Collective Neutrino Oscillations *in* Supernovae

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Outline

- Neutrino mixing and "self-coupling"
- Why do collective oscillations occur?
- Where do collective oscillations occur?

Neutrino Oscillations in SNe

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$

mass matrix

electron density

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_{\mathrm{F}}\,\mathrm{diag}[n_e, 0, 0] + \mathsf{H}_{\nu\nu}$$

neutrino energy

ν–ν forward scattering (self-coupling)

Neutrino Oscillations in SNe

$$H^{\nu\nu}_{\alpha\beta} = \sqrt{2}G_{\mathrm{F}}\Big[\sum_{\nu'}(1-\hat{\mathbf{p}}'\cdot\hat{\mathbf{p}})n_{\nu'}\,\langle\nu_\beta|\psi_{\nu'}\rangle\langle\psi_{\nu'}|\nu_\alpha\rangle$$
$$-\sum_{\bar{\nu}'}(1-\hat{\mathbf{p}}'\cdot\hat{\mathbf{p}})n_{\bar{\nu}'}\,\langle\bar{\nu}_\beta|\psi_{\bar{\nu}'}\rangle\langle\psi_{\bar{\nu}'}|\bar{\nu}_\alpha\rangle\Big]$$

Fuller et al, Astrophys. J 322:795, 1987

Nötzold & Raffelt, Nucl. Phys. B307:924, 1988

Pantaleone, Phys. Rev. D46:510, 1992

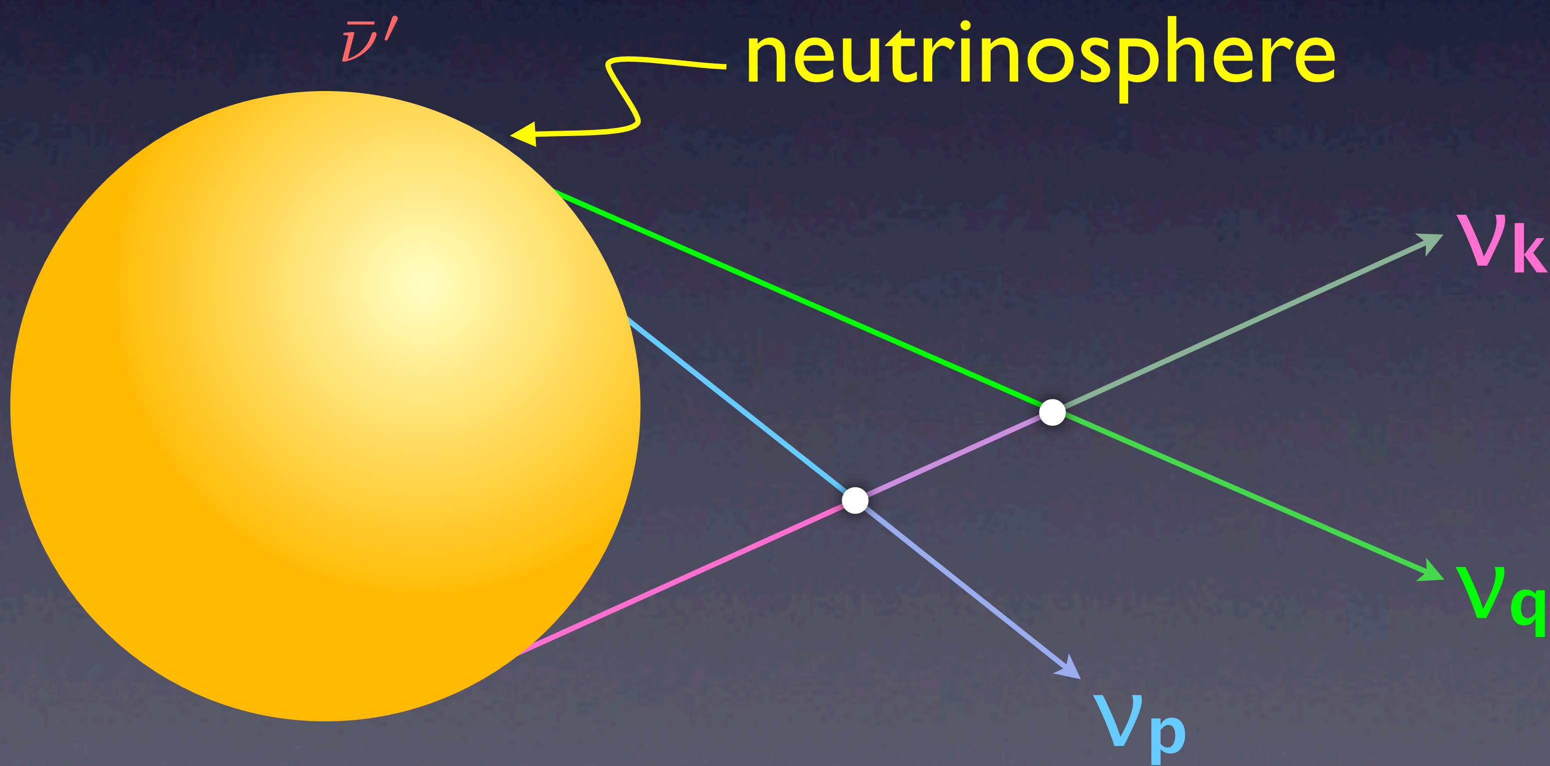
Sigl & Raffelt, Nucl. Phys. B406:423, 1993

Qian & Fuller, Phys. Rev. D51:1479, 1995

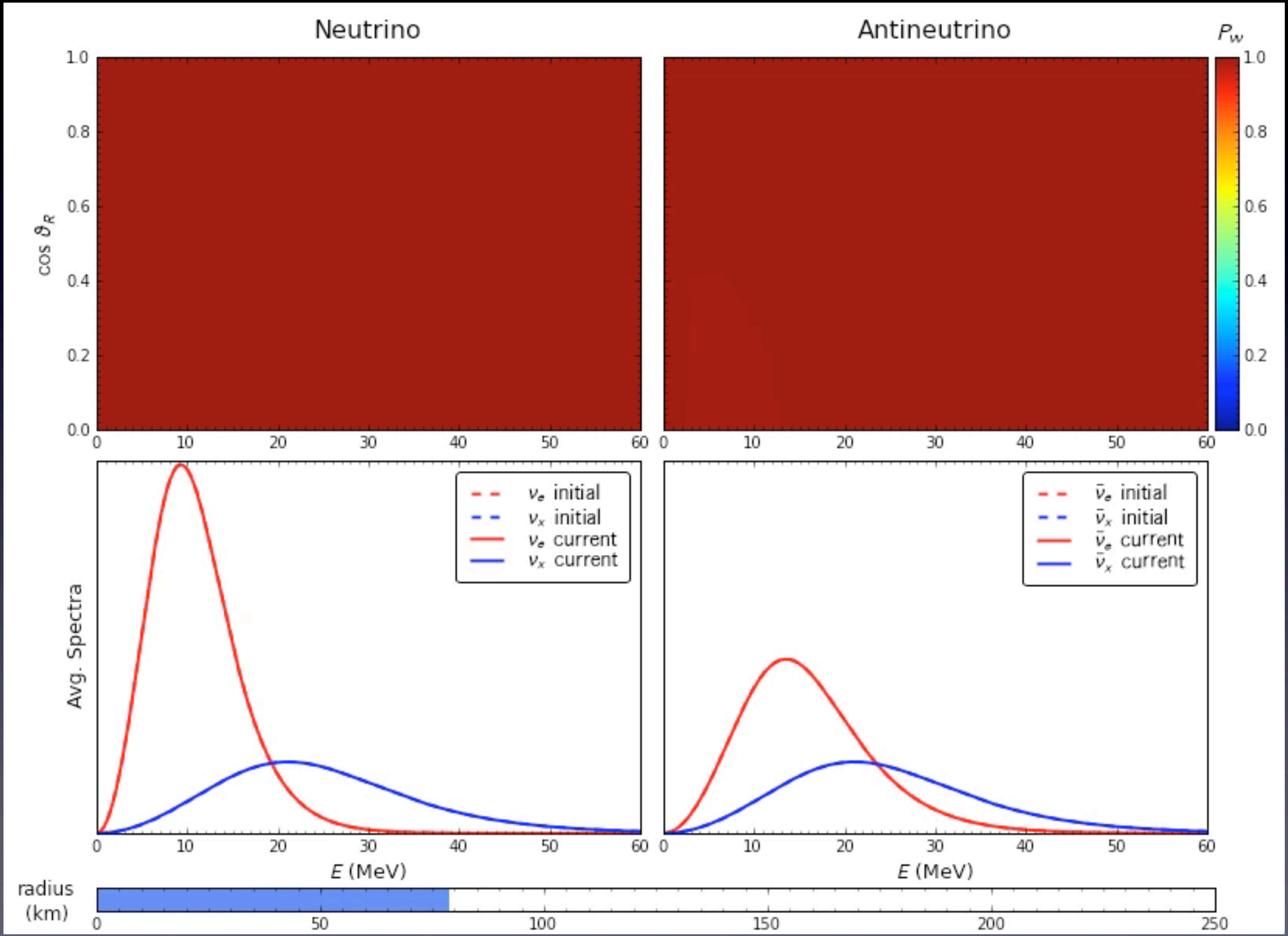
Neutrino Oscillations in SNe

$$H_{\alpha\beta}^{\nu\nu} = \sqrt{2}G_{\mathrm{F}}\left[\sum_{\nu'}(1-\hat{\mathbf{p}}'\cdot\hat{\mathbf{p}})n_{\nu'}\,\langle\nu_\beta|\psi_{\nu'}\rangle\langle\psi_{\nu'}|\nu_\alpha\rangle\right.$$

