Oscillation Effects and Time Variation of the Supernova Neutrino Signal

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Core-collapse SN and neutrinos

Beyond ~1000 km the ν self-interactions become negligible.

Neutrino flavor oscillations continue to occur and are modulated by the presence of matter, the MSW effect.

As the star explodes the density profile changes.

The altered profile leads to variations of the neutrino flavor composition that emerges from the SN.

A detector here on Earth that can differentiate between flavors will observe an evolving neutrino signal.

From the time variation of the signal we can learn valuable information about the explosion and neutrinos.

Core-collapse SN

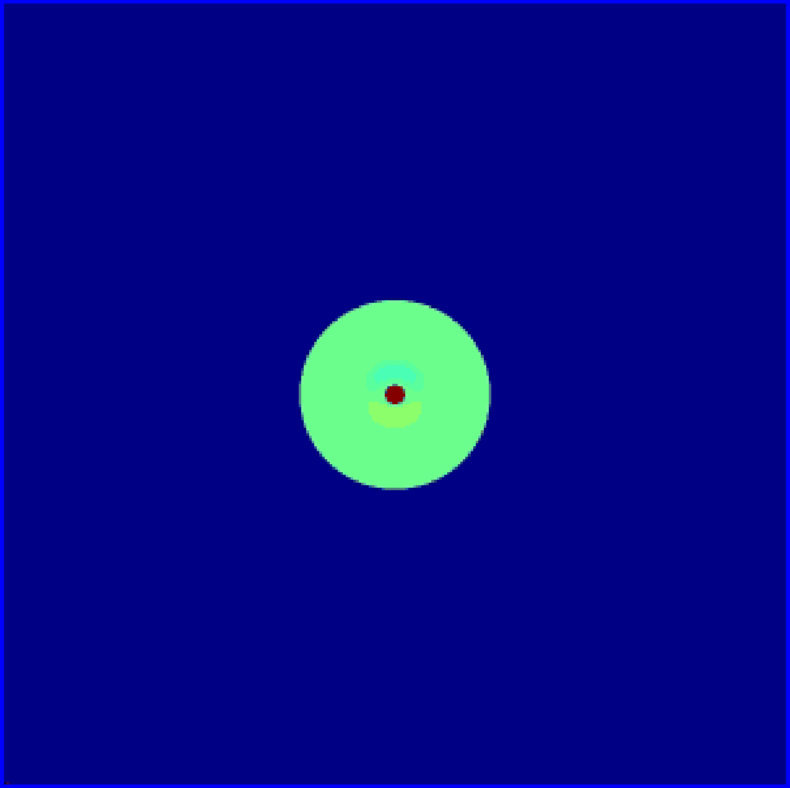
SN simulations are complex and computationally intensive.

Models of Iron core collapse SN do not currently explode (but may do soon): the shock stalls at $r \sim 200$ km.

Somehow the stalled shock is revived and the star explodes.

Pulsar velocities and the observation of polarized light indicate SN are aspherical.

Blondin, Mezzacappa & DeMarino, ApJ, **584**, 971 (2003) found that perturbations in the shape of the standing accretion shock can generate large $\ell = 1$, $\ell = 2$ moments.



http://www.phy.ornl.gov/tsi/pages/simulations.html

For our purposes what is occurring in the core is irrelevant.

The first neutrino resonance is at $r \sim 10^9$ cm for $E_\nu \sim 10$ MeV, the second is even further out.

All we need from a simulation is the density profile between 10^8 cm $< r < 10^{10}$ cm.

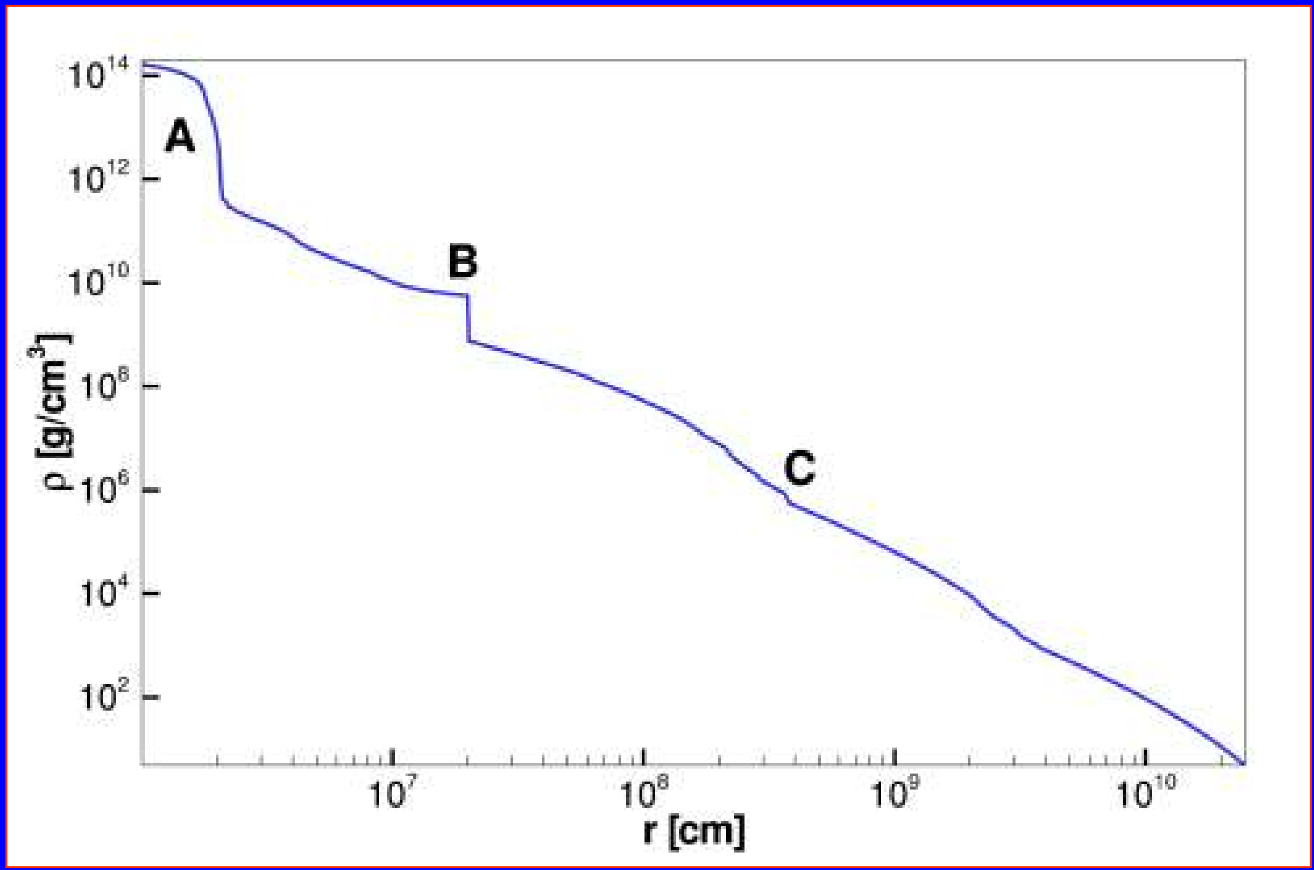
For this reason we employ a simplistic approach where we artificially create the explosion in the stellar mantel.

Our simulations use VH-1: a PPM hydro code.

We mapped into the code a 13.2 $M_{\odot}$ progenitor from Heger.

(www.ucolick.org/alex/stellarevolution)

On to the front we spliced a standing accretion shock profile and a γ = 2.5 polytropic core containing 3 $M_{\odot}$.

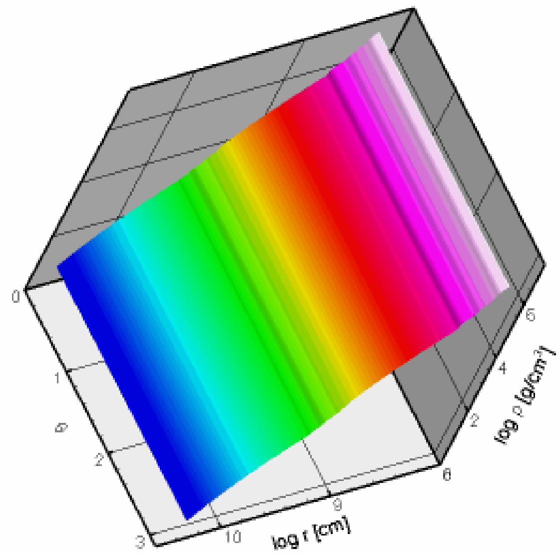


We heated the material according to:

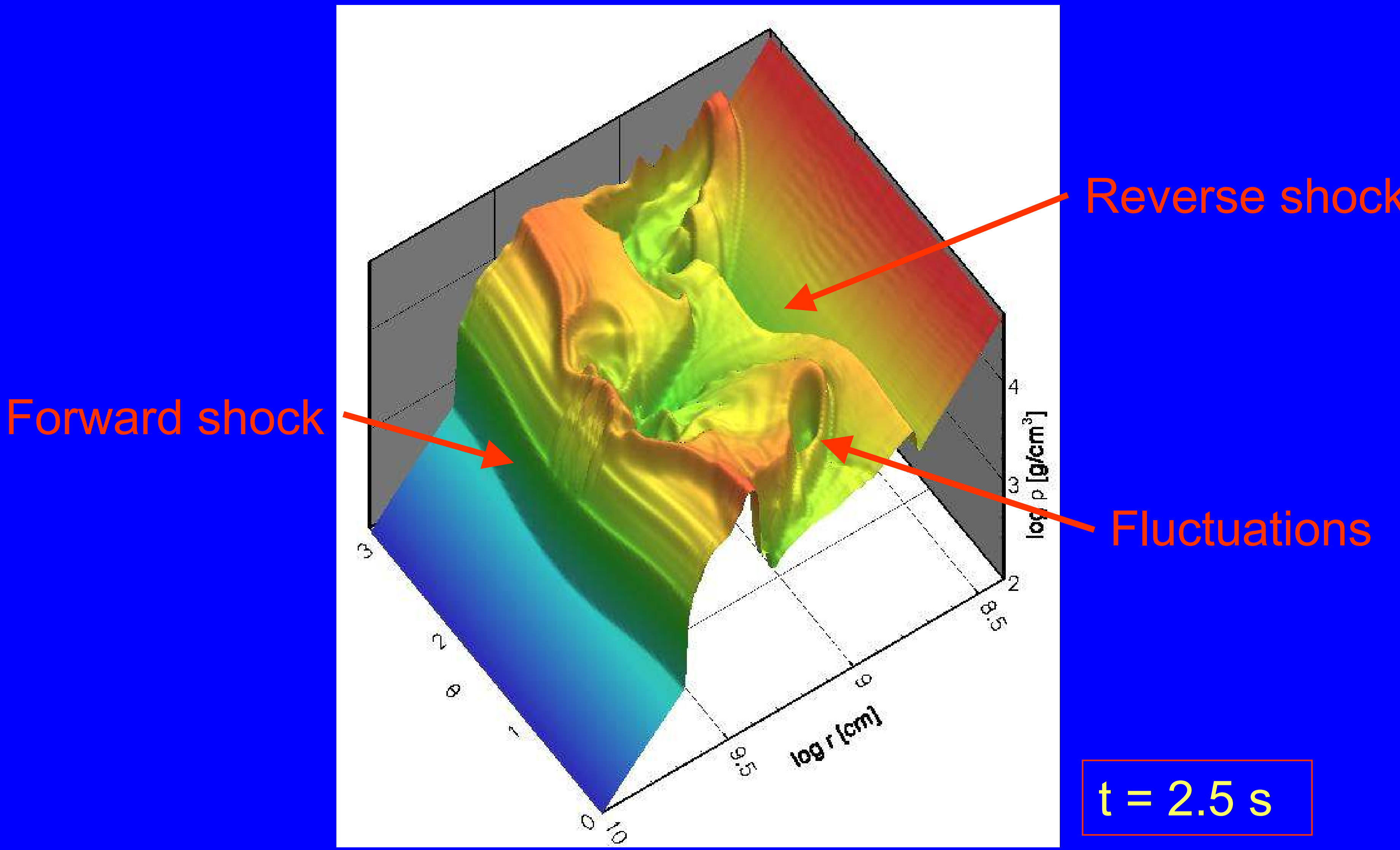
$$\frac{d^2 Q}{dt\,dV} \propto \frac{\rho}{r^2}\left(\frac{r-r_g}{r}\right) t\, e^{-t/\tau}\left(1+\tfrac{1}{2}\sin^2\theta\right)$$

r_g is the gain radius: r_g = 100 km.

