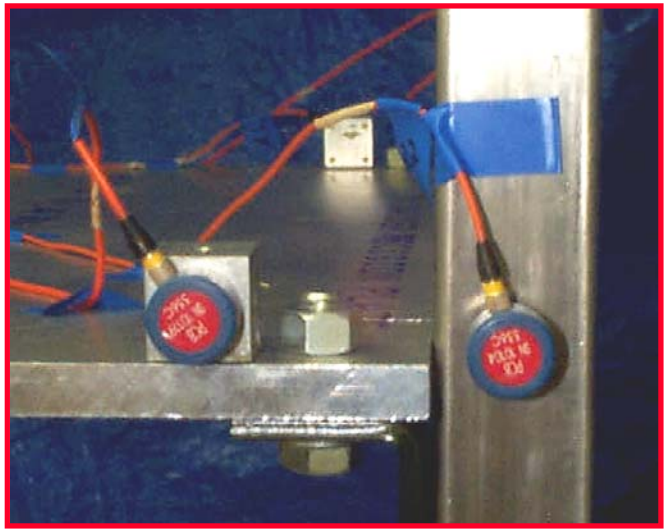
Structural Condition I (undamaged)

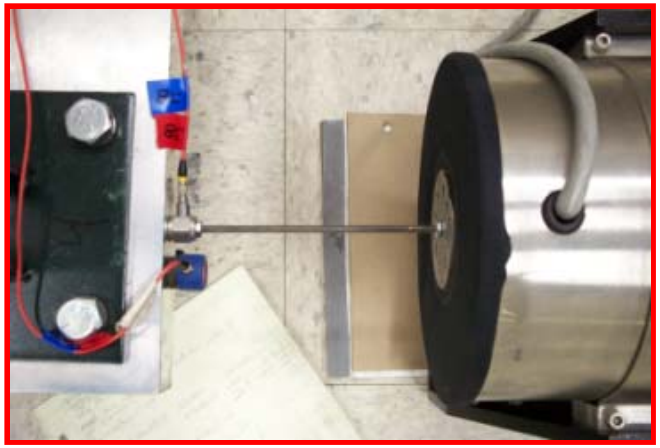
Structural Condition II (damaged)

Damage Classification using $\sigma(\varepsilon_y)/\sigma(\varepsilon_x)$

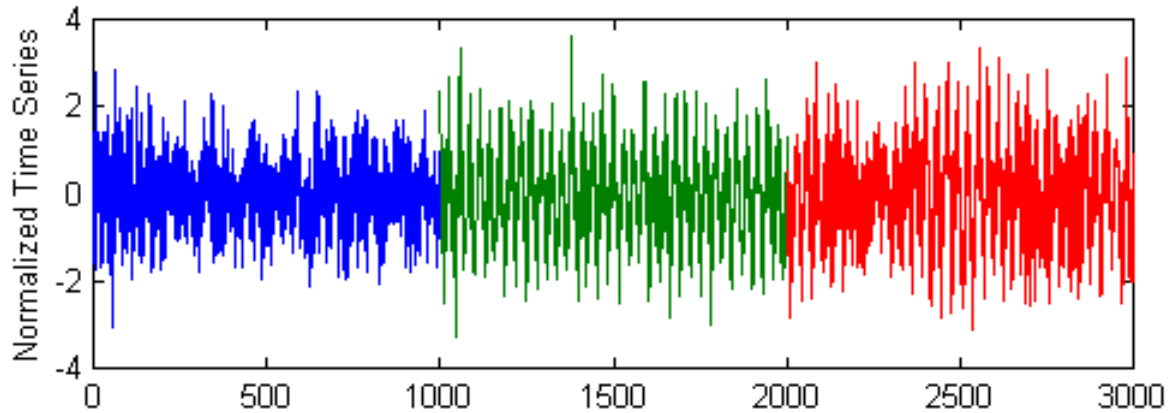


A Moment Resisting Frame Structure Model



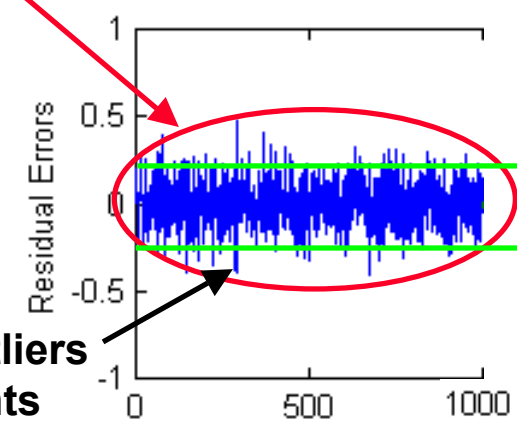


Establishment of Decision Boundaries



Undamaged (Training) | Undamaged (Testing) | Damaged (Testing)