$$\left.-\sum_{\bar{\nu}'}(1-\hat{\mathbf{p}}'\cdot\hat{\mathbf{p}})n_{\bar{\nu}'}\,\langle\bar{\nu}_\beta|\psi_{\bar{\nu}'}\rangle\langle\psi_{\bar{\nu}'}|\bar{\nu}_\alpha\rangle\right]$$



$$\delta m^2 = -3\times 10^{-3}\,\mathrm{eV}^2,\ \theta_\mathrm{v} \ll 1,\ L_\nu = 10^{51}\,\mathrm{erg/s}$$
$$\langle E_{\nu_e}\rangle = 11\,\mathrm{MeV},\ \langle E_{\bar\nu_e}\rangle = 16\,\mathrm{MeV},\ \langle E_{\nu_x,\bar\nu_x}\rangle = 25\mathrm{MeV}$$



Why?

Neutrino Flavor Isospin

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}\psi_\nu = \mathsf{H}\psi_\nu$$
$$= -\vec{H}\cdot\frac{\vec{\sigma}}{2}\psi_\nu$$

$$\frac{\mathrm{d}}{\mathrm{d}\lambda}\vec{s} = \vec{s}\times\vec{H}$$

$$\begin{bmatrix} e^- \\ \nu_e \end{bmatrix} \quad \begin{bmatrix} \mu/\tau \\ \nu_\mu/\nu_\tau \end{bmatrix}$$

Weak Isospin

Flavor Isospin

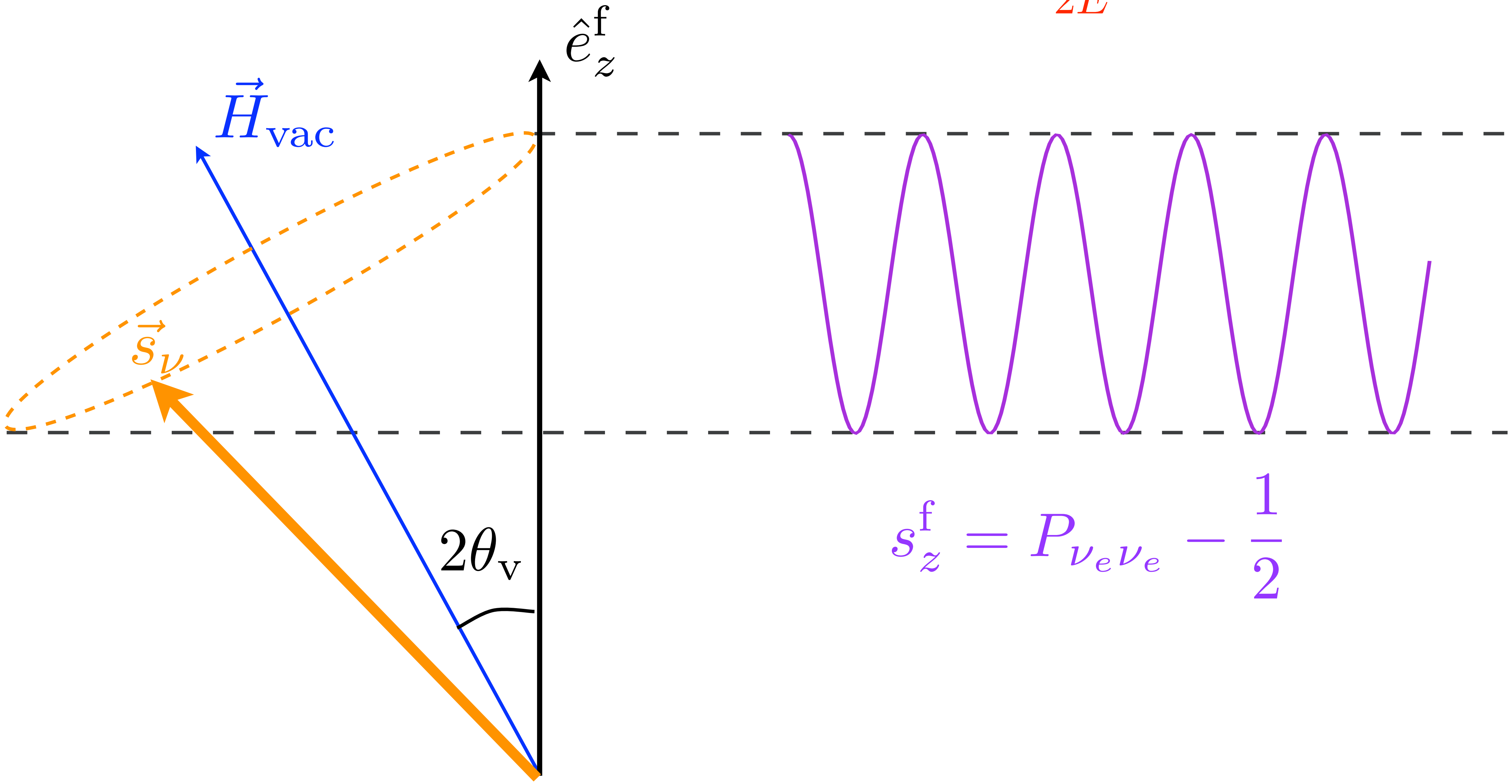
	e-flavor	τ'-flavor	maximally mixed
$\vec{s}_\nu \equiv \psi_\nu^\dagger \frac{\vec{\sigma}}{2}\psi_\nu$	$\uparrow$	$\downarrow$	$\rightarrow$
$\vec{s}_{\bar{\nu}} \equiv (\sigma_y\psi_{\bar{\nu}})^\dagger \frac{\vec{\sigma}}{2}(\sigma_y\psi_{\bar{\nu}})$	$\downarrow$	$\uparrow$	$\rightarrow$

Vacuum Oscillations

$$\vec{H} = \omega \vec{H}_{\rm vac}$$

$$\vec{H}_{\rm vac} \equiv -\hat{e}_x^{\rm f} \sin 2\theta_{\rm v} + \hat{e}_z^{\rm f} \cos 2\theta_{\rm v}$$

$$\omega \equiv \pm \frac{\delta m^2}{2E}$$

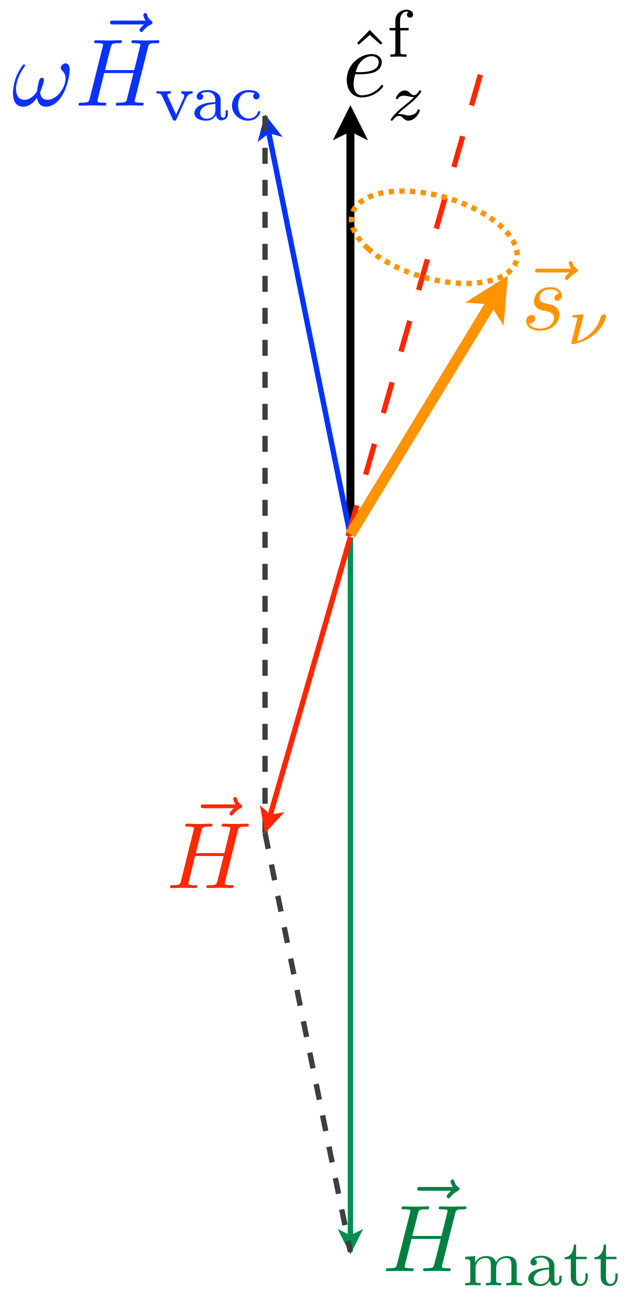


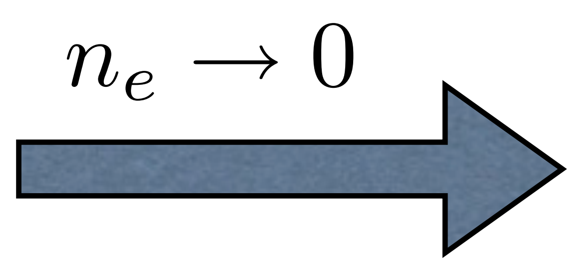
$$s_z^{\rm f} = P_{\nu_e \nu_e} - \frac{1}{2}$$