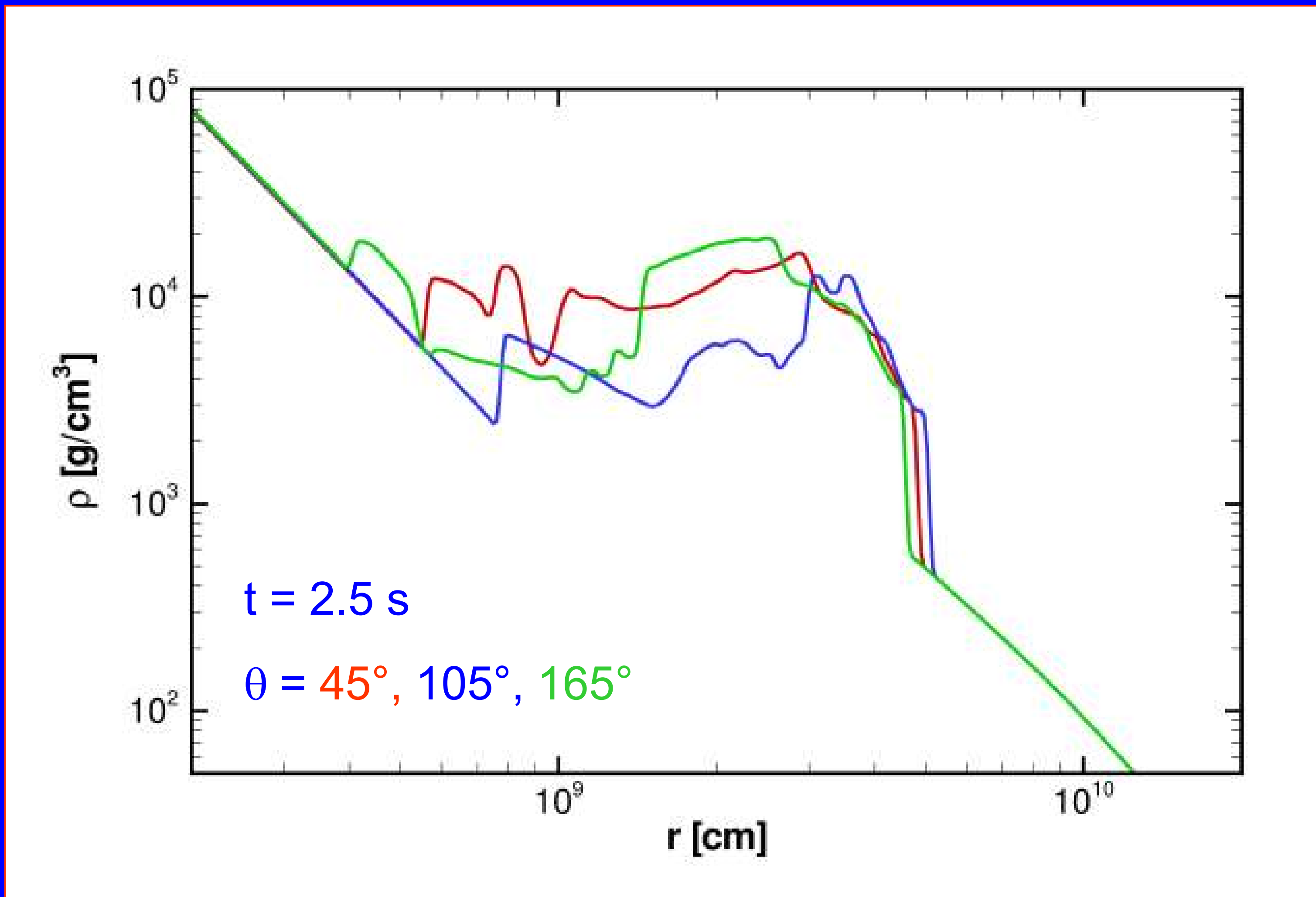
We constant of proportionality was set so that we achieved an explosion energy of $Q = 3 \times 10^{51}$ ergs.


log r [cm]
log ρ [g/cm^3]
θ
0
1
2
3
10
9
8
2
4
6

The 2D simulation has many prominent features that will affect neutrino passage through the SN.



The profile varies with θ.



1D models

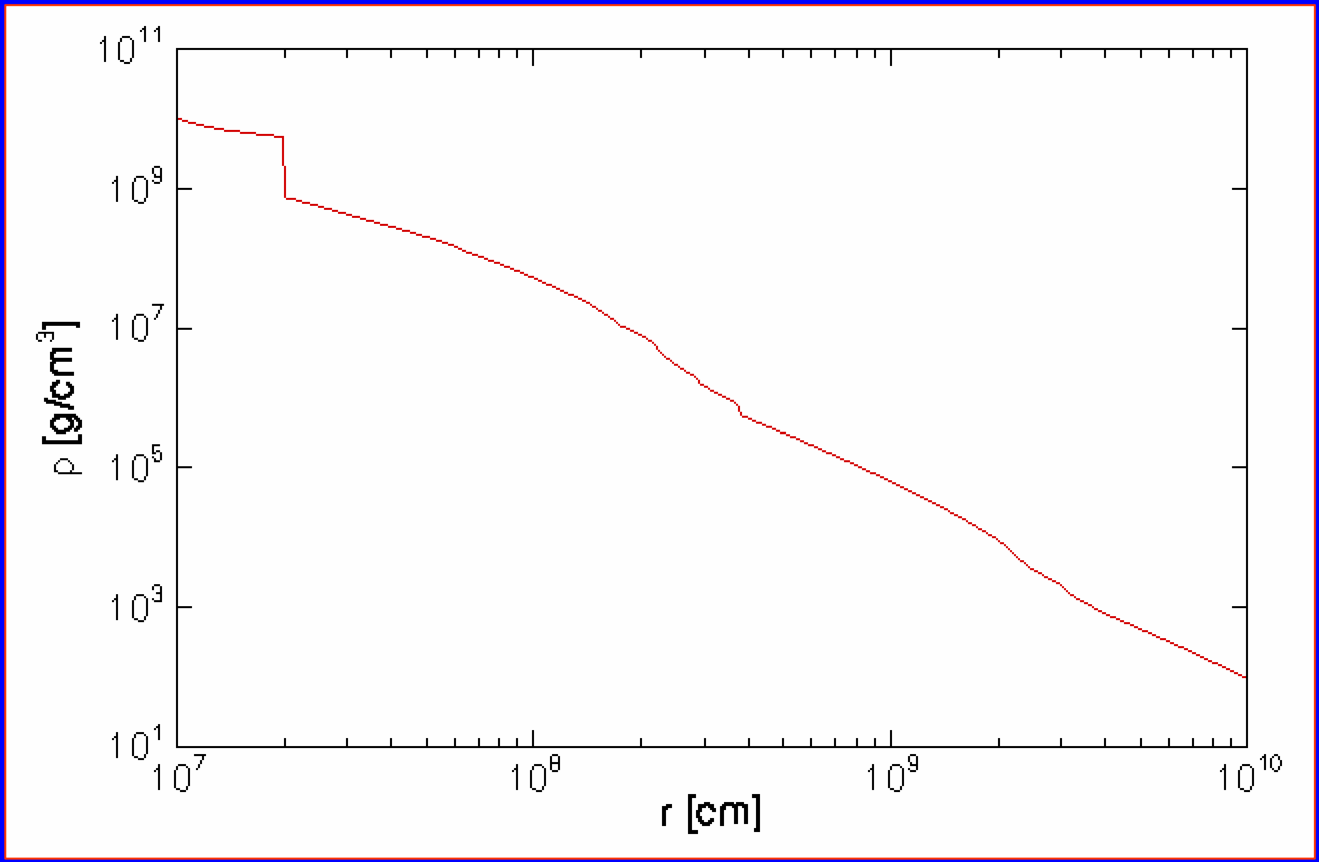
We supplemented our 2D model with numerous 1D models.

These are quicker to compute and allow us to focus upon particular features.

Again, to make the star explode, we heated the material according to:

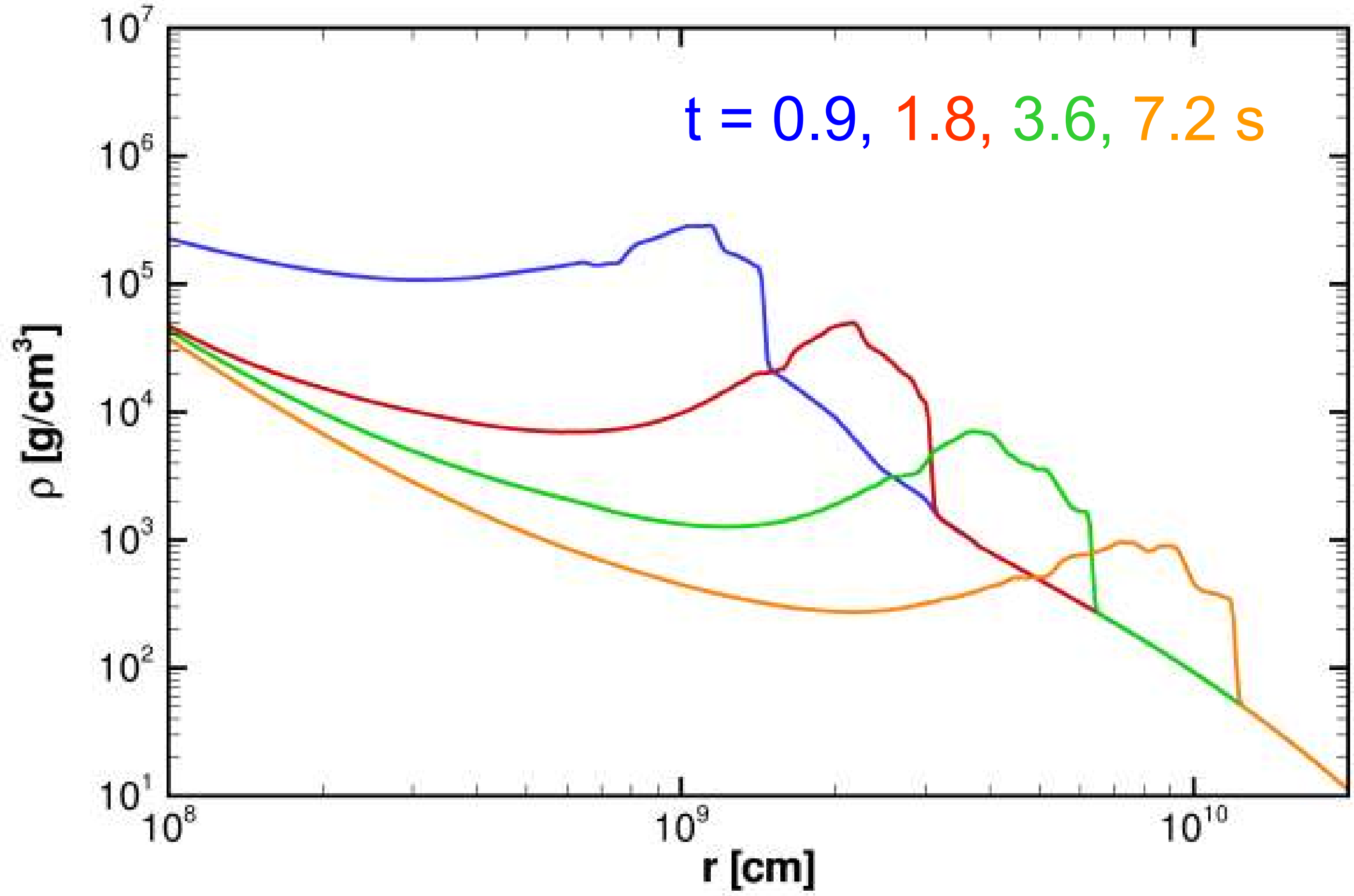
$$\frac{d^2Q}{dt\,dV} \propto \frac{\rho}{r^2}\left(\frac{r - r_g}{r}\right) t\, e^{-t/\tau}$$

and tracked the amount of energy, *Q*, we inserted.