Unsupervised Learning: Use only the baseline data for training

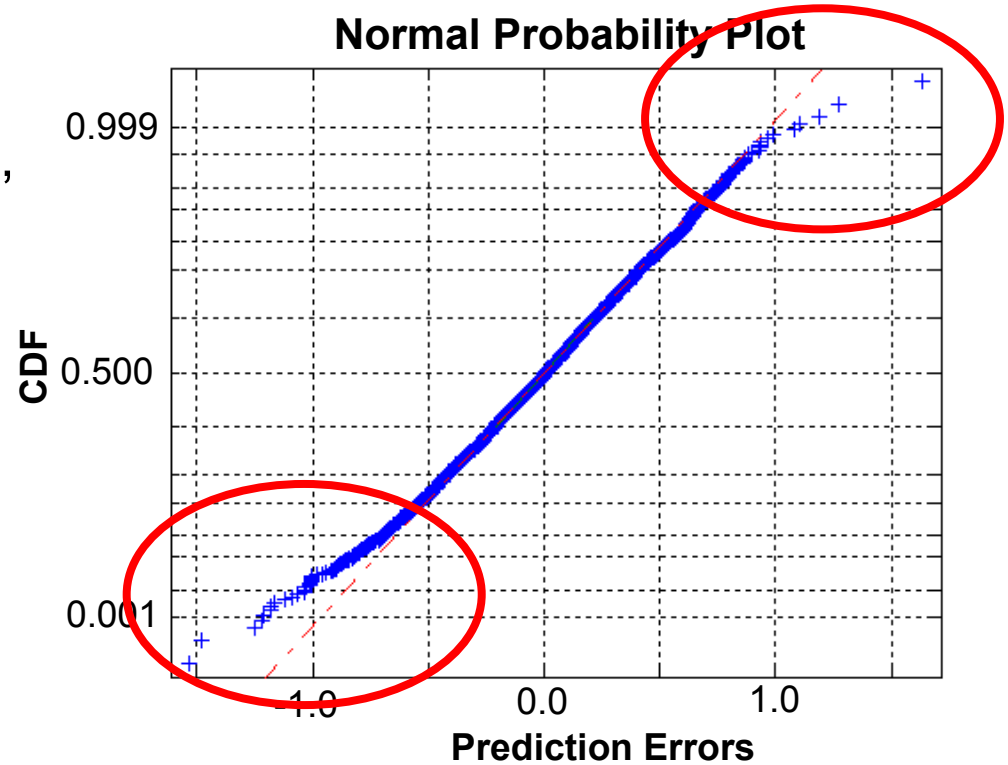


From Normal # of outliers: 48

# of Expected outliers = 1% of 1000 points = 10 outliers

Normality Assumption of Data

- Let's look at the baseline prediction errors to see whether they have a normal distribution or not.
- A normal probability plot graphically assesses whether the data come from a normal distribution.
- If the data are normal, the plot will be linear. Otherwise, there would be curvature in the plot.
- The central population of data seems to fit to the normal distribution well, but the tails do not.



Why Extreme Value Statistics ?

- In general, the distribution type of the parent data is unknown, and there are infinite numbers of candidate distributions.

- There are only three types of distributions for extreme (maximum or minimum) values regardless the distribution type of the parent data [Fisher and Tippett, 1928].

- That means, the model selection for the extreme values becomes much easier, because there are only three models to choose.(Gumbel, Weibull, Frechet distributions)

Feasible Cumulative Density Functions for Maxima

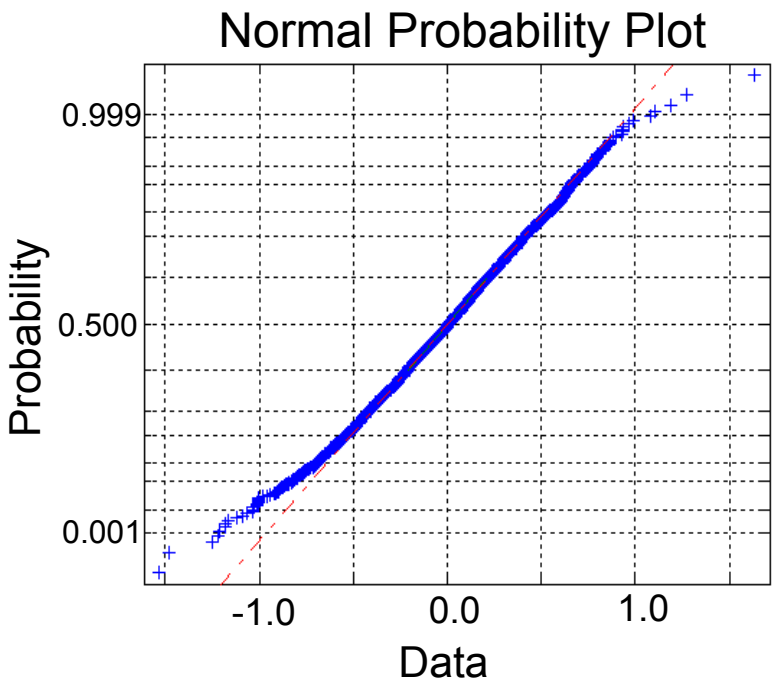
From Castillo [1988]:

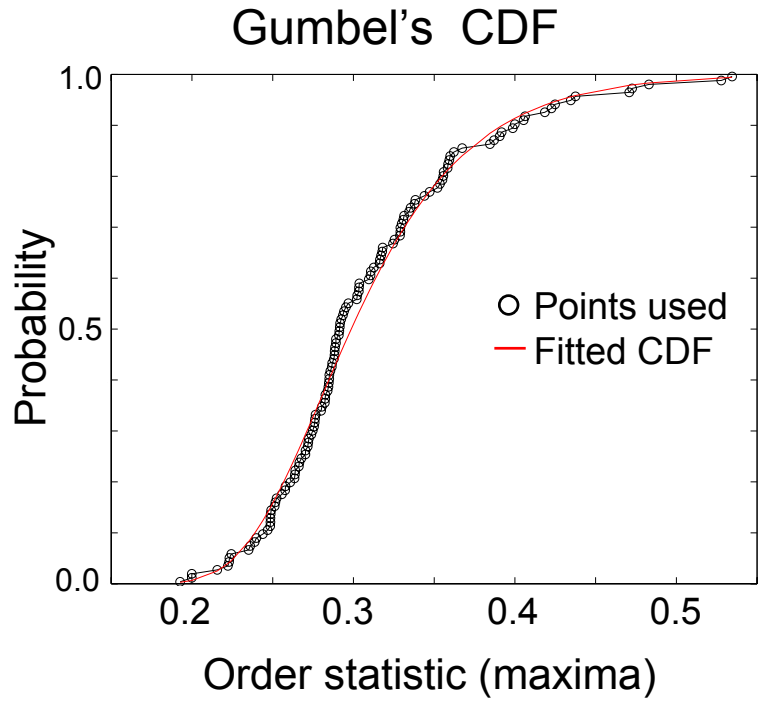
- Gumbel $$F(x) = \exp\left[-\exp\left(-\frac{x-\lambda}{\delta}\right)\right] \quad -\infty < x < \infty,\ \delta > 0$$

- Weibull: $$F(x) = \begin{cases} 1 & \text{if } x \geq \lambda \\ \exp\left[-\left(\frac{\lambda - x}{\delta}\right)^{\beta}\right] & \text{otherwise} \end{cases}$$

- Frechet: $$F(x) = \begin{cases} \exp\left[-\left(\frac{\delta}{x-\lambda}\right)^{\beta}\right] & \text{if } x \geq \lambda \\ 0 & \text{otherwise} \end{cases}$$