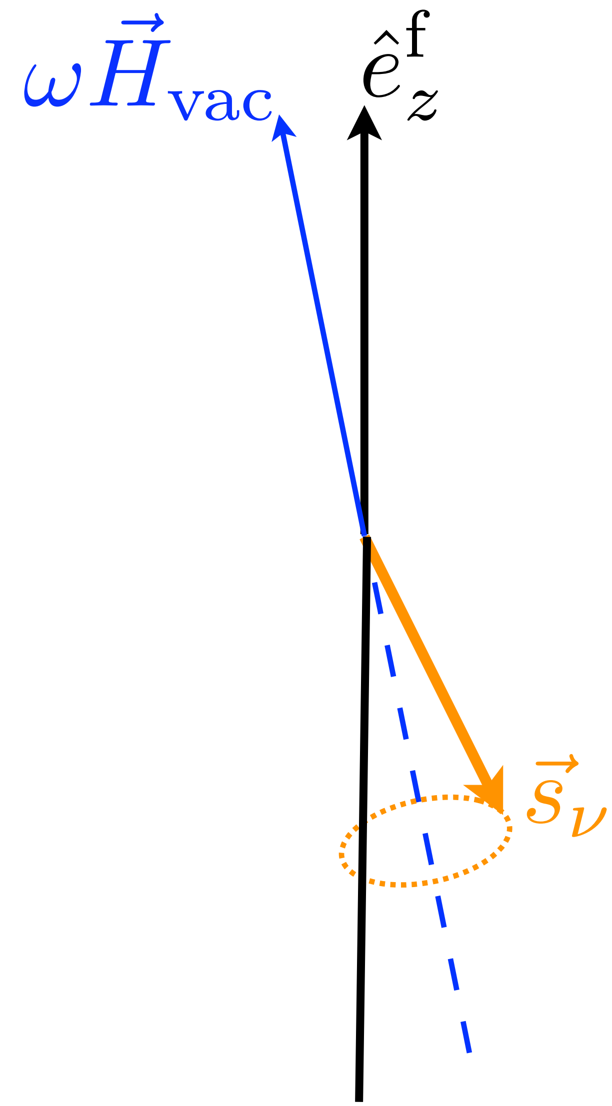
MSW Mechanism

$$\vec{H} = \omega\vec{H}_{\mathrm{vac}} + \vec{H}_{\mathrm{matt}}$$

$$\vec{H}_{\mathrm{matt}} \equiv -\hat{e}_z^{\mathrm{f}}\sqrt{2}G_{\mathrm{F}}n_e$$







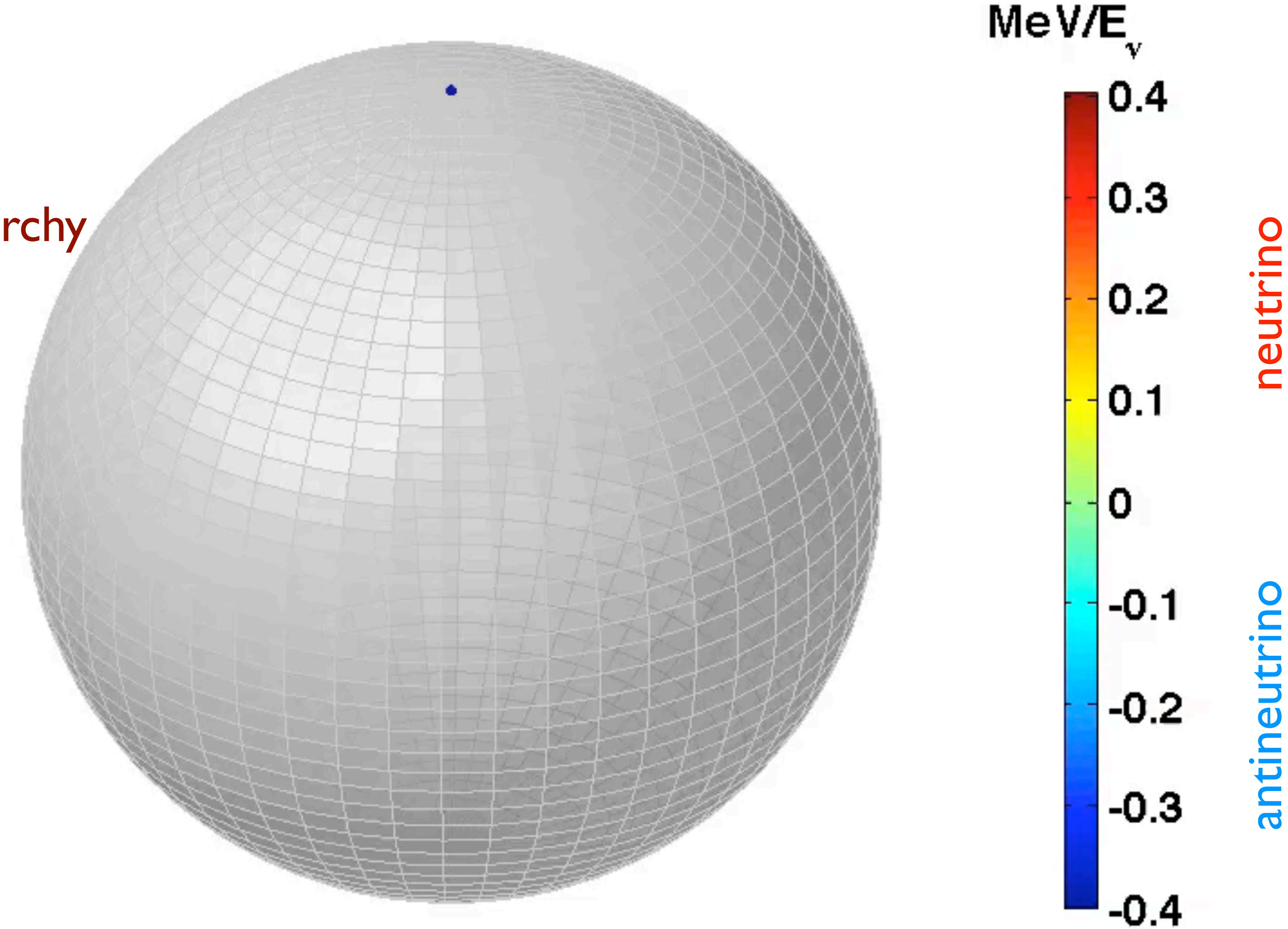
$$\omega = \frac{\delta m^2}{2E},\ \delta m^2 > 0$$