$Q = 1.66 \times 10^{51}$ erg

Weak explosions possess only a forward shock.

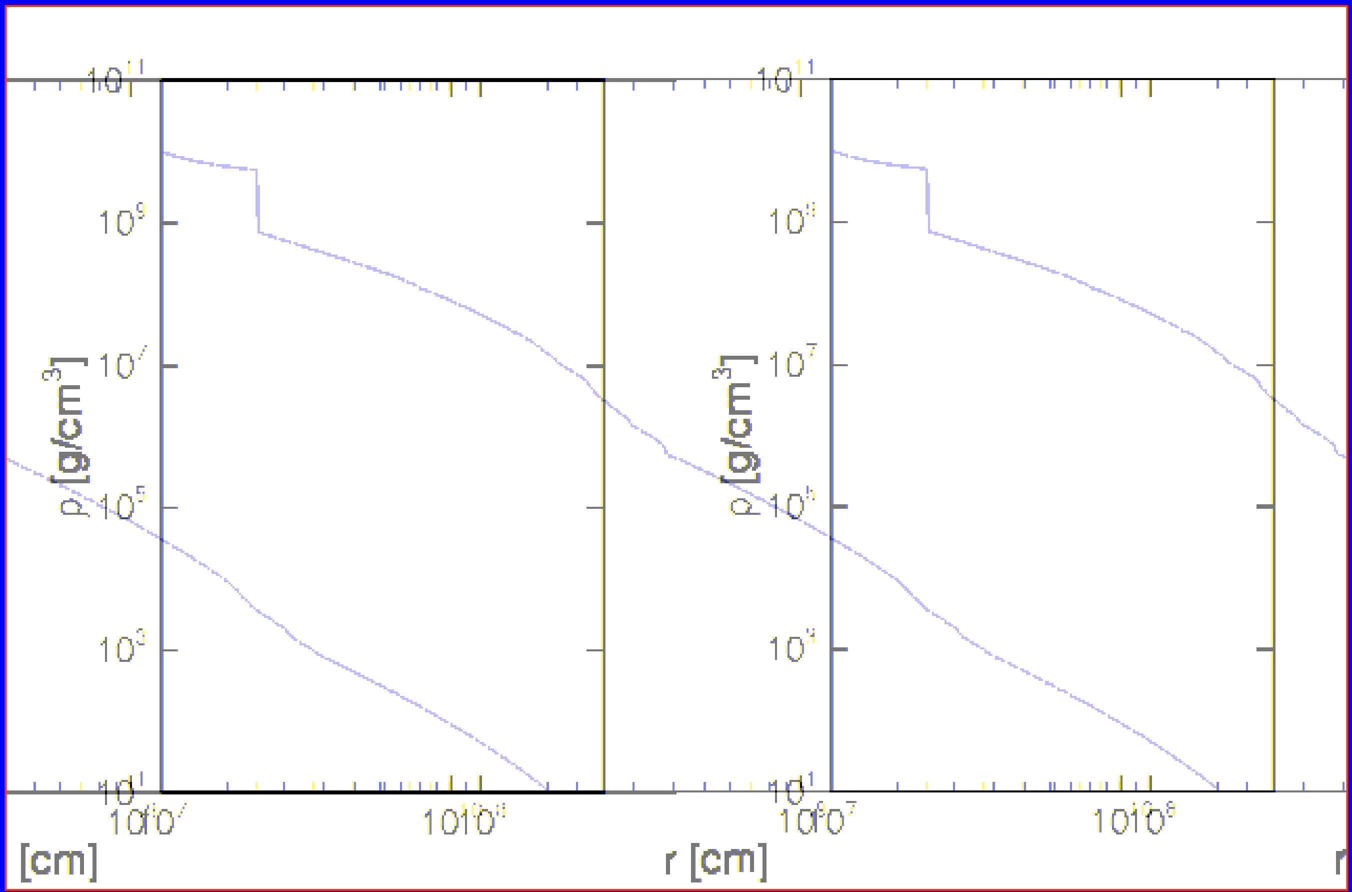


$Q = 3.07 \times 10^{51}$ erg

As the deposited energy increases a cavity forms behind the shock.

Yet more energy leads to the formation of a reverse shock.

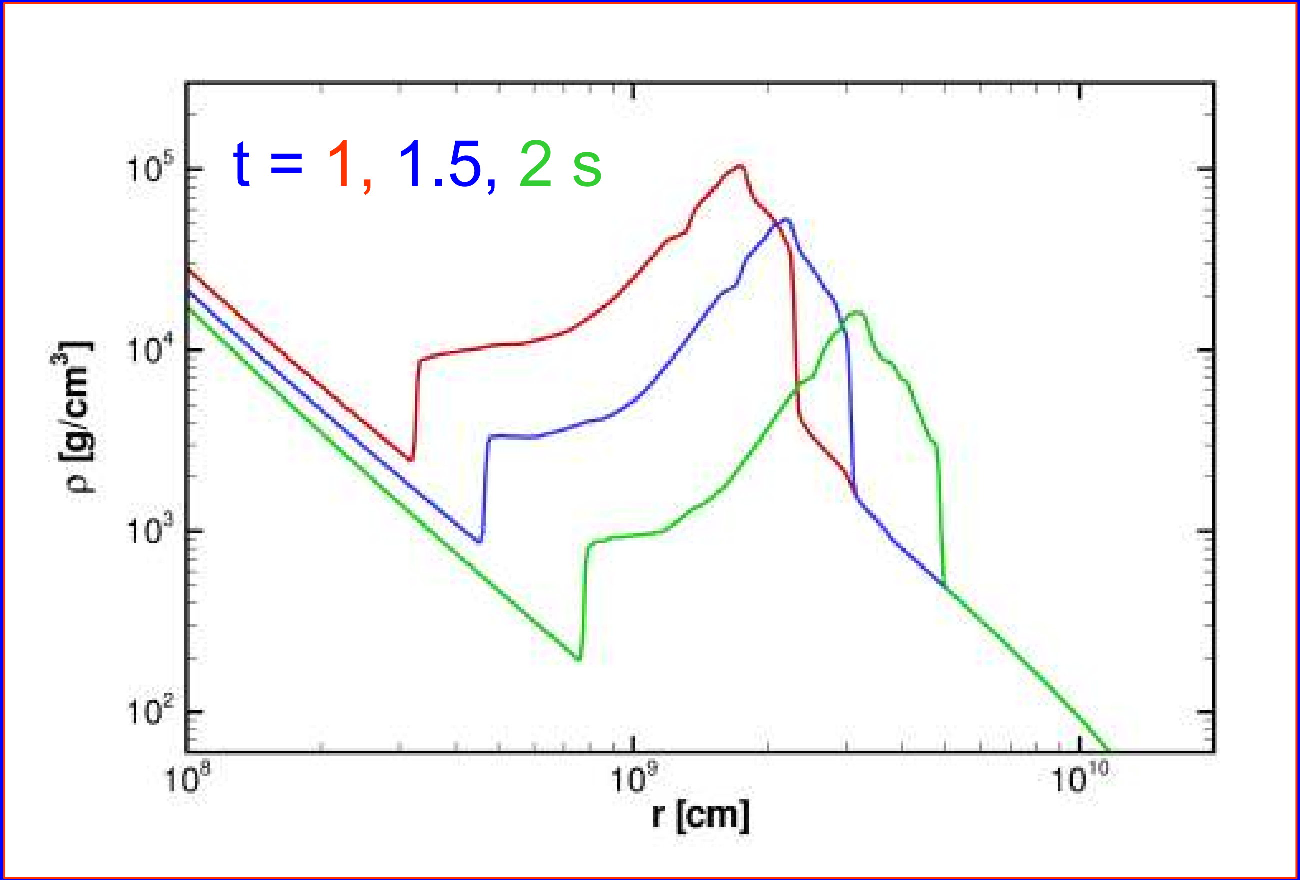
The reverse shock eventually stalls as the wind abates and then retreats back to the core where it is reflected.



$Q = 3.36 \times 10^{51}$ erg

Seen also by Thomas et al, JCAP, **9** (2004) 15 and Arcones, Janka & Scheck, A&A, **467** (2007) 1227.

For very large Q the reverse shock can penetrate to lower densities than that in front of the forward shock.



$Q = 4.51 \times 10^{51}$ erg

Actual SN may struggle to reach such energies.

Calculating the MSW effect

The neutrinos do something weird as they propagate through the SN.

Neutrinos born as, say, electron type are not necessarily electron type when they reach the detector.

There are two different sets of basis states:

- the mass eigenstates $|\nu_1\rangle$, $|\nu_2\rangle$, $|\nu_3\rangle$ of the Hamiltonian.
- the flavor states $|\nu_e\rangle$, $|\nu_\mu\rangle$, $|\nu_\tau\rangle$ produced in Weak interactions and seen in detectors.

The relationship between the bases is described by the mixing matrix

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta} & c_{12}c_{23} - s_{12}s_{13}s_{23}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta} & c_{13}c_{23} \end{pmatrix}$$

In matter the electron flavor picks up a potential energy.

$$V_{ee}(\boldsymbol{r},t) = \sqrt{2}G_F \frac{Y_e(\boldsymbol{r},t)\rho(\boldsymbol{r},t)}{m_N}$$

For neutrinos of a particular energy there is a density at which the flavor content varies most rapidly.

There are two resonances:

- the 'H' resonance with the atmospheric $|\delta m^2| \sim 3 \times 10^{-3}$ eV^2 but unknown sign, and the unknown, but small, θ_{13},
- the 'L' resonance with the solar mixing parameters.

Because the sign of the atmospheric δm^2 is not known the hierarchy is uncertain.

The wavefunction at Earth is related to the wavefunction emitted by the proto-neutron star by a matrix S.

$$\left|\nu(\oplus)\right\rangle = S(\oplus)\left|\nu(0)\right\rangle \qquad \left|\bar{\nu}(\oplus)\right\rangle = \bar{S}(\oplus)\left|\bar{\nu}(0)\right\rangle$$

In the matter basis we can write S as

$$S = \begin{pmatrix} \alpha_L & \beta_L & 0 \\ -\beta_L^* & \alpha_L^* & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \alpha_H & \beta_H \\ 0 & -\beta_H^* & \alpha_H^* \end{pmatrix} \qquad \bar{S} = \begin{pmatrix} \bar{\alpha}_H & 0 & \bar{\beta}_H \\ 0 & 1 & 0 \\ -\bar{\beta}_H^* & 0 & \bar{\alpha}_H^* \end{pmatrix}$$

and we define three crossing probabilities

$$P_H = 1 - \left|\alpha_H\right|^2 = \left|\beta_H\right|^2 \qquad P_L = 1 - \left|\alpha_L\right|^2 = \left|\beta_L\right|^2$$

$$\bar{P}_H = 1 - \left|\bar{\alpha}_H\right|^2 = \left|\bar{\beta}_H\right|^2$$

The flux of matter/mass state *i* at Earth is

$$F_i(\oplus) = \sum_j |S_{ij}|^2 F_j(0) \quad \overline{F}_i(\oplus) = \sum_j |\overline{S}_{ij}|^2 \overline{F}_j(0)$$

Any coherence is lost so the flux of flavor state α is

$$F_\alpha = \sum_i |U_{\alpha i}|^2 F_i \quad \overline{F}_{\overline{\alpha}} = \sum_{\bar{i}} |\overline{U}_{\overline{\alpha}\bar{i}}|^2 \overline{F}_{\bar{i}}$$

The flavor fluxes at Earth can be written in terms of the emitted flavor spectra by introducing the survival probabilities

$$F_{\nu_e}(d,E) = \frac{1}{4\pi d^2}\left[p(E)\Phi_{\nu_e}(0,E) + \left(1-p(E)\right)\Phi_{\nu_x}(0,E)\right]$$

$$F_{\overline{\nu}_e}(d,E) = \frac{1}{4\pi d^2}\left[\overline{p}(E)\Phi_{\overline{\nu}_e}(0,E) + \left(1-\overline{p}(E)\right)\Phi_{\nu_x}(0,E)\right]$$

$$4F_{\nu_x}(d,E) = \frac{1}{4\pi d^2}\left[\begin{array}{l} \left(1-p(E)\right)\Phi_{\nu_e}(0,E) + \left(1-\overline{p}(E)\right)\Phi_{\overline{\nu}_e}(0,E) \\ \qquad + \left(2+p(E)+\overline{p}(E)\right)\Phi_{\nu_x}(0,E) \end{array}\right]$$

Putting it all together the electron neutrino and electron antineutrino survival probabilities, *p* and $\bar{p}$, are

$$p = \begin{cases} |U_{e1}|^2 P_L P_H + |U_{e2}|^2 (1-P_L) P_H + |U_{e3}|^2 (1-P_H) & NH \\ |U_{e1}|^2 P_L (1-P_H) + |U_{e2}|^2 (1-P_L)(1-P_H) + |U_{e3}|^2 P_H & IH \end{cases}$$

$$\bar{p} = \begin{cases} |\bar{U}_{e1}|^2 (1-\bar{P}_H) + |\bar{U}_{e3}|^2 \bar{P}_H & NH \\ |\bar{U}_{e1}|^2 \bar{P}_H + |\bar{U}_{e3}|^2 (1-\bar{P}_H) & IH \end{cases}$$

These formula can be simplified:

- the L resonance is always adiabatic so P_L is always zero
- $|U_{e3}|$ is small.