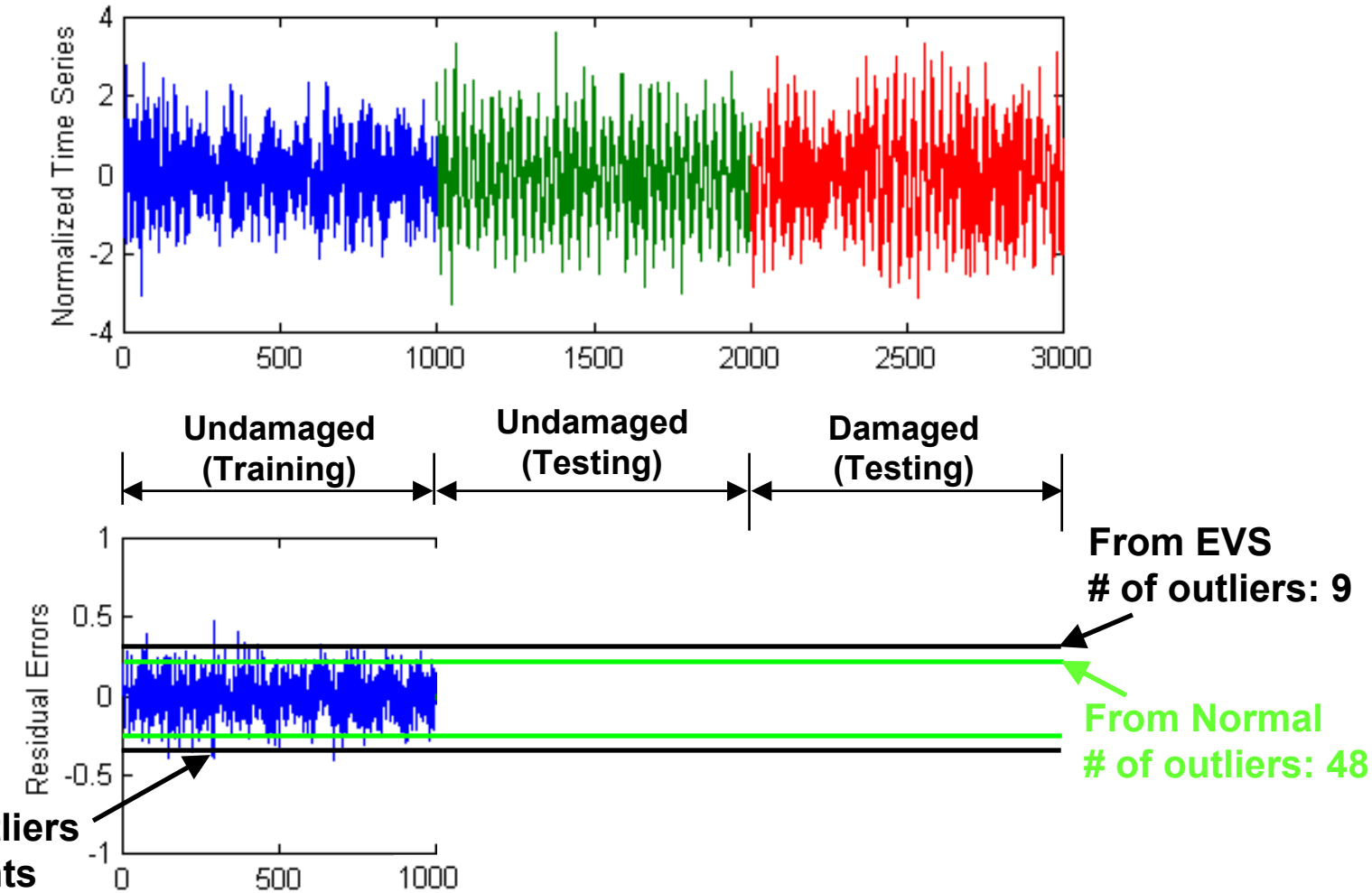
Fitting to Gumbel Distribution



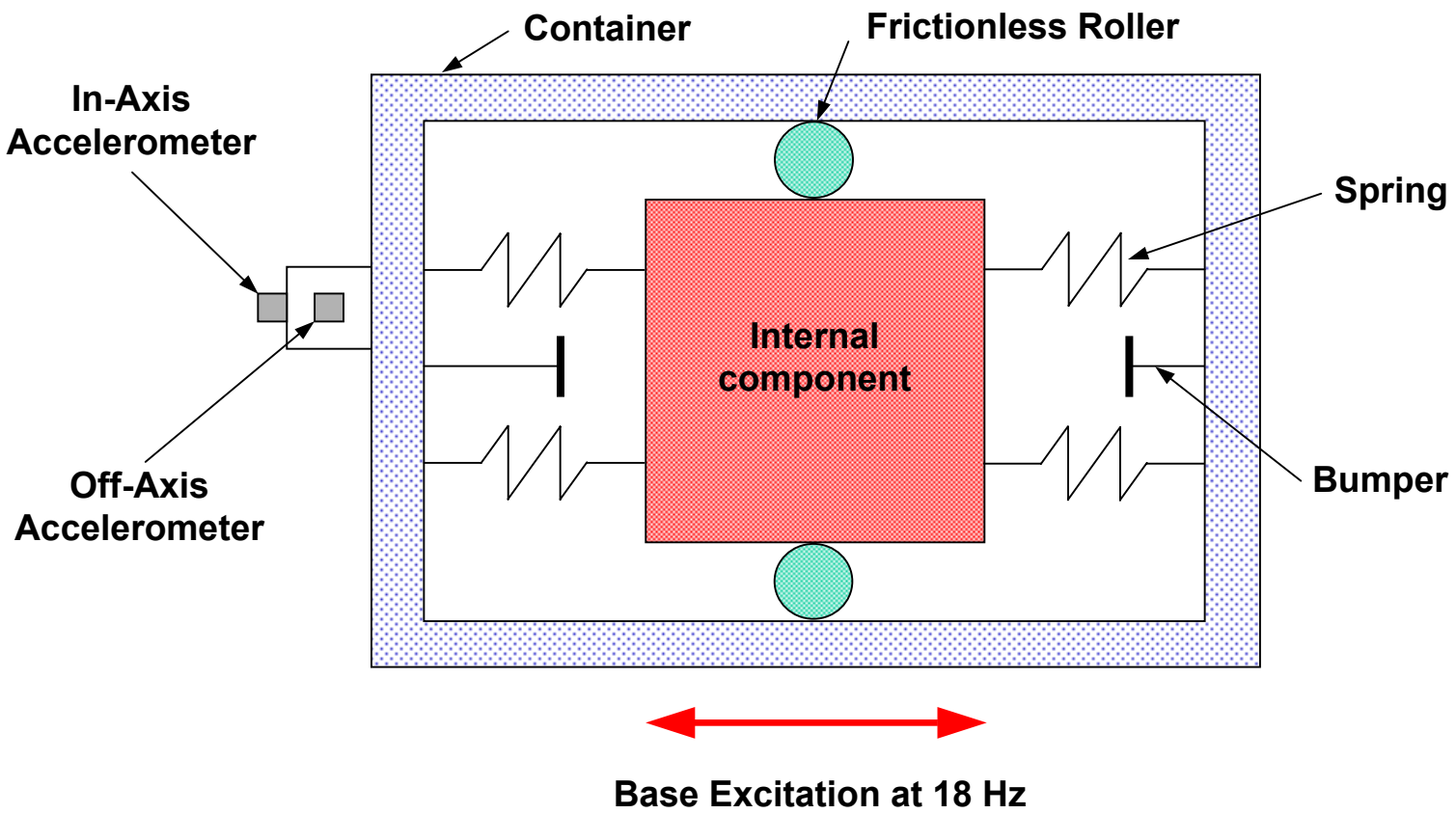


- Divide the original times series with 8192 data points into 128 time series with 64 points.
- Compute the maximum value from each block and fit the 128 maxima to a Gumbel distribution.