neutrino, normal hierarchy

MSW Mechanism

$\delta m^2 < 0$

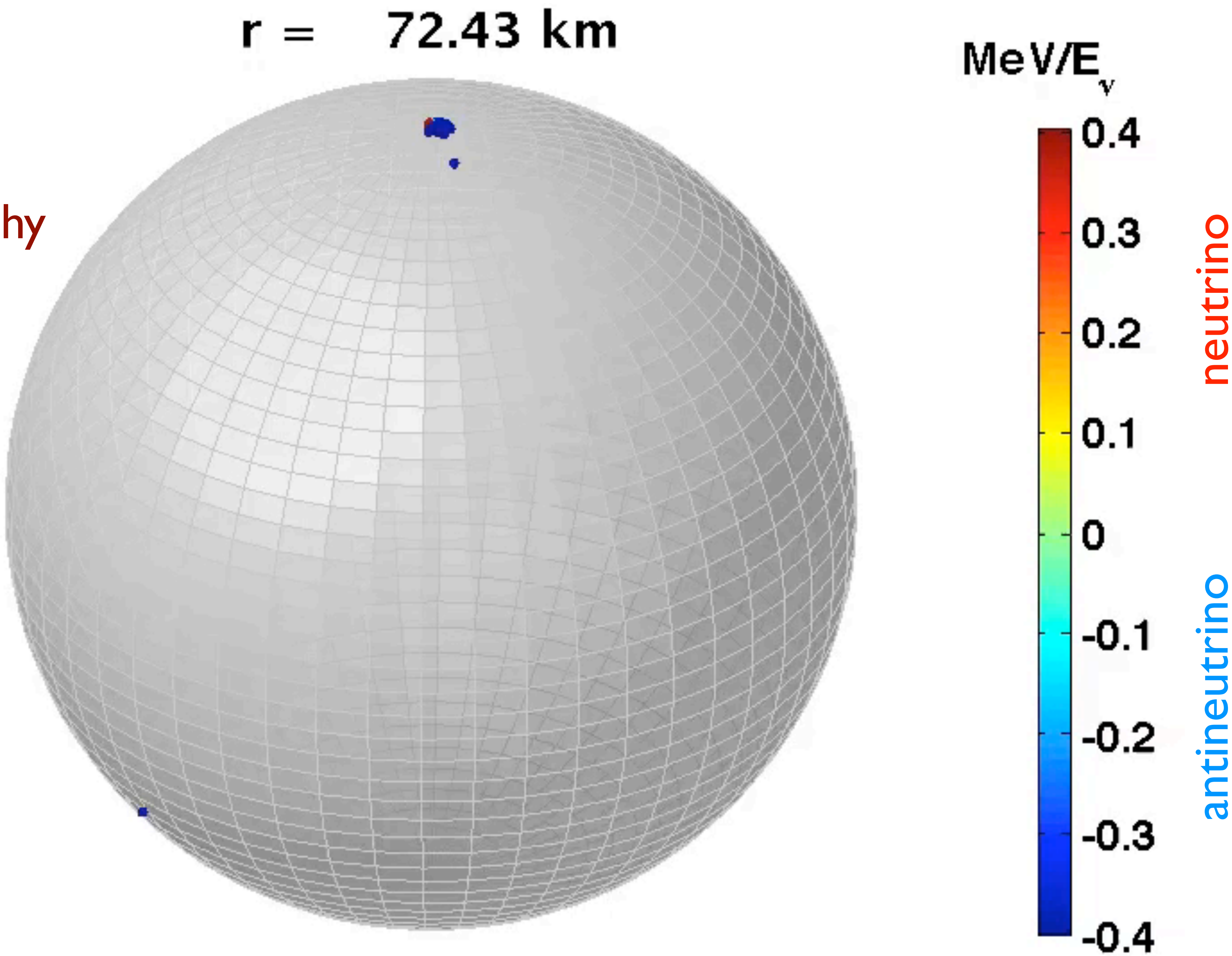
inverted hierarchy



Collective Oscillations

$\delta m^2 < 0$

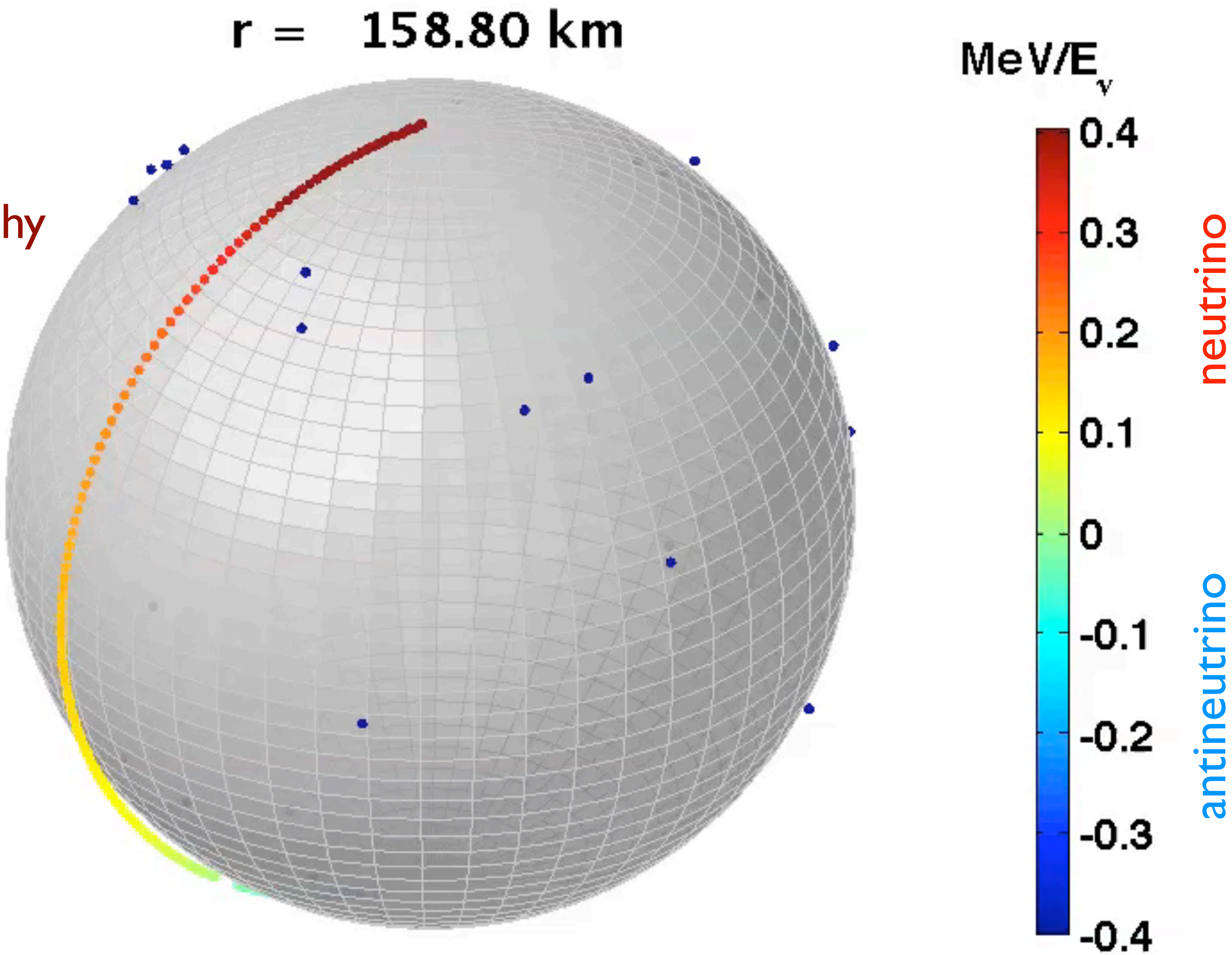
inverted hierarchy



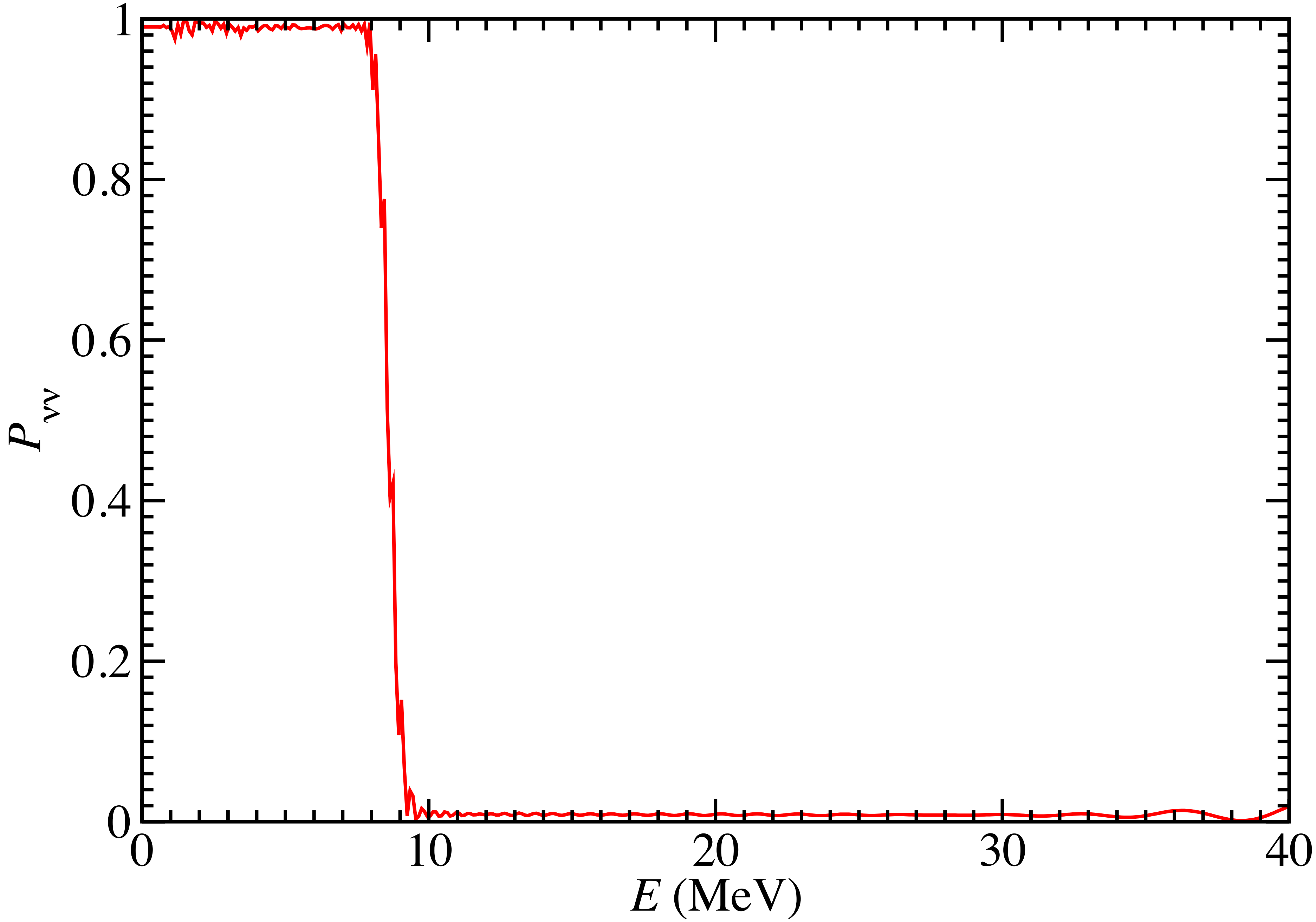
Collective Oscillations

$\delta m^2 < 0$

inverted hierarchy



Collective Oscillations



Collective Oscillations

homogeneous, isotropic neutrino gas

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{s}_\omega = \vec{s}_\omega \times \vec{H}_\omega$$