	Normal	Inverted
p	$P_H \sin^2\theta_\odot$	$\sin^2\theta_\odot$
$\bar{p}$	$\cos^2\theta_\odot$	$\bar{P}_H \cos^2\theta_\odot$

The forward shock

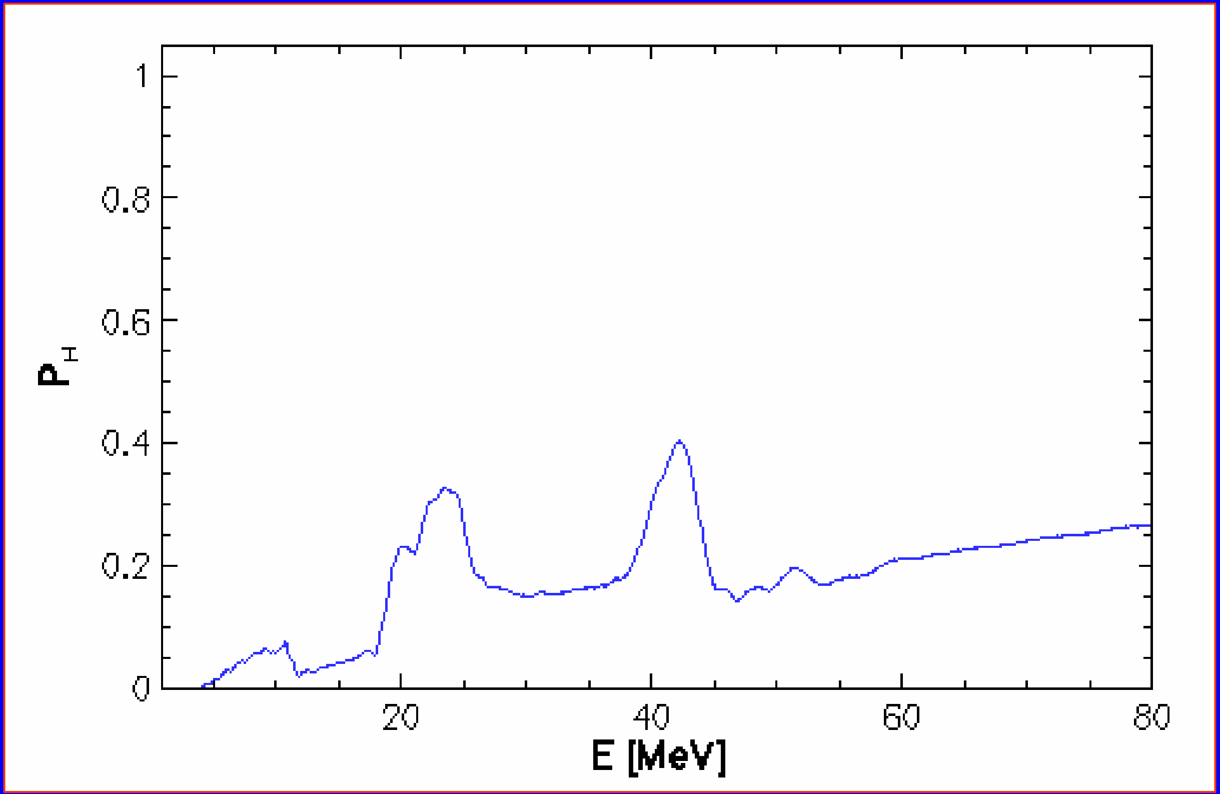
Schirato & Fuller, astro-ph/0205390, showed that the revived forward shock can reach the H resonance.

The forward shock leads to the first change in the ν signal.

$Q = 1.66 \times 10^{51}$ erg

$\delta m^2 = 3 \times 10^{-3}$ eV^2

$\sin^2 2\theta_V = 4 \times 10^{-4}$



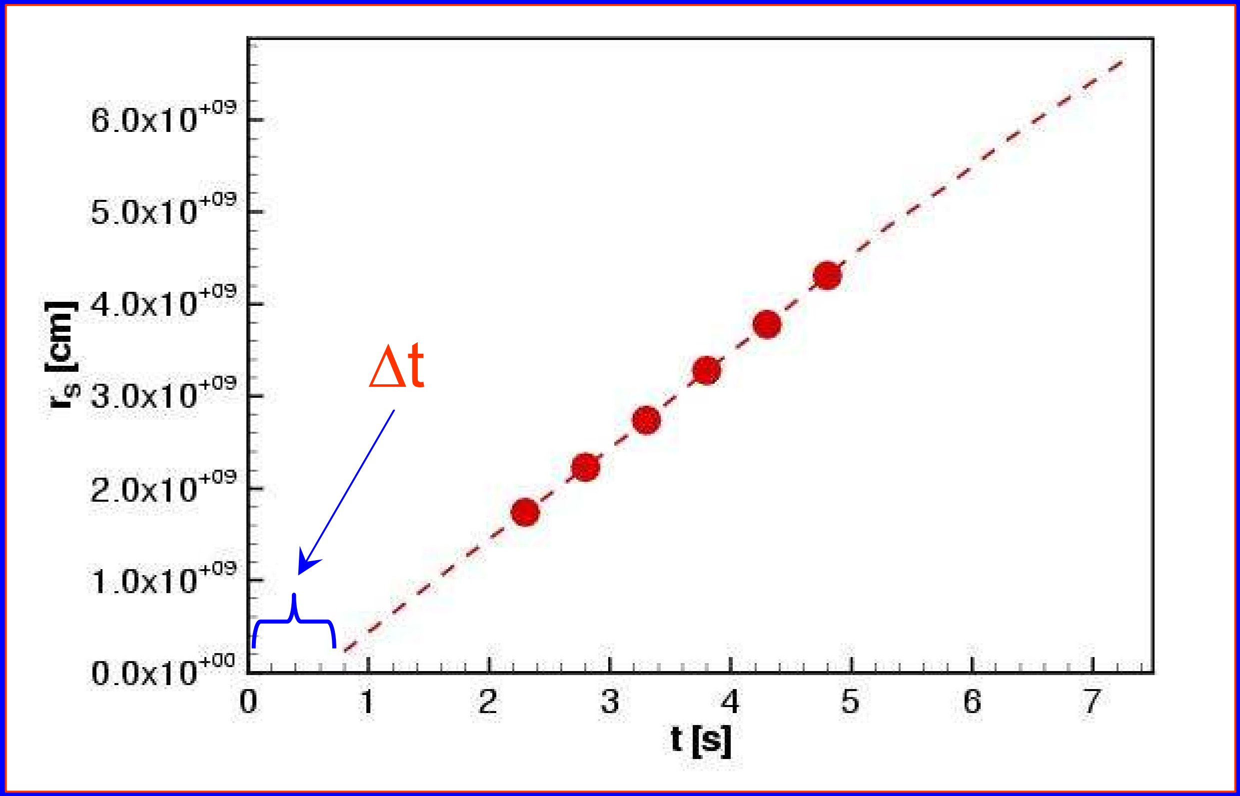
The shock acts as a (semi-transparent) window allowing us to see ~30% of the ν_e, or ~70% of the anti-ν_e, flux.

The ‘width’, ΔE, of the shock feature indicates the density jump and is related to the strength of the shock.

$$\frac{\Delta E}{E_S - \Delta E} = \frac{\Delta\rho}{\rho} = \frac{2(M^2-1)}{2+(\gamma-1)M^2}$$

M is the Mach number and γ the ratio of specific heats, E_S is the highest energy affected by the shock.

If we know the progenitor we can determine the shock position and measure the shock speed.



It may be possible to extrapolate the shock position back to the proto-neutron star.

A noticeable time lag will indicate that the shock stalled and for how long.

Reverse Shock

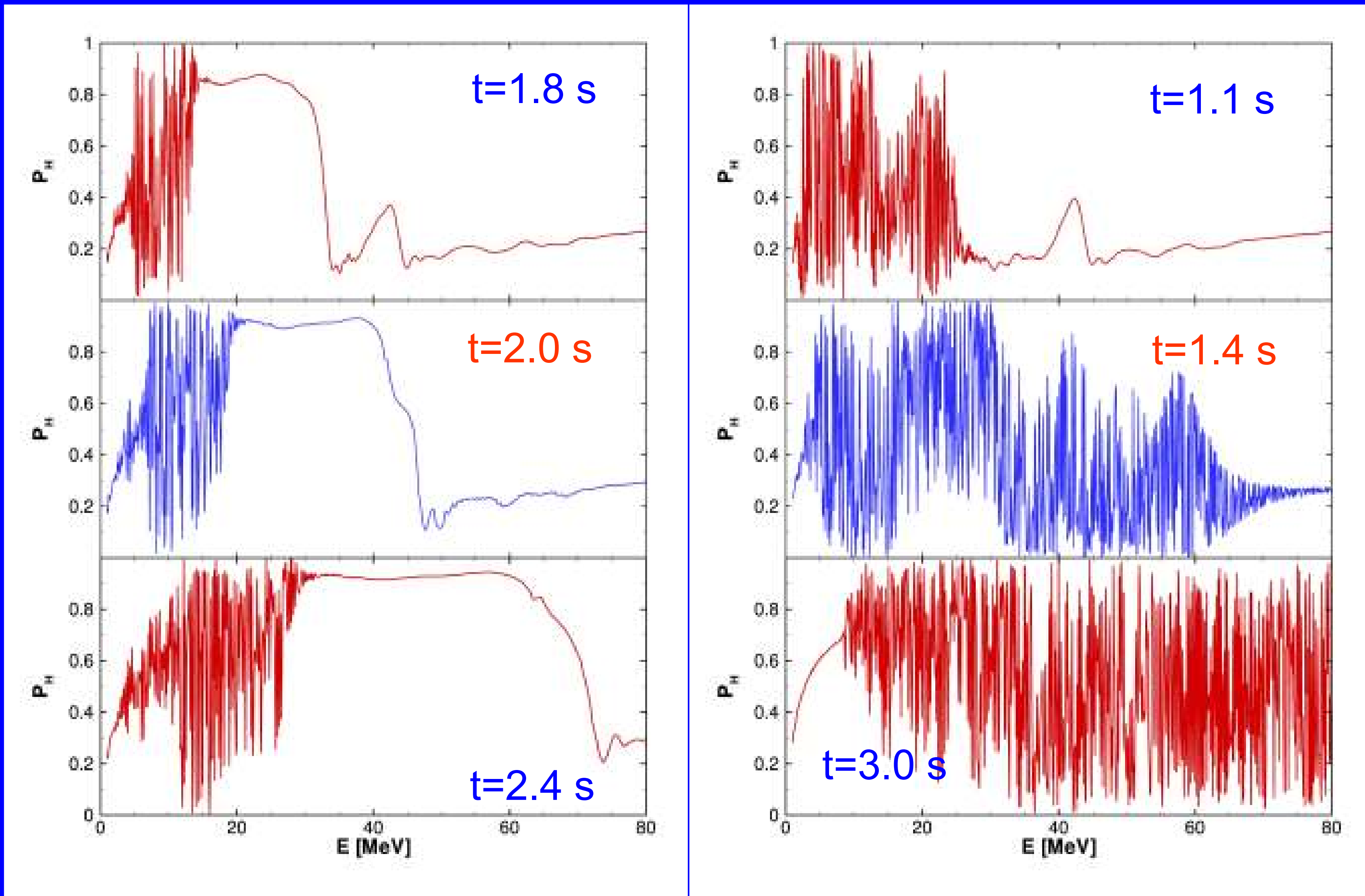
Tomàs et al, JCAP, **9**, 15 (2004) pointed out the reverse shock will also affect the neutrinos.