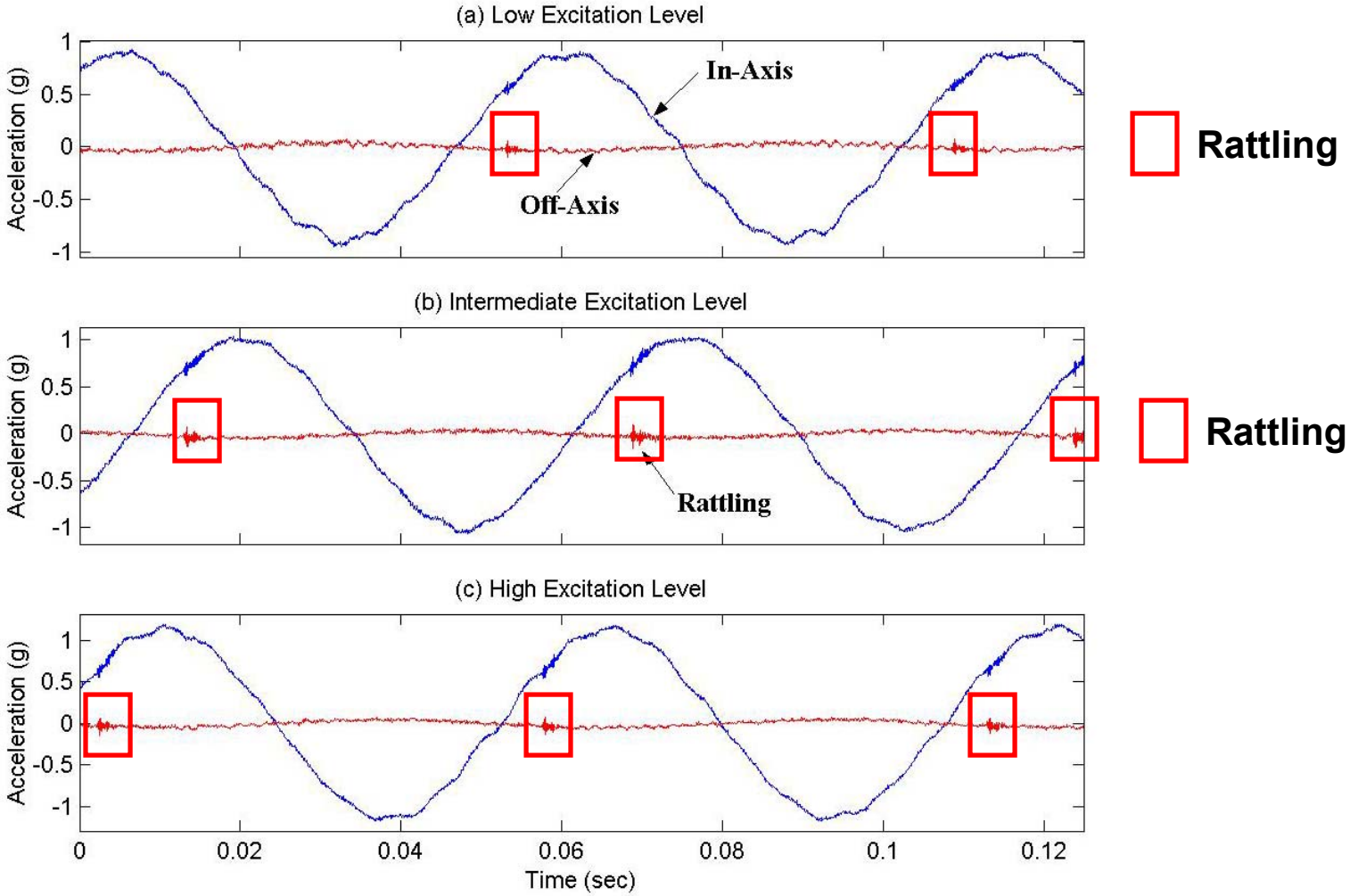
Establishment of Decision Boundaries



Detection of Rattling



Acceleration Response



Discontinuity Detection via Holder Exponent

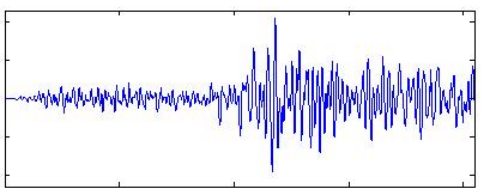
- Definition: The Holder Exponent is a measure of the regularity of the signal. The regularity of the signal is the number of continuous derivatives that the signal possesses.

- Objective: Identify discontinuity in signals that can be caused by certain types of damage.

- Application: Examples of damage that might induce discontinuity into the dynamic response signal include:
  - Opening and closing of cracks
  - A loose joint that is allowing contact (rattle) to occur

Holder Exponent Analysis

Time Signal



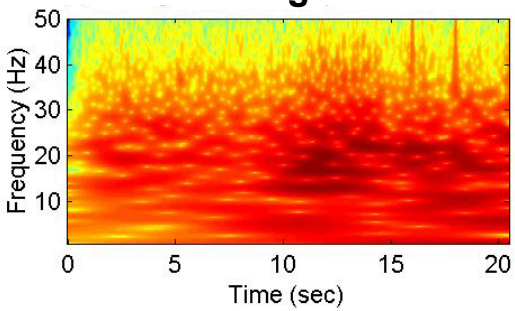
Wavelet Transform

$$Wf(u,s) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{s}} \psi^* \left(\frac{t-u}{s} \right) dt$$