$$= \vec{s}_\omega \times (\omega\vec{H}_{\mathrm{vac}} + \cancel{\vec{H}}_{\mathrm{matt}} + \vec{H}_{\nu\nu})$$

cause collective oscillations

neutral (HD et al, PRD, 2005)

special axis

disrupt collective oscillations

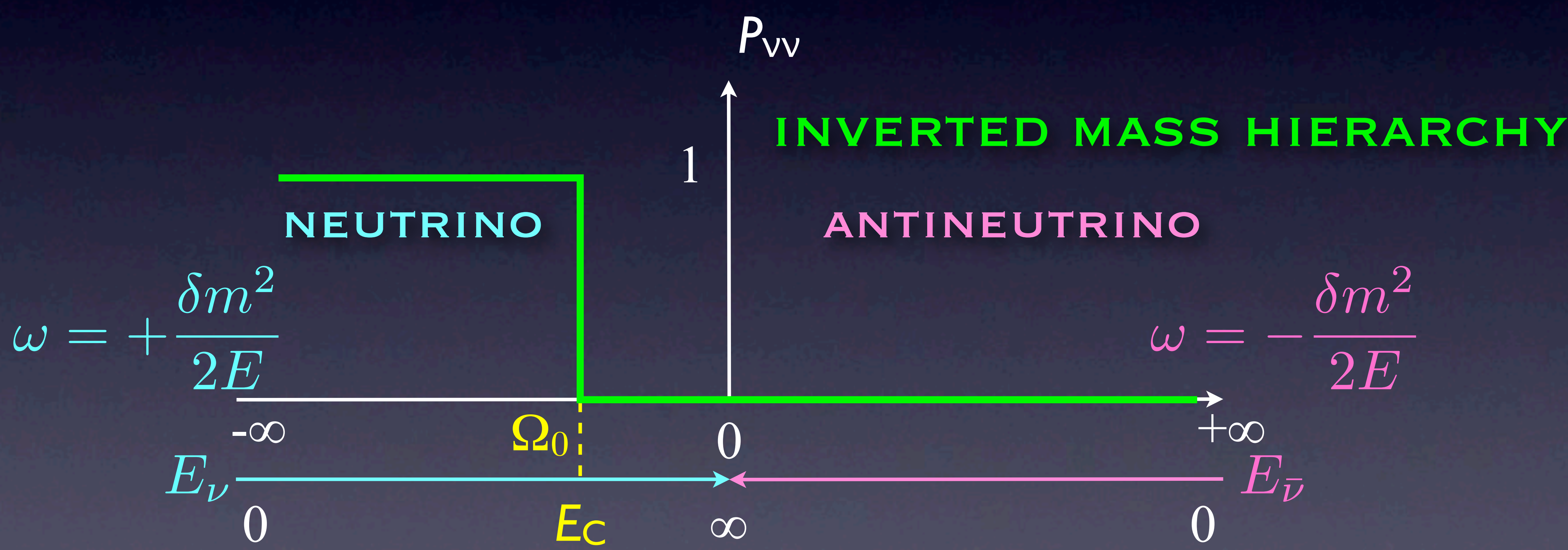
$$\vec{H}_{\nu\nu} = -\mu\int_{-\infty}^{\infty}\mathrm{d}\omega'\, f(\omega')\,\vec{s}_{\omega'}$$

distribution function

coupling strength, $\propto$ neutrino density

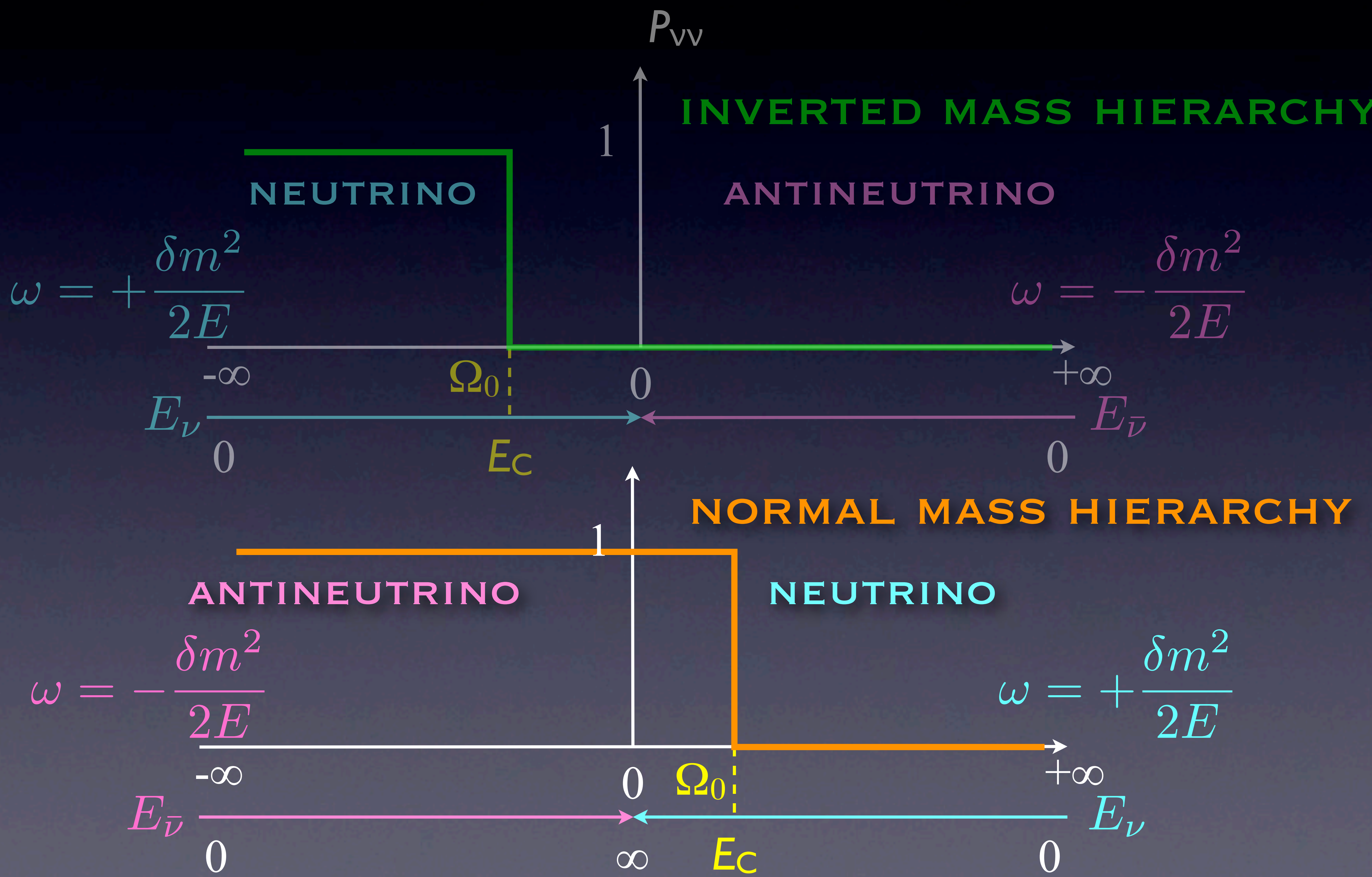
Collective Oscillations

if collective precession exists until $n_\nu \to 0$:



HD et al, PRD (2006)

Collective Oscillations



HD et al, PRD (2006)

Where?