Dasgupta & Dighe, astro-ph/0510219 showed that phase effects can occur due to the multiple resonances.

- see also Fogli et al., PRD, **68** (2003), 033005,
- and Kneller & McLaughlin, PRD, **73** (2006) 056003

Phase effects lead to oscillations of the crossing probability as a function of the neutrino energy.

$\delta m^2 = 3 \times 10^{-3}\ eV^2$, $\sin^2 2\theta_{13} = 4 \times 10^{-4}$



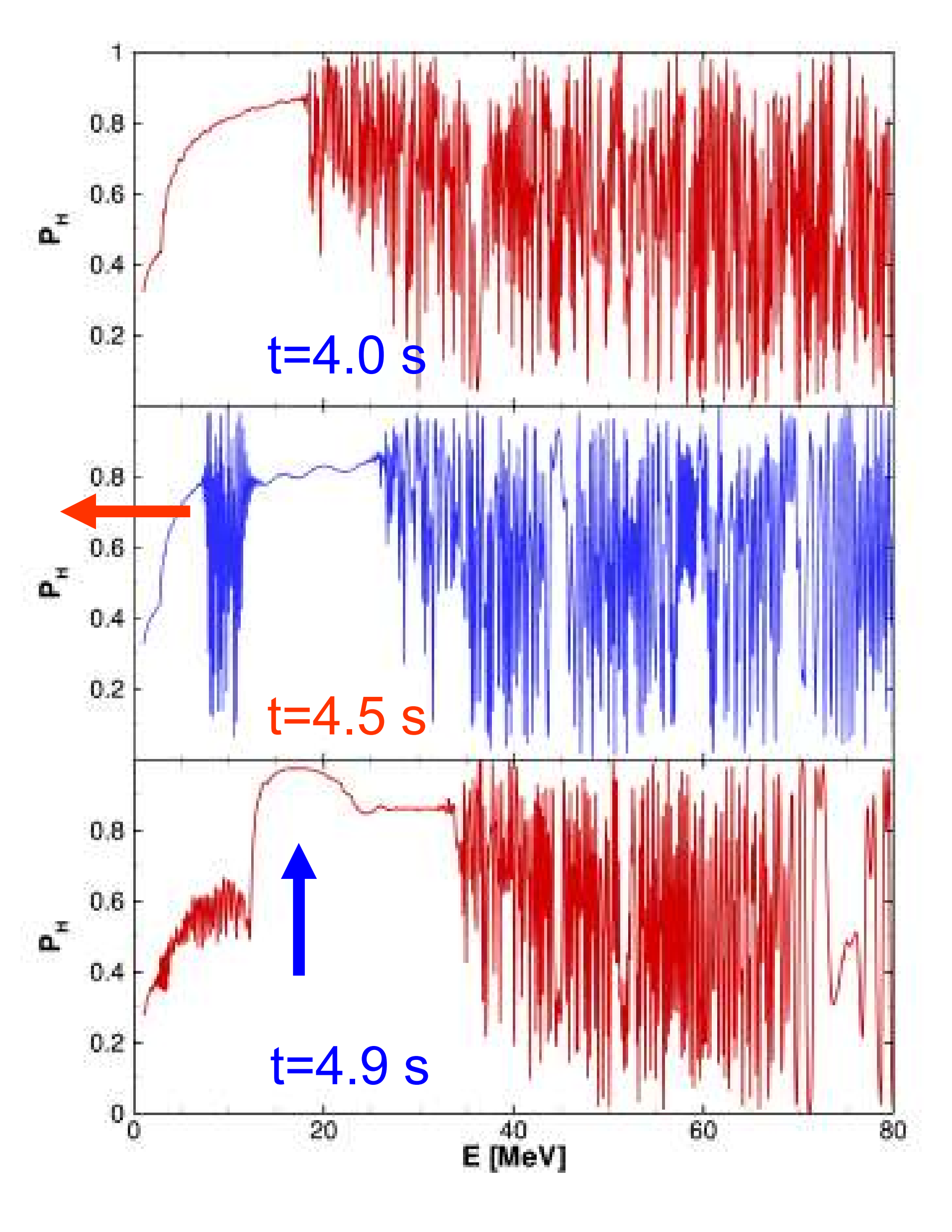
$Q = 3.36 \times 10^{51}$ erg

$Q = 4.51 \times 10^{51}$ erg

Eventually the reverse shock turns around.

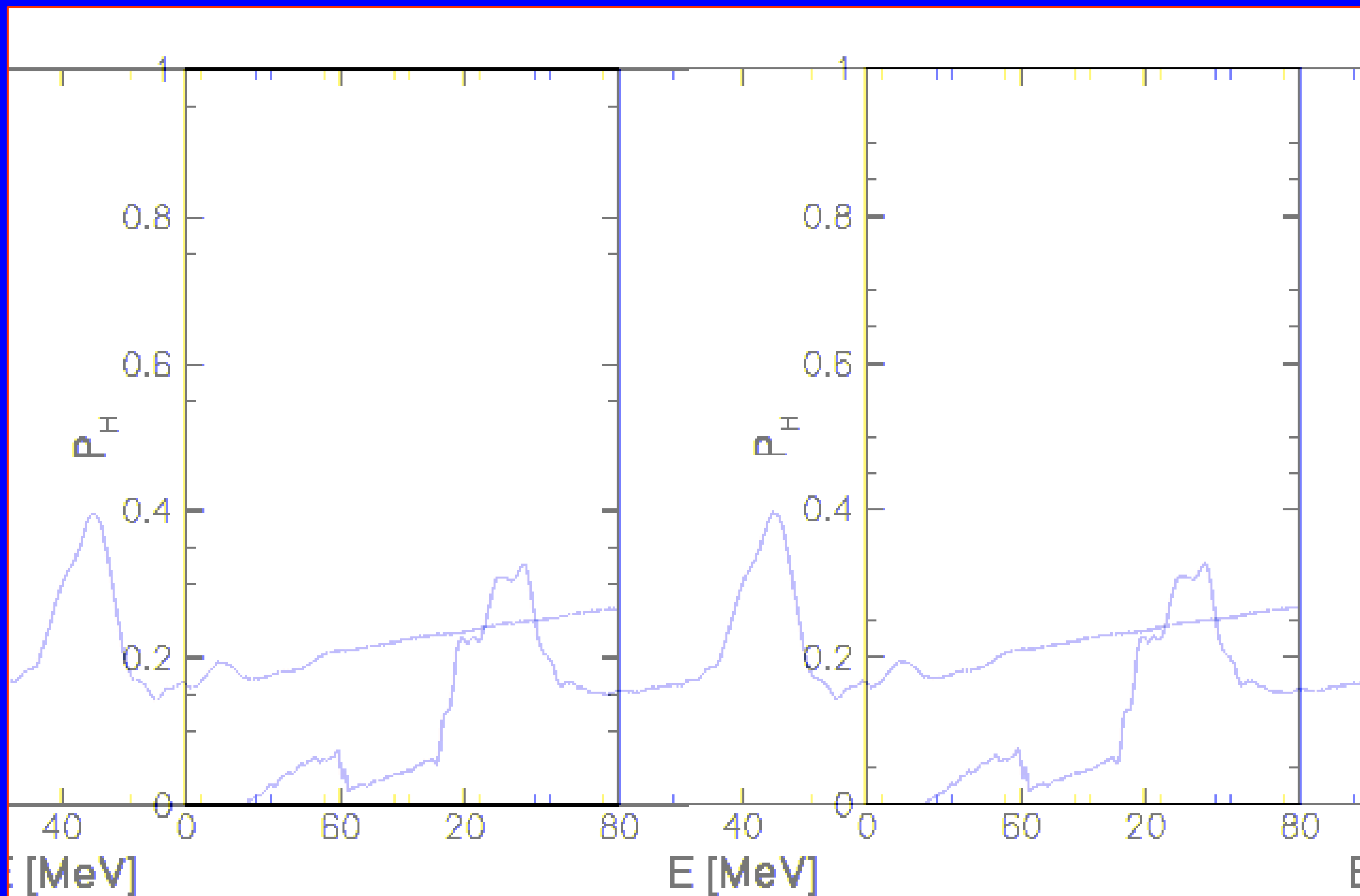
It's effects move down through the spectrum.

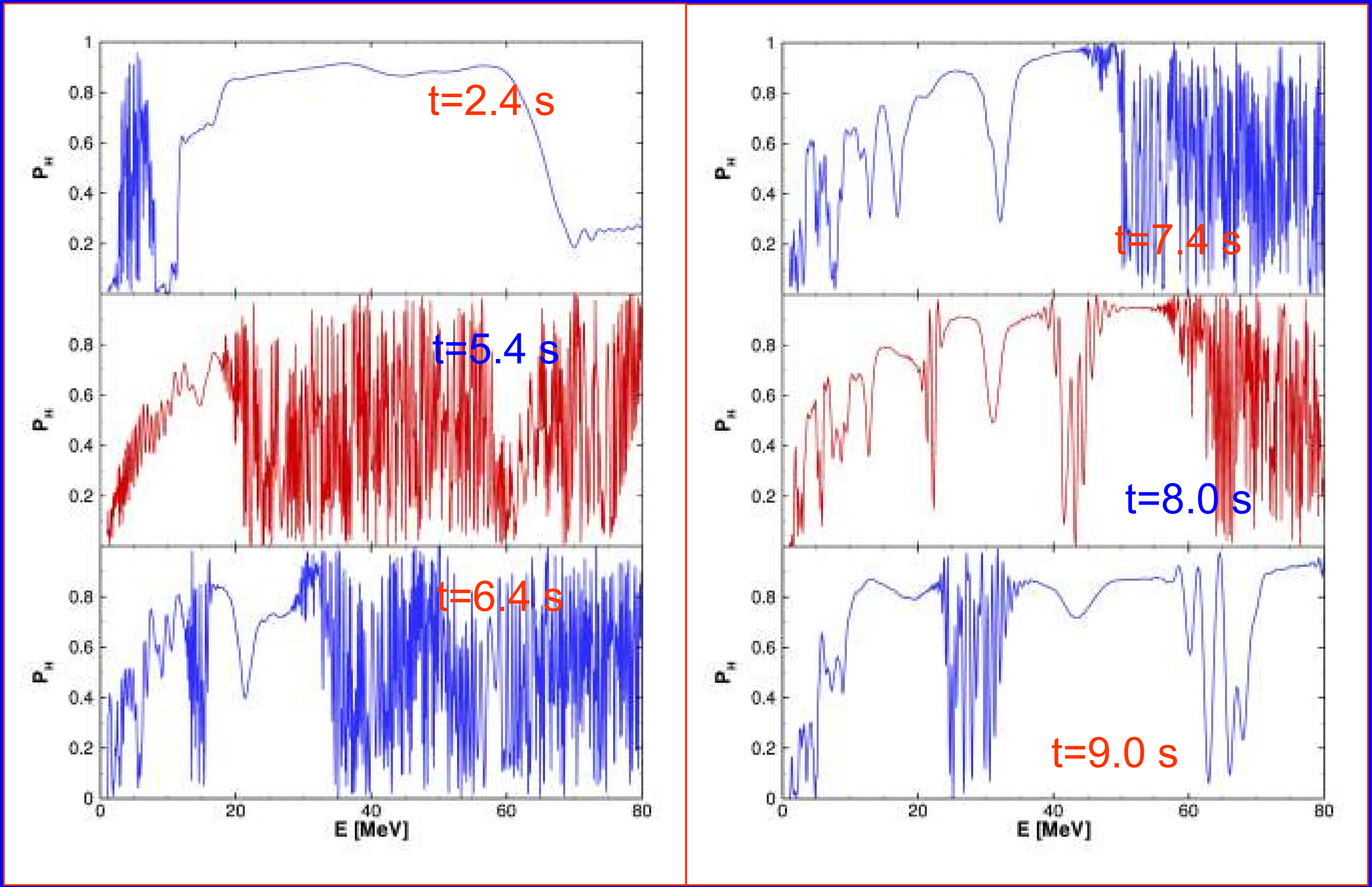
And then it was reflected.