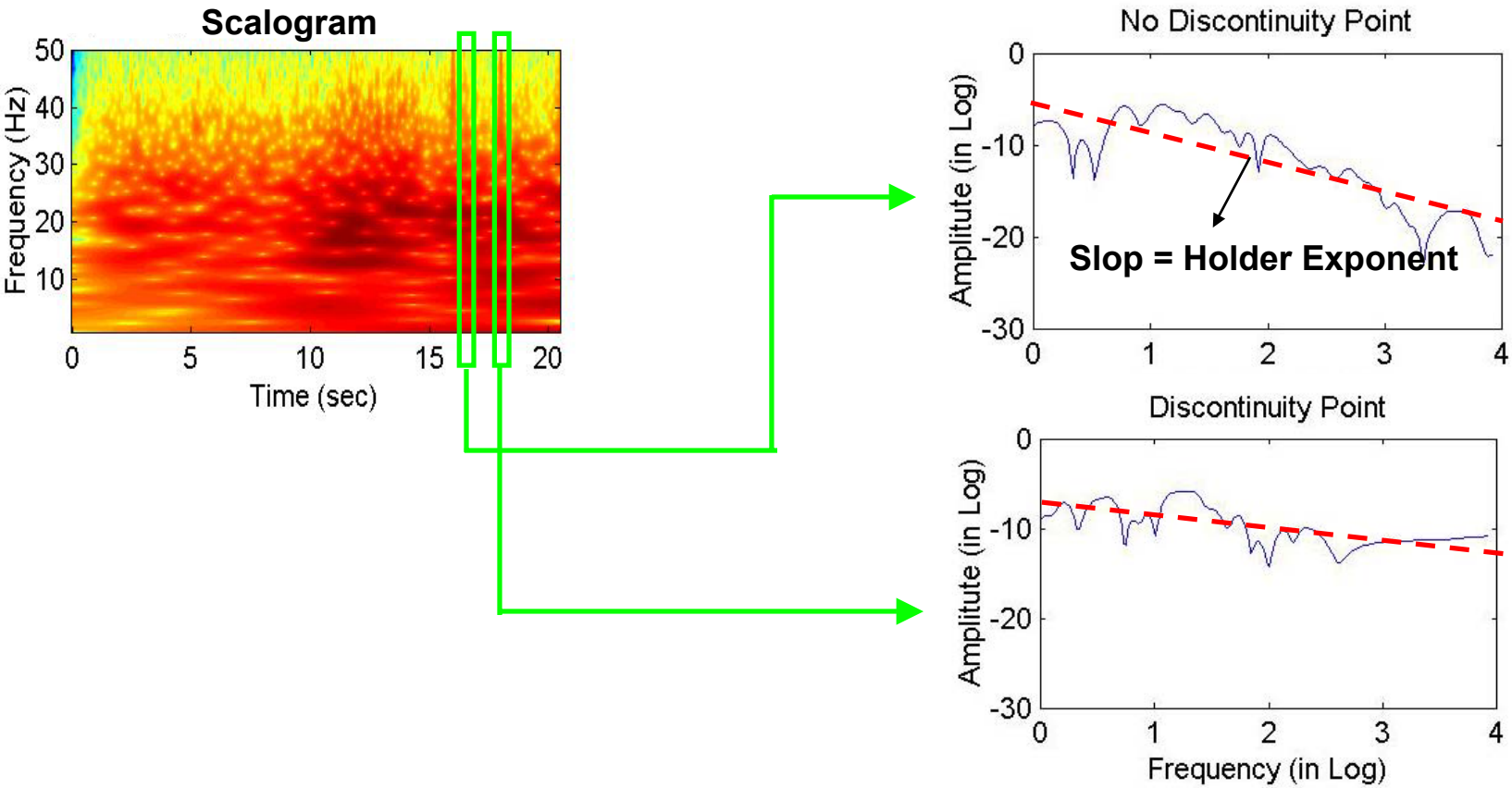
Translation

Dilation

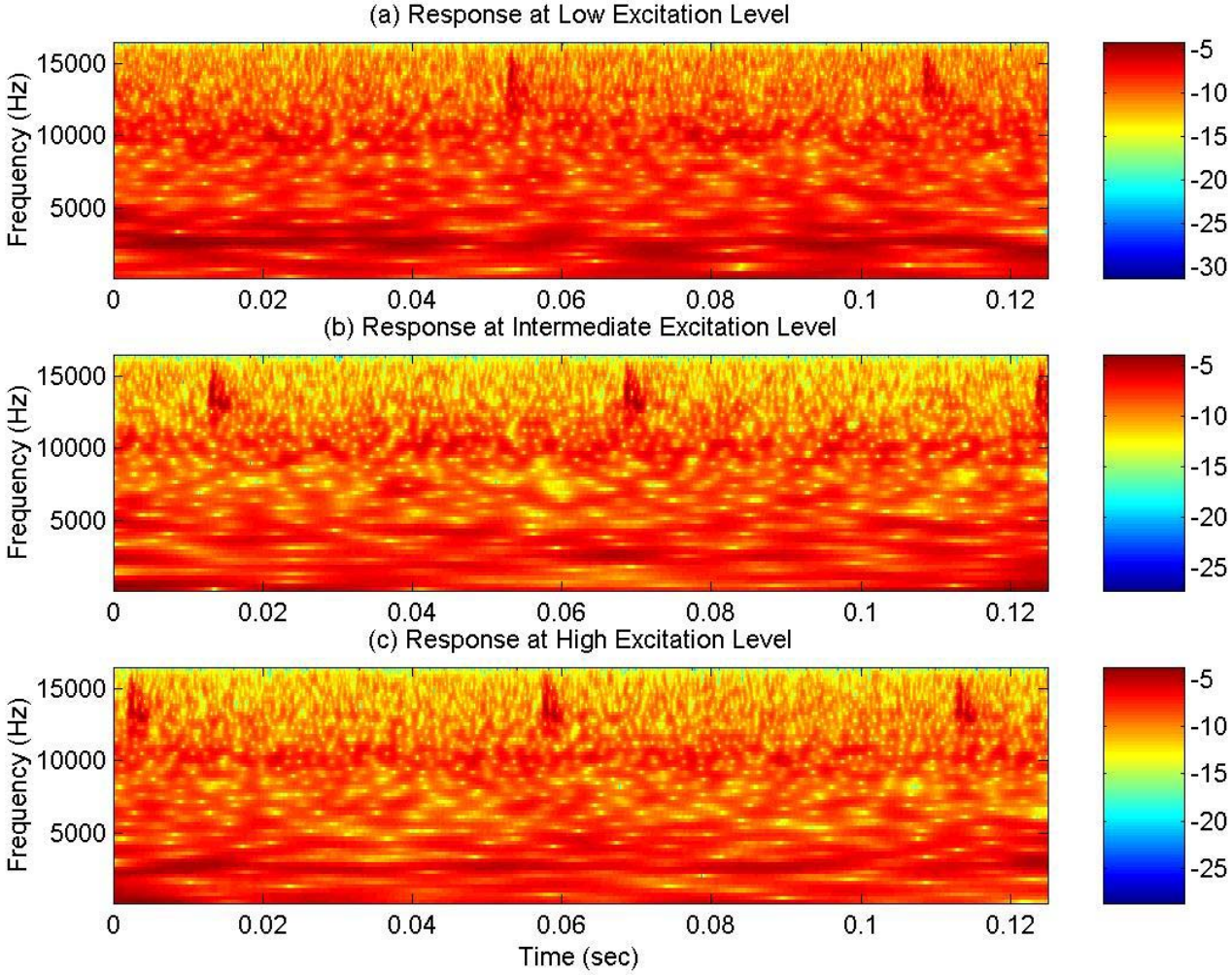
Scalogram



Holder Exponent Analysis

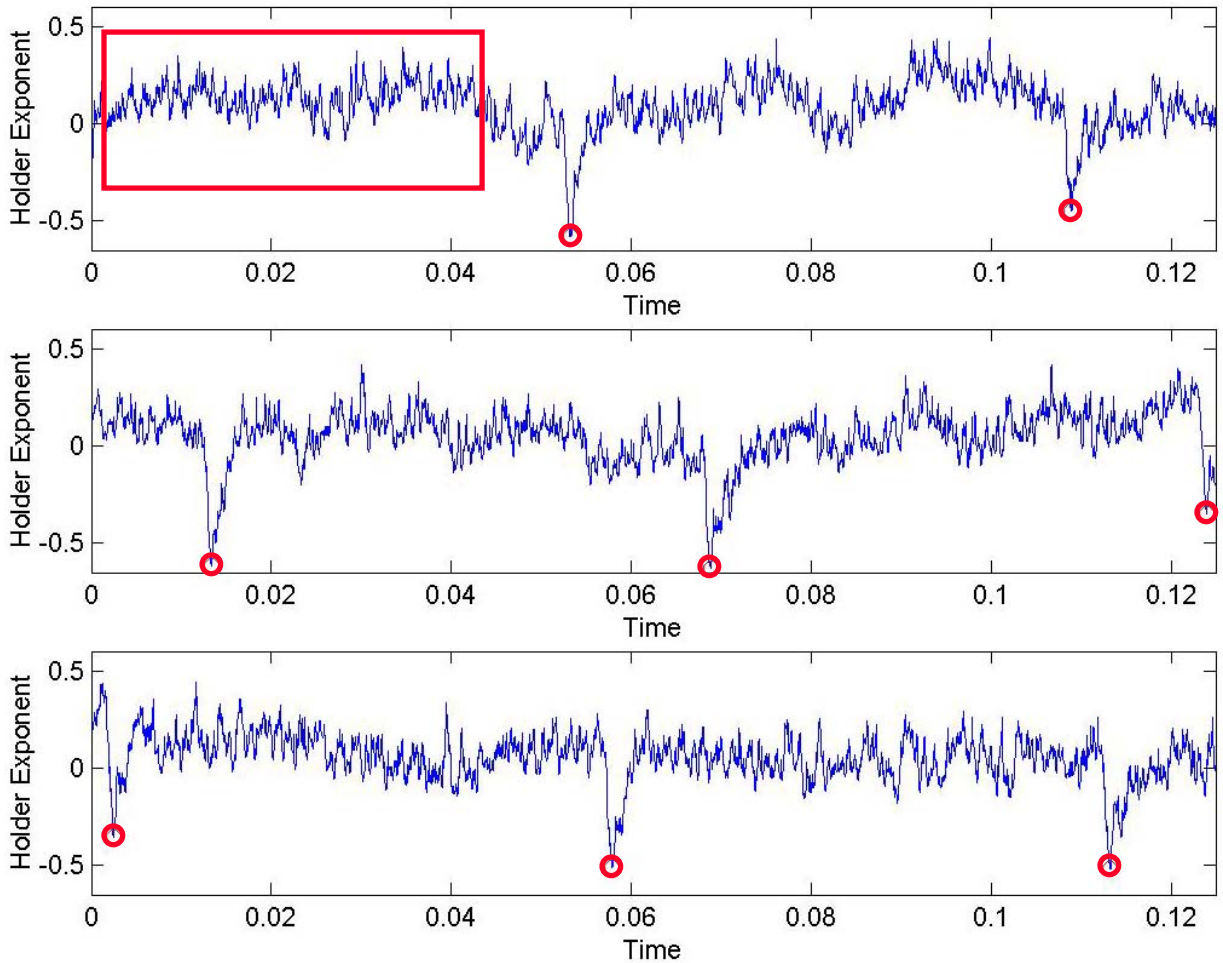


Scalogram from Wavelet Transform



Holder Exponent Analysis

(1) Find the max drop (2) Threshold = C x max drop (3) Discontinuity > Threshold



Summary

- Cast structural health monitoring problems in the framework of statistical pattern recognition.
- Developed various signal-based damage detection algorithms.
- Embed damage detection algorithms into on-board microprocessors.
- Address data normalization issue explicitly.
- Decision making is based on rigorous statistical modeling.
- Provide a suite of data interrogation algorithms for structural health monitoring in the format of GUI software called DIAMOND II (patent pending).