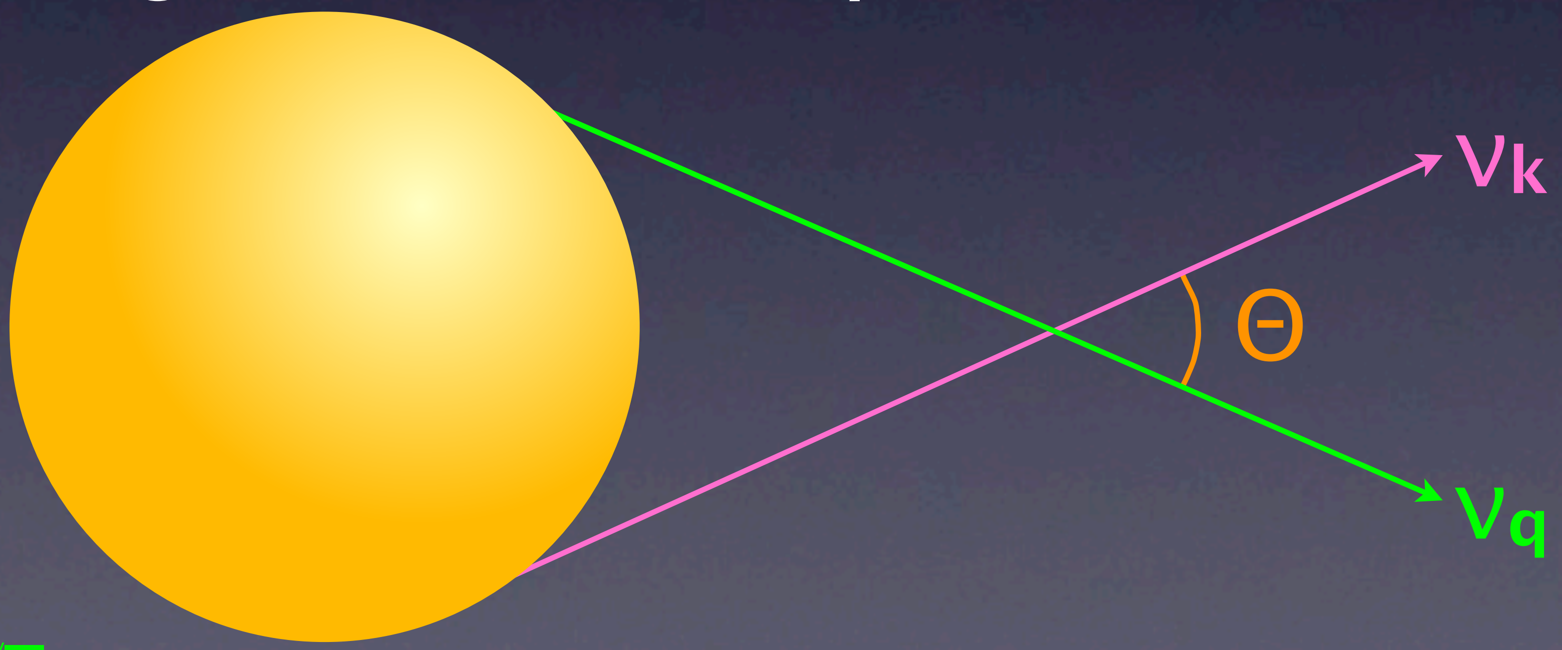
Neutrino Oscillations

$$\mathsf{H} = \frac{\mathsf{M}^2}{2E} + \sqrt{2}G_\mathrm{F}\,\mathrm{diag}[n_e, 0, 0] + \mathsf{H}_{\nu\nu}$$

- vacuum term: $\Lambda_{vac}=\Delta m^2/2E$
- matter (electron) density: $\Lambda_{mat}=\sqrt{2}G_F n_e$
- neutrino density: $\Lambda_\nu=\sqrt{2}G_F(n_\nu-n_{\bar{\nu}})$

Collective Oscillations

- Λ_{vac} depends on neutrino energy $\Rightarrow$ $\Delta\Lambda_{vac}$: dispersion in energies
- $\Delta\Lambda_{vac}$ ($\sim \Lambda_{vac}$) $\lesssim$ Λ_ν $\langle 1-\cos\Theta \rangle$: neutrinos with different energies oscillate in phase



$\Lambda_{vac}=\Delta m^2/2E$ $\quad$ $\Lambda_\nu=\sqrt{2}G_F(n_\nu-n_{\bar{\nu}})$

Collective Oscillations

- Λ_{vac} depends on neutrino energy $\Rightarrow$ $\Delta\Lambda_{vac}$: dispersion in energies
- $\Delta\Lambda_{vac}$ ($\sim \Lambda_{vac}$) $\lesssim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$: neutrinos with different energies oscillate in phase

$$\Lambda_\nu \langle 1-\cos\Theta \rangle \propto r^{-2}\, r^{-2} = r^{-4}$$

$\Lambda_{vac} = \Delta m^2/2E$ $\Lambda_\nu = \sqrt{2} G_F (n_\nu - n_{\bar{\nu}})$

Self-Suppression

- $\Delta\Lambda_{vac}$ ($\sim \Lambda_{vac}$) $\ll \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$: synchronization; no significant oscillations unless experiencing MSW resonance
  (Pastor et al 2001, 2002)

- Criterion for significant collective oscillations:
  $\underline{\Delta\Lambda_{vac} (\sim \Lambda_{vac}) \sim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle}$
  (Duan, Fuller & Qian, 2005)

$\Lambda_{vac} = \Delta m^2/2E$ $\Lambda_\nu = \sqrt{2} G_F (n_\nu - n_{\bar{\nu}})$

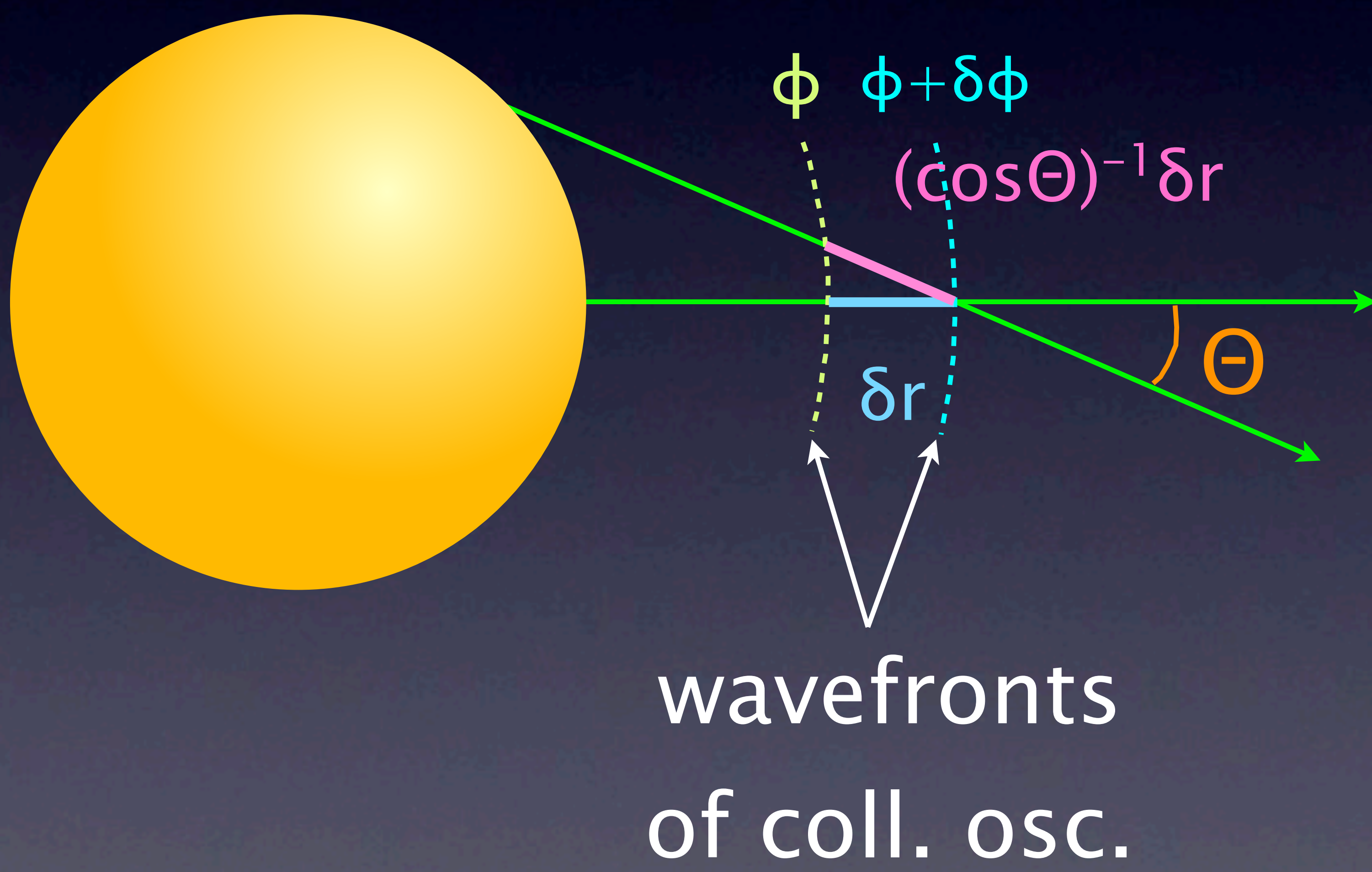
Matter Suppression

- $\Lambda_{mat} \gtrsim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$: suppression of collective oscillations?
- No. Uniform matter distribution does not suppress collective oscillations in the homogeneous and isotropic neutrino gas.
(Duan, Fuller & Qian, 2005)

$\Lambda_{mat} = \sqrt{2} G_F n_e$ $\Lambda_\nu = \sqrt{2} GF(n_\nu - n_{\bar{\nu}})$