$Q = 4.51 \times 10^{51}$ erg

In 2D phase effects are again present but at late times they are qualitatively different.





However you expect P_H ~ ½.

Friedland & Gruzinov, arXiv:astro-ph/0607244

Detector Signals

The number and size of neutrino detectors have both increased in scale since 1987.

Many are water Cherenkov detectors that are best suited for electron antineutrinos.

Other technologies capable of detecting the other flavors have been demonstrated.

To determine what a given detector will observe we need three extra pieces of information:

- the initial fluxes,
- the cross sections for the reactions in the detector,
- the energy resolution of the detector.

Initial Fluxes

We adopt an emitted spectra of the form

$$\Phi_\nu(0,E) = \frac{L_\nu(\alpha+1)^{(\alpha+1)}}{\langle E_\nu\rangle^2\Gamma(\alpha+1)}\left(\frac{E}{\langle E_\nu\rangle}\right)^\alpha \exp\left(-(\alpha+1)\frac{E}{\langle E_\nu\rangle}\right)\exp\left(-t/\tau\right)$$

Keil, Raffelt, Janka, ApJ, **590** (2003), 871

α is the pinch parameter, L_ν is the luminosity, $\langle E_\nu\rangle$ is the mean energy and τ a decay timescale.

The luminosities are set to:

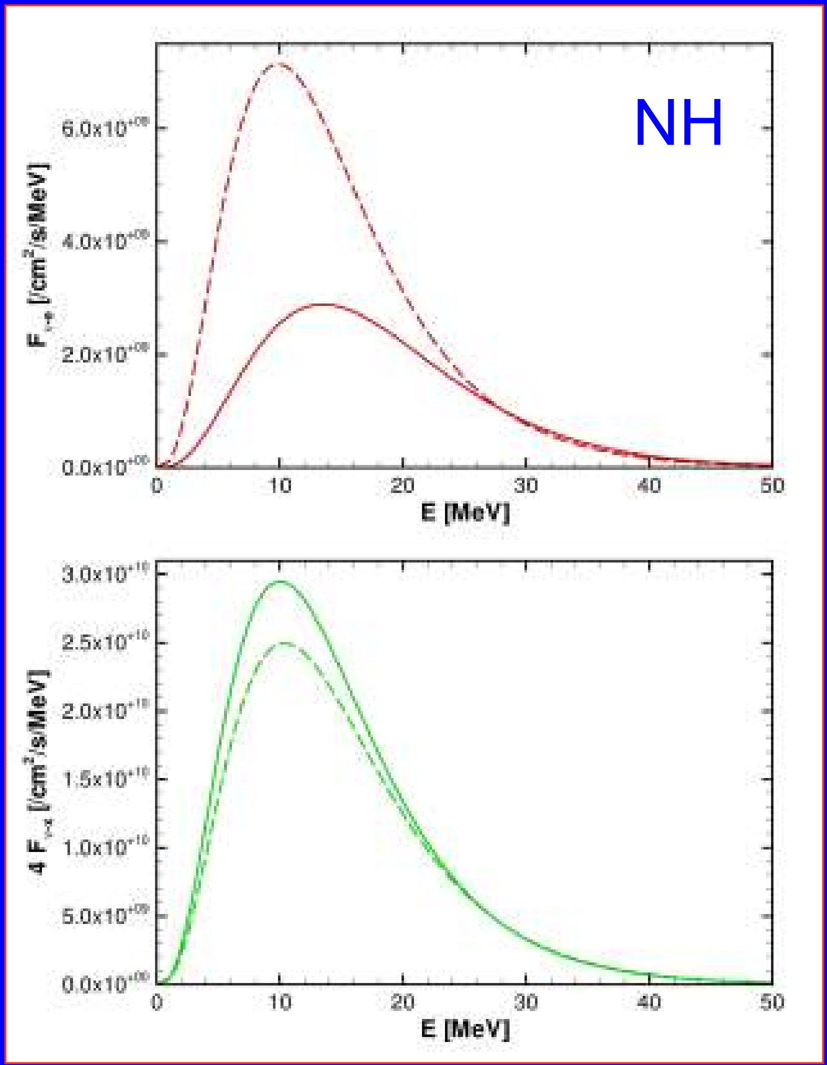
$L_e = L_{\bar{e}} = 6 \times 10^{52}$ erg/s, $L_x = 2 \times 10^{52}$ erg/s,

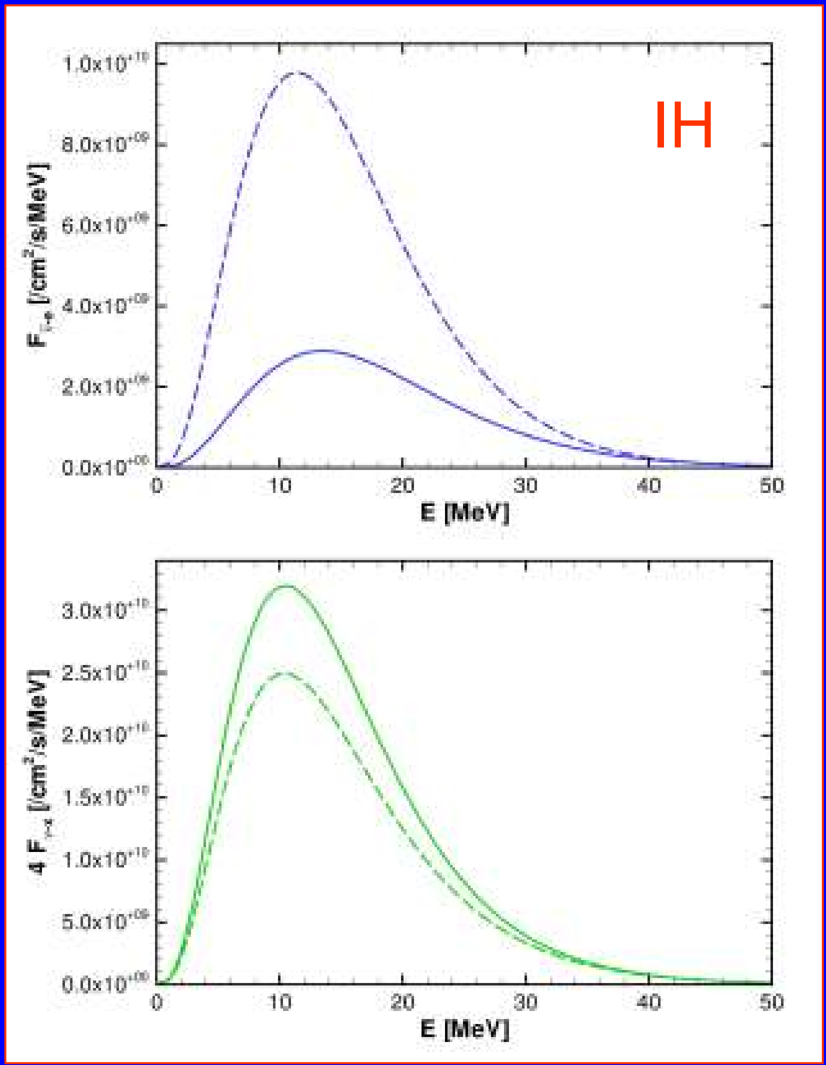
and a hierarchy of mean energies:

$\langle E_e\rangle = 12$ MeV, $\langle E_{\bar{e}}\rangle = 15$ MeV, $\langle E_x\rangle = 18$ MeV.

For either hierarchy there are two extremes for the fluxes:

- completely adiabatic propagation, $P_H(E) = 0$,
- completely non-adiabatic propagation, $P_H(E) = 1$.





For each flavor and each hierarchy there is a particular energy where the two curves cross.

These are the critical energies.

At the critical energies the adiabaticity is irrelevant: the flux is independent of P_H.

The differences between the two curves is largest for energies $\lesssim$ 40 MeV.

Variations of the adiabaticity for neutrinos with these energies will lead to the largest changes in the flux.

Water Cerenkov Detectors

Water Cerenkov detectors are the dominant type of neutrino detection technology.

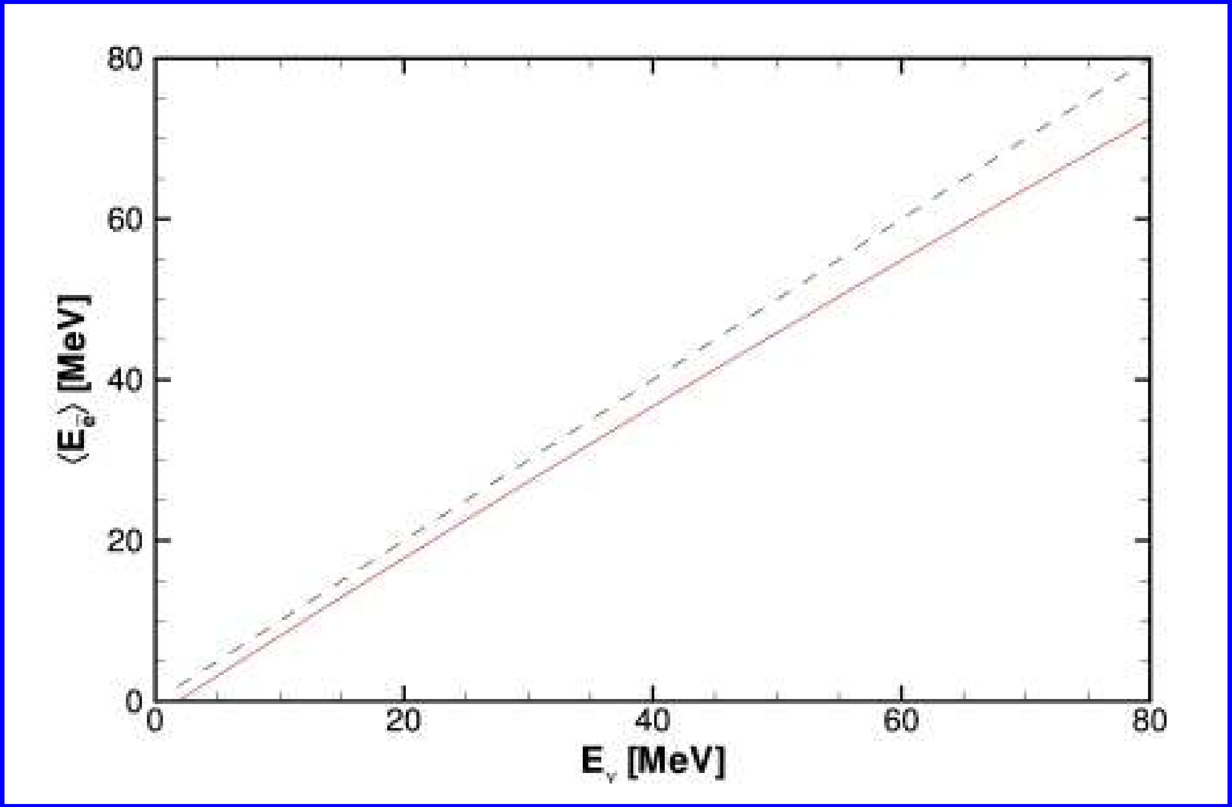
They are best suited to detect the positrons produced via inverse-β reactions by $\bar{\nu}_e$ upon protons.