Matter Suppression

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}\lambda}|\psi_{\nu,\mathbf{p}}\rangle = \hat{H}|\psi_{\nu,\mathbf{p}}\rangle$$



Matter Suppression

- Dispersion in $\Lambda_{mat} d\lambda/dr$: $\Delta\Lambda_{mat}$
- Criterion for significant collective oscillations: $\underline{\Delta\Lambda_{vac} + \Delta\Lambda_{mat} \sim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle}$
- $\Delta\Lambda_{mat} \gtrsim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle \Rightarrow$ suppression of collective oscillations (Esteban-Pretel et al, 2008)
- $\Delta\Lambda_{mat} \propto r^{-2}\,\rho(r) \Rightarrow$ suppression only at early-time and/or very close to NS

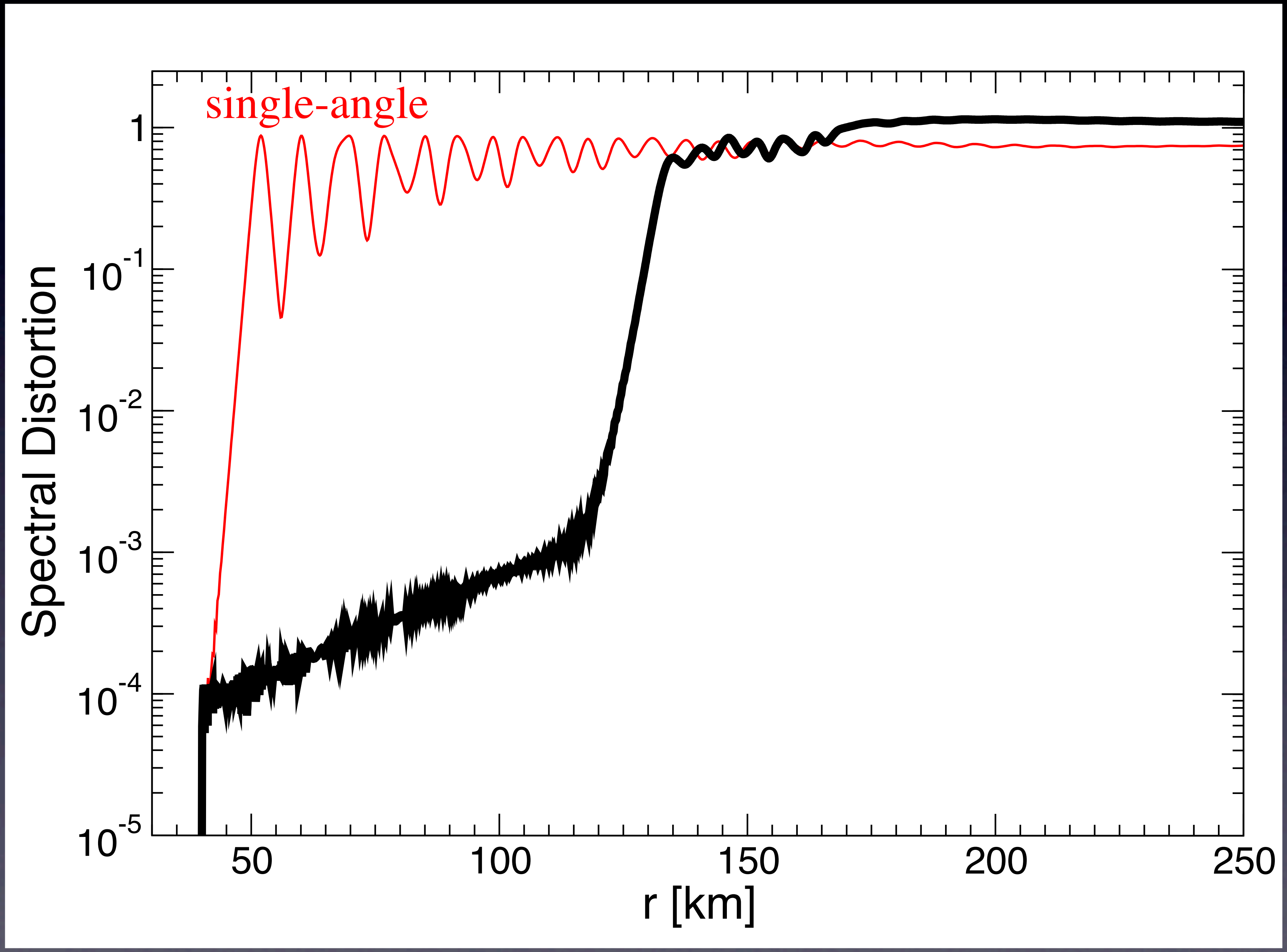
$\Lambda_{vac} = \Delta m^2/2E$ $\Lambda_{mat} = \sqrt{2} G_F n_e$ $\Lambda_\nu = \sqrt{2} GF(n_\nu - n_{\bar{\nu}})$

Multiangle Suppression

- $\Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$ depends on neutrino emission angle ⇒ dispersion in angles $\Delta\Lambda_\nu$ ($\sim\Lambda_\nu \langle 1\text{-}\cos\Theta \rangle$)
- Criterion for significant collective oscillations: $\underline{\Delta\Lambda_{vac} + \Delta\Lambda_{mat} + \Delta\Lambda_\nu \sim \Lambda_\nu \langle 1\text{-}\cos\Theta \rangle}$
or $\Delta\Lambda_{vac} \sim \Delta\Lambda_\nu$ for low matter density
(Duan & Friedland, 2010; Banerjee, Dighe & Raffelt, 2011)

$\Lambda_{vac} = \Delta m^2/2E$ $\Lambda_\nu = \sqrt{2} G_F (n_\nu - n_{\bar{\nu}})$

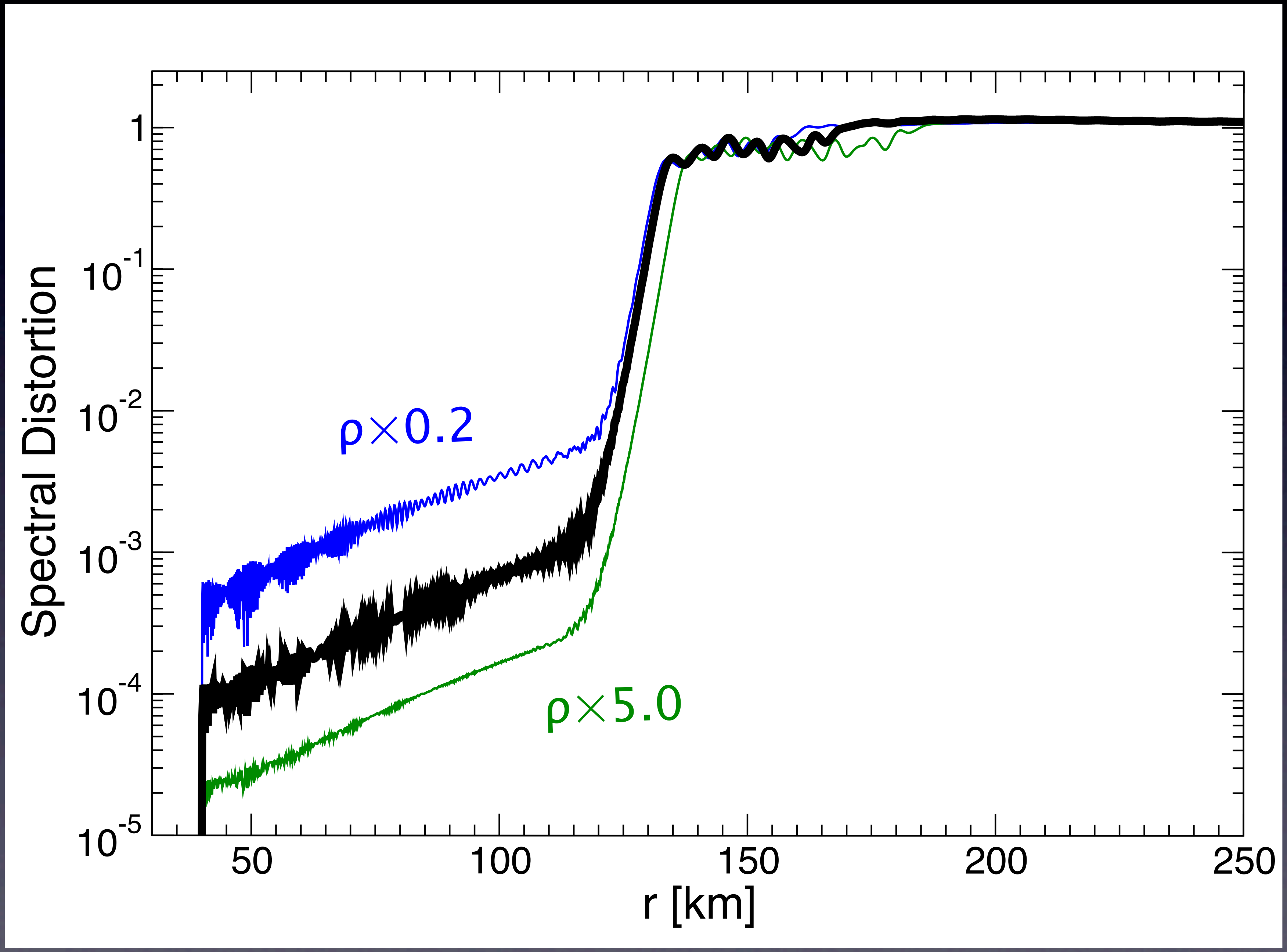
Multiangle Suppression



Using late-time spectra from Monte Carlo simulations (Keil et al 2002)