There is some sensitivity to ν_e produced by $O(\nu_e,e)F$.

Temporal variations of the positron spectrum (beyond simply fading) would require an inverse hierarchy.

The positron energy and the antineutrino energy are closely correlated.

Features in the $\bar{\nu}_e$ spectrum at a particular energy will show up in the $\bar{e}$ spectrum at a similar energy.

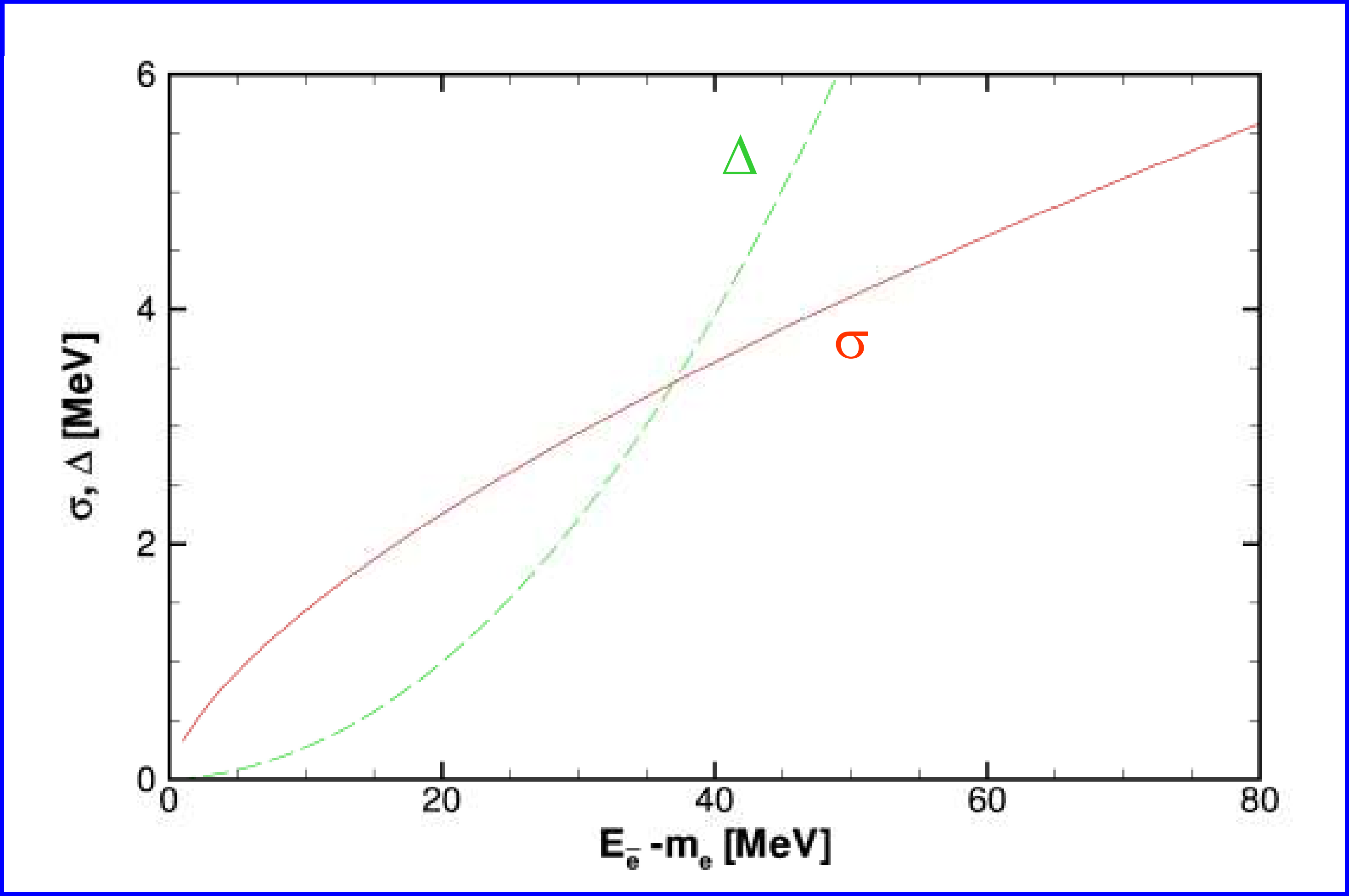


But the correlation isn't perfect and a range, Δ, of antineutrino energies can give the same positron energy.

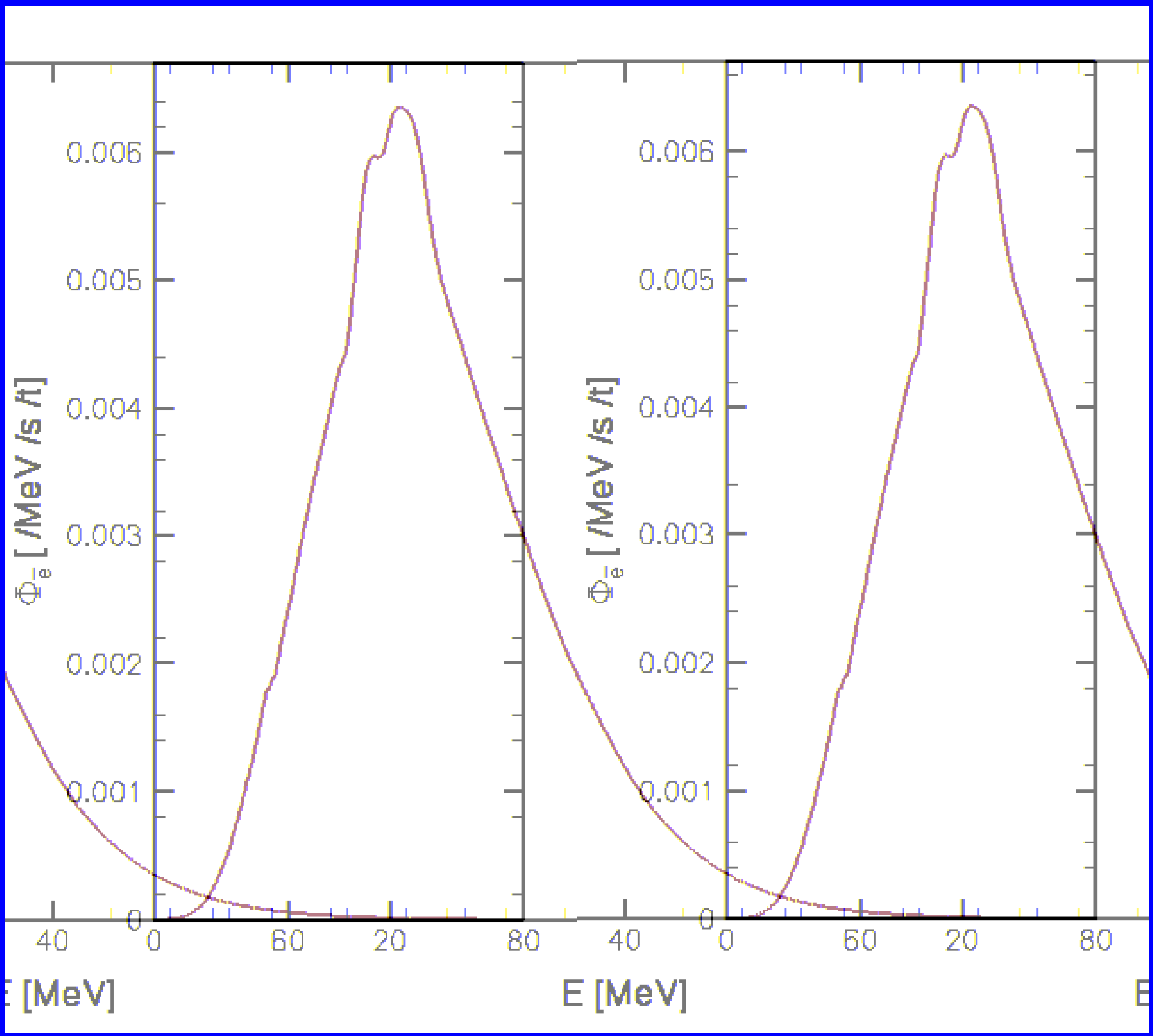
The energy resolution, σ, of the detector smears the observed positron spectrum.

We use the Super-K energy resolution.

Hosaka et al., PRD, **73** (2006) 112001

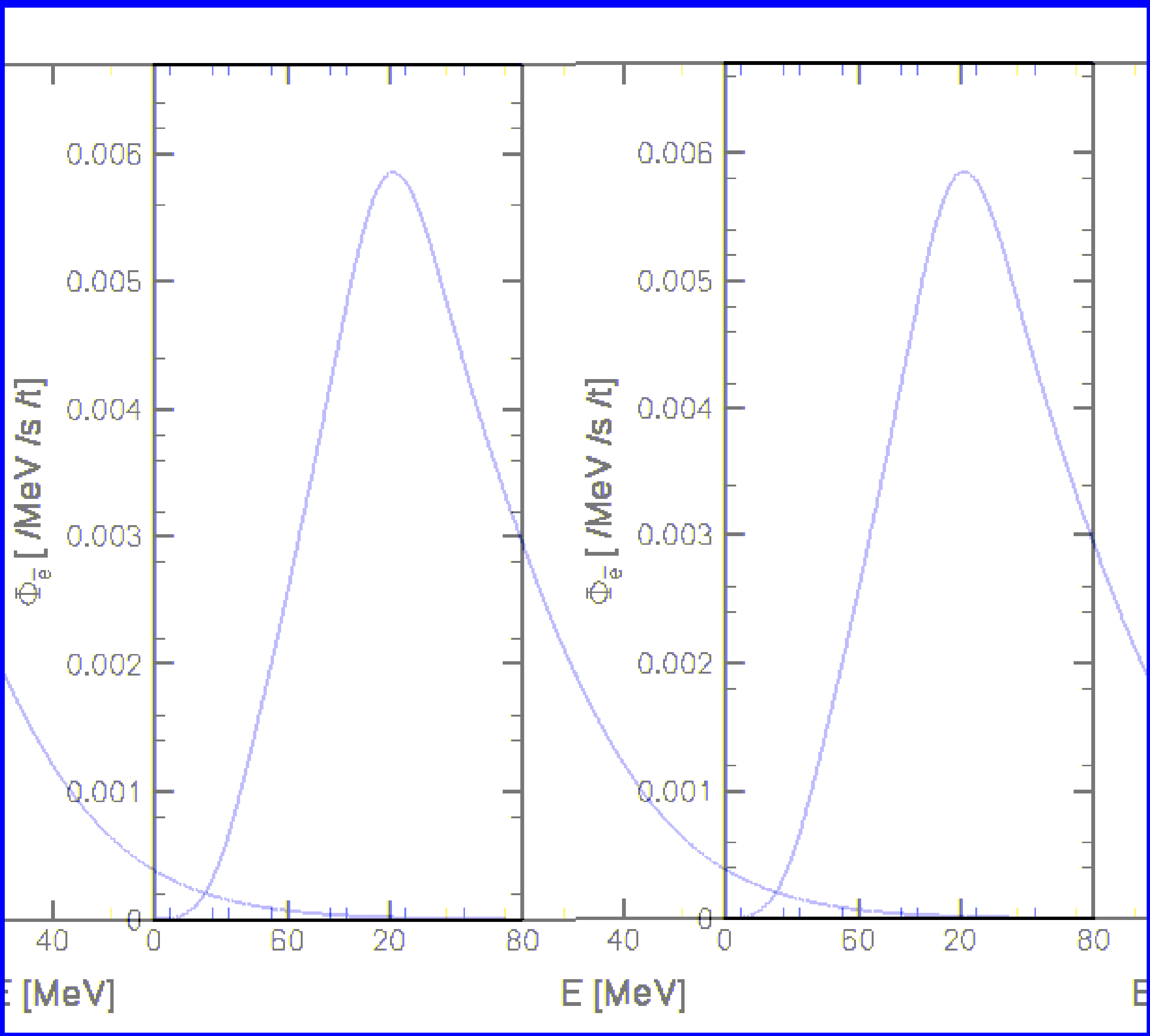


The shock appears clearly in the case of the weak explosion and perfect, σ = 0, detector.