Duan & Friedland, PRL 106, 091101 (2011)

Multiangle Suppression



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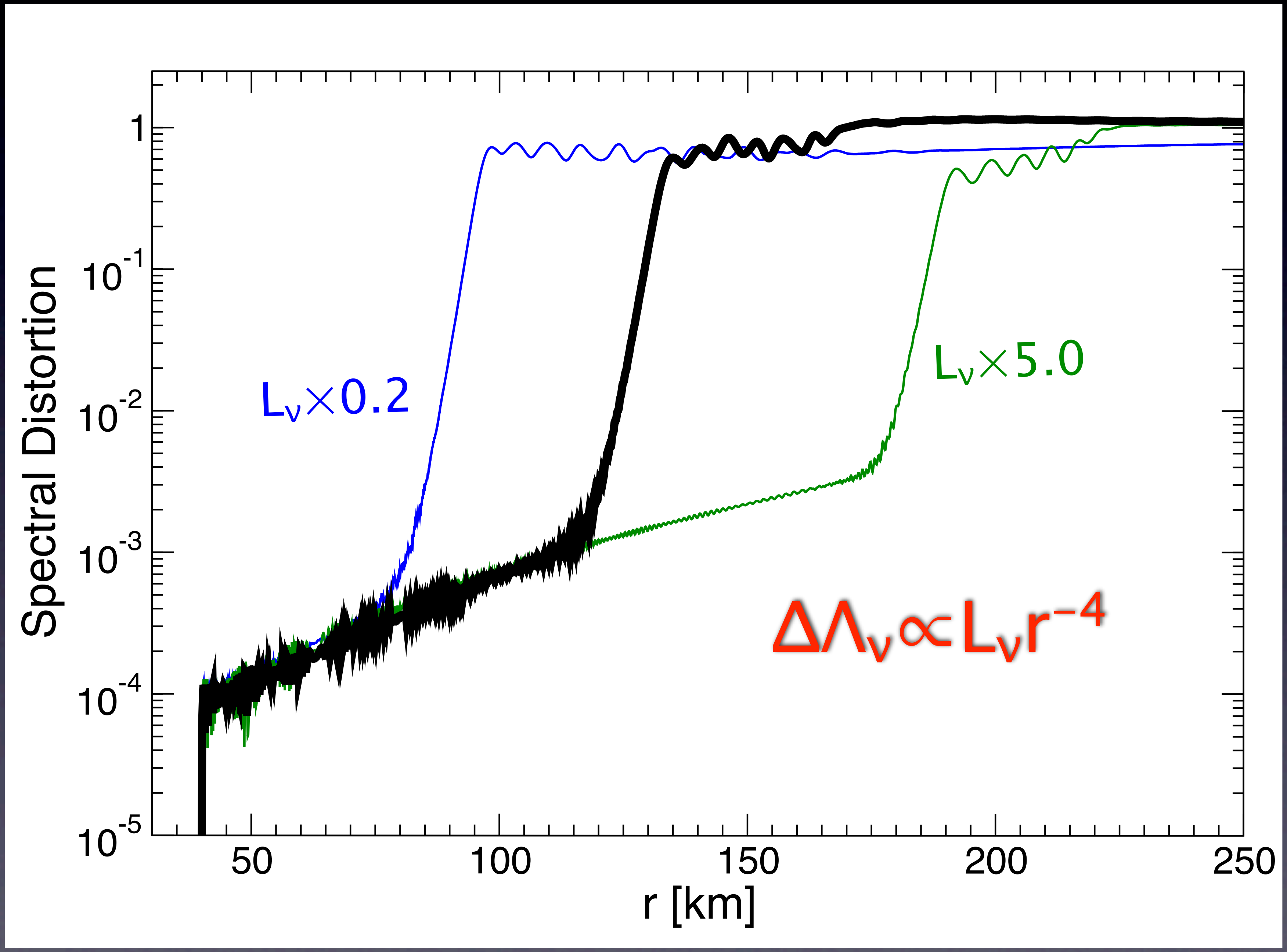

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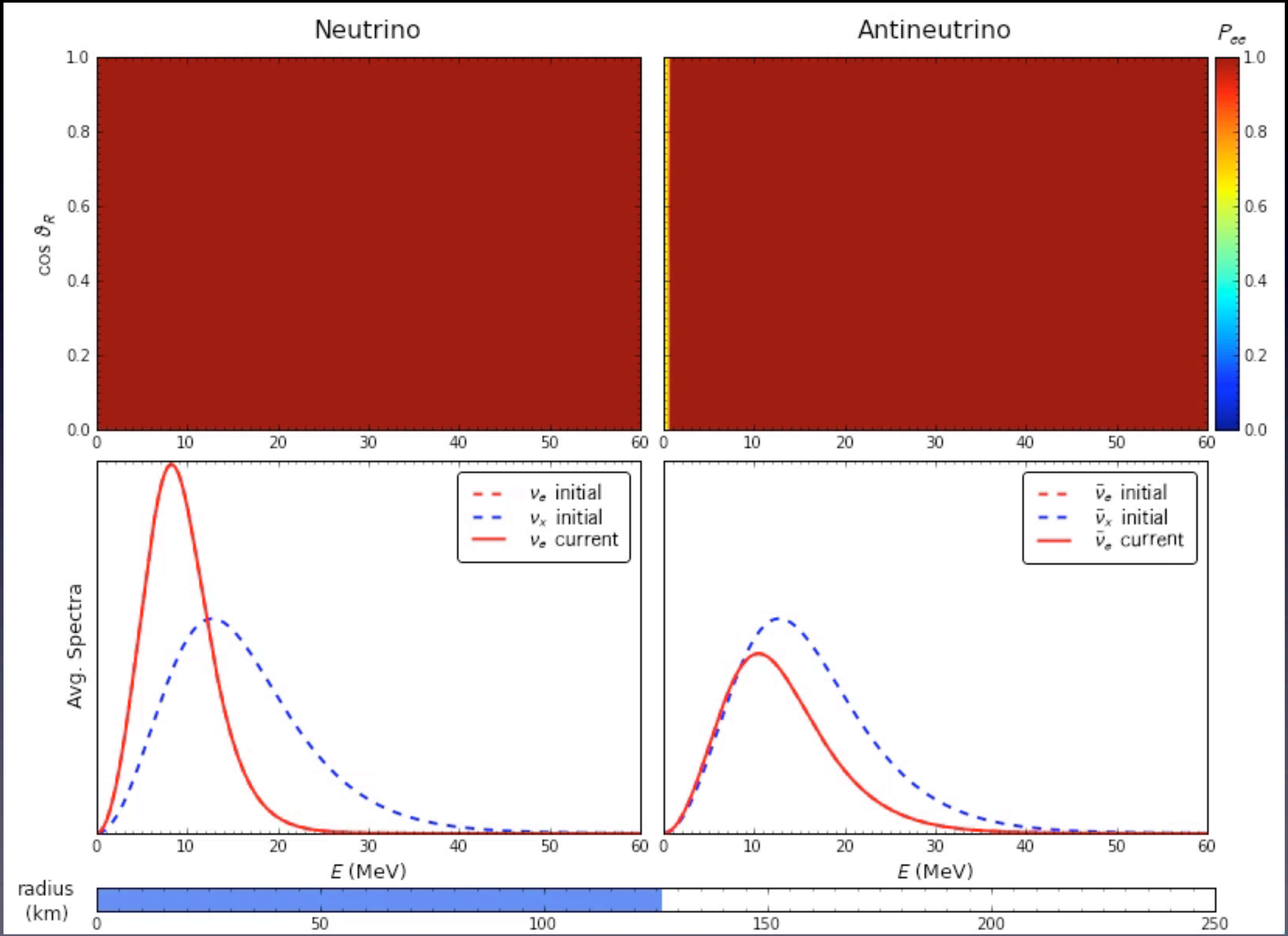
$$\langle L_{\nu_e}\rangle = 4.1\,\mathrm{foe},\ \langle L_{\bar\nu_e}\rangle = 4.3\,\mathrm{foe},\ \langle L_{\nu_x,\bar\nu_x}\rangle = 7.9\,\mathrm{foe}$$

$$\langle E_{\nu_e}\rangle = 9.4\,\mathrm{MeV},\ \langle E_{\bar\nu_e}\rangle = 13.0\,\mathrm{MeV},\ \langle E_{\nu_x,\bar\nu_x}\rangle = 15.8\mathrm{MeV}$$



Summaries

- Collective neutrino oscillations can occur in dense neutrino media — a result of intrinsic symmetry.
- Significant collective oscillations occur when neutrino densities are “moderate”.
- Be careful with the single-angle approximation.
- More than one collective modes exist — phase transition?