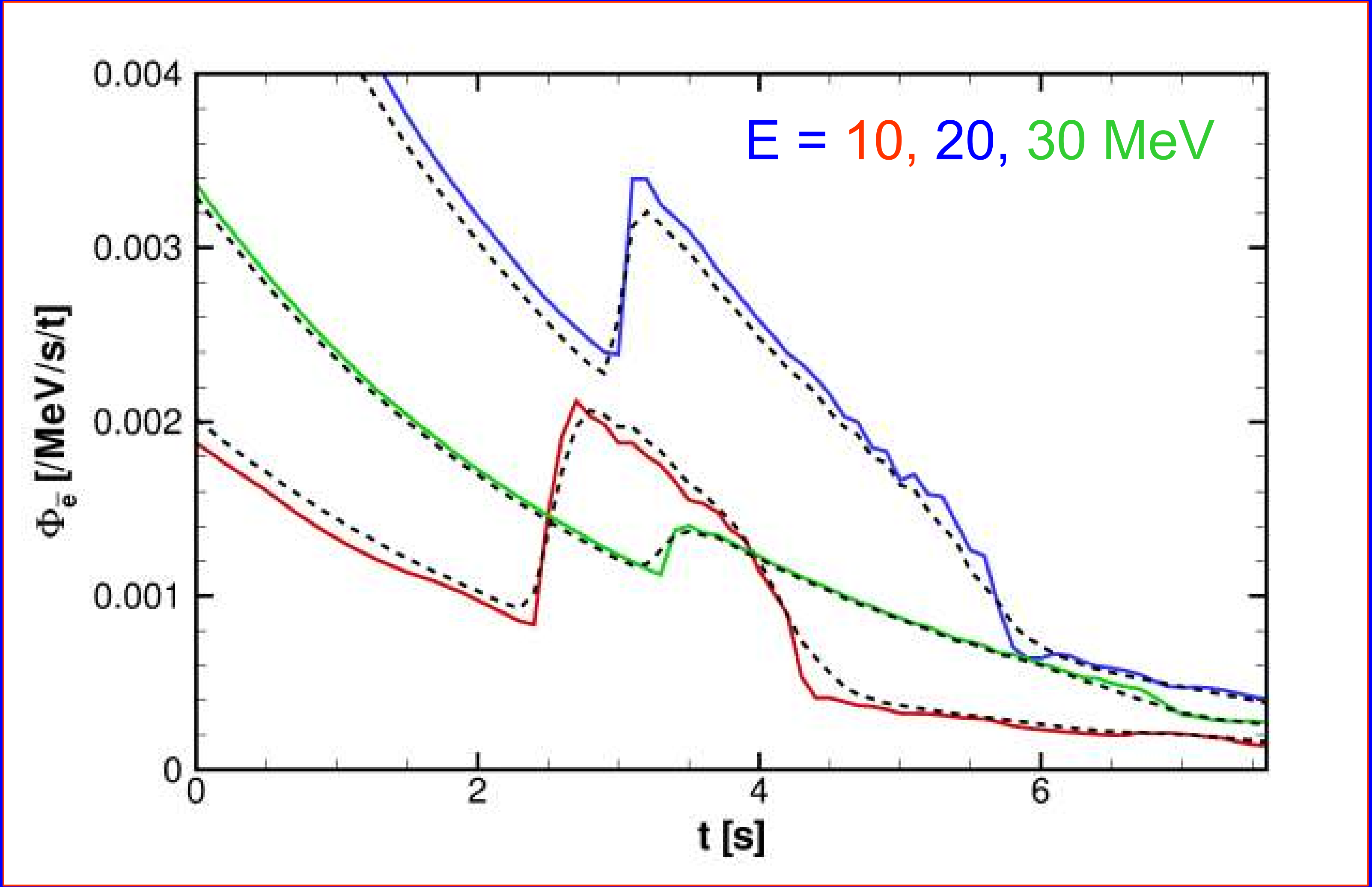
$Q = 1.66 \times 10^{51}$ erg

Even when convolved with the detector resolution the shock still appears in the spectrum.



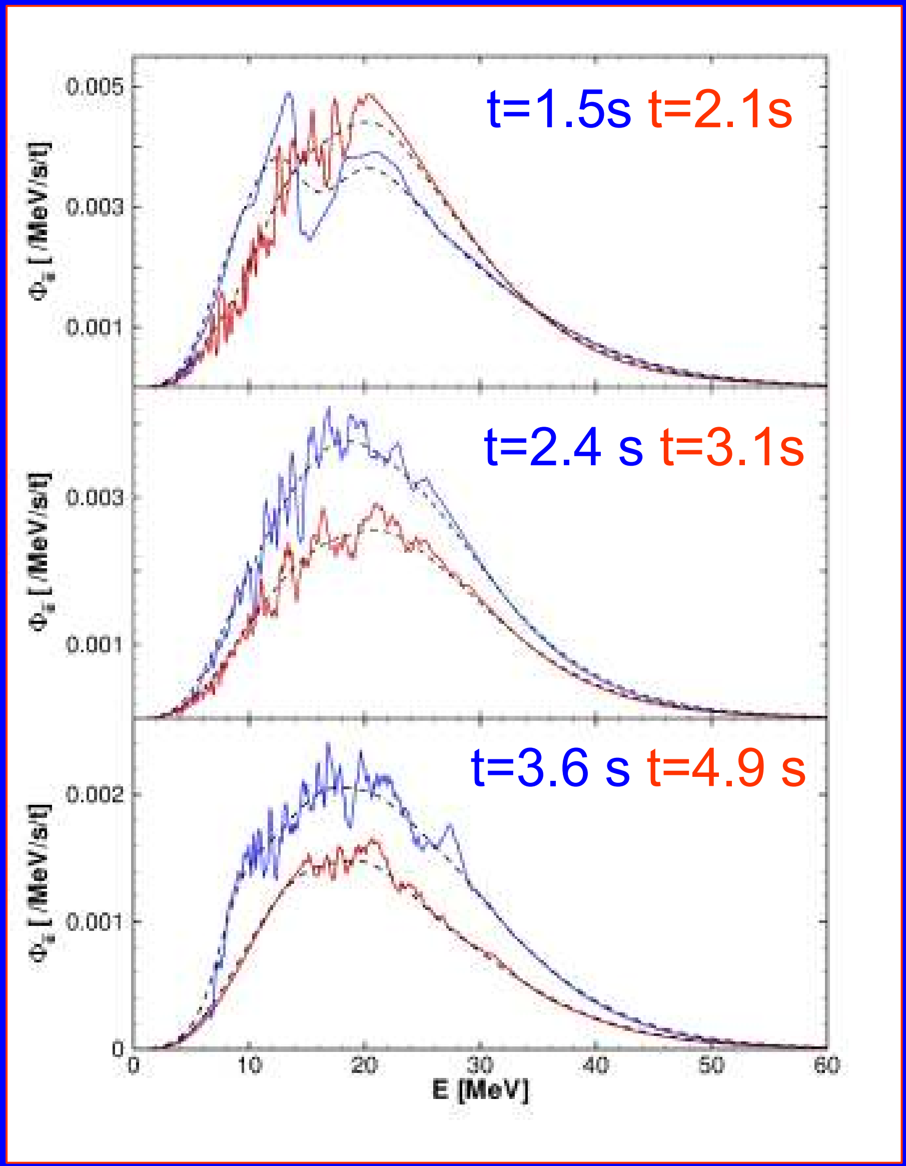
Q = 1.66 x 10^{51} erg

For fixed positron energy the general trend is $\exp(-t/\tau)$ and the shock is a temporary change in the spectrum.



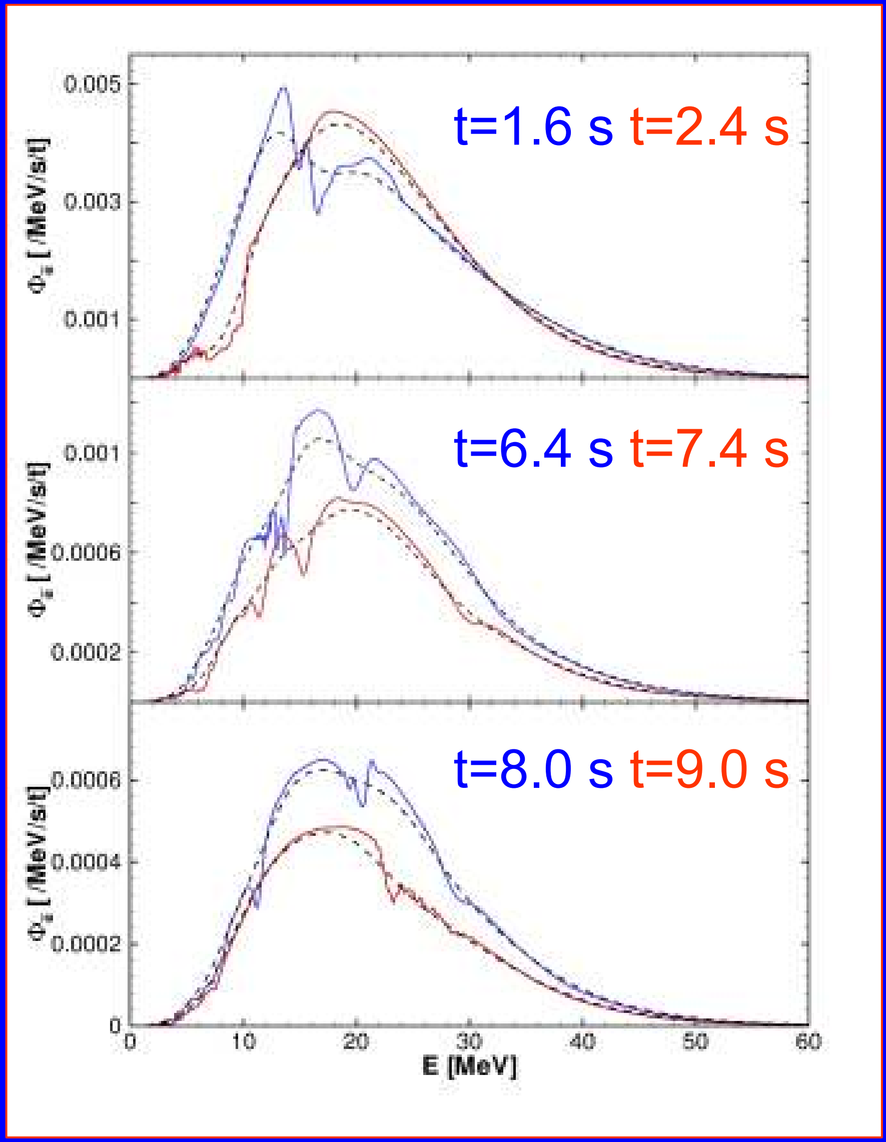
$Q = 1.66 \times 10^{51}$ erg

For more powerful explosions the spectrum begins to show evidence of phase effects (for perfect detector resolution).



Q = 3.36 x 10^{51} erg

And for anisotropic SN the spectrum also hints at phase effects and variability at late times (for $\sigma = 0$).



In summary

- If θ_{13} is not too small, the neutrino flux will vary with time due to changes in the density profile.
- The forward shock is the first feature to affect the neutrino signal.
  - Knowing the progenitor profile one can determine the shock speed and, possibly, the shock revival timescale.
  - Current detector technology can observe this feature
- High frequency phase effects indicate the presence of multiple shocks / resonances.
  - Does the reverse shock turn around?
  - Can the phase effects be resolved?
- Aspherical profiles lead to low frequency phase effects at late times.
  - But is the signature of asphericity $P_H \sim ½$?