Sterile Neutrinos : Some Mixing History & 6X6 Matrix Approach

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Int. J. Mod. Phys. **A20** (2005) 6373 [**hep-ph/0307245**]
(with M. Garbutt)

Quark-Lepton Symmetry demands 3 sterile Weyl spinor partners for the active neutrinos

$$\begin{bmatrix} U \\ D \end{bmatrix}_{\mathcal{L}} \Leftrightarrow \begin{bmatrix} \nu \\ \ell \end{bmatrix}_{\mathcal{L}}$$

$$[U]_{\mathcal{R}} = [U^c]_{\mathcal{L}} \Leftrightarrow [\nu]_{\mathcal{R}} = [\nu^c]_{\mathcal{L}}$$

$$[D]_{\mathcal{R}} = [D^c]_{\mathcal{L}} \Leftrightarrow [\ell]_{\mathcal{R}} = [\ell^c]_{\mathcal{L}}$$

But do they have to be

very massive

as in the classical see-saw?

No!

[e.g., Kusenko (Nu2006)

et al., hep-ph/0405198]

Whatever protects gauge boson hierarchy may also protect neutrino masses.

See R. Slansky {*Phys. Rept*. **79** (1981) 1}

How Large Are the Neutrino Mixing Angles?

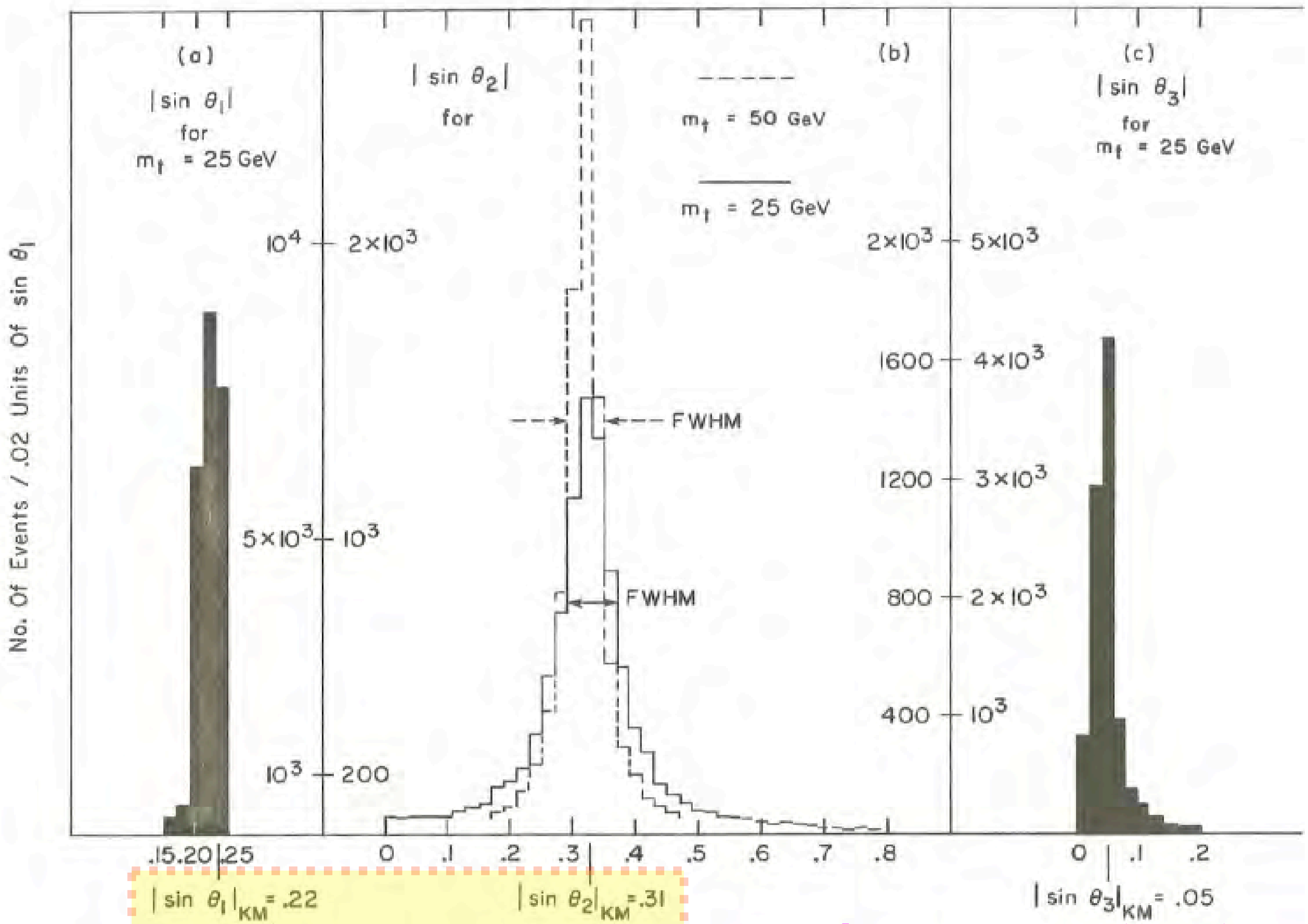
Phys. Rev. D24 (1981) 236 "predicted" large flavor mixing

Off-diagonal 3X3 Dirac blocks set to CKM rotation from diagonal of [up] quark masses.

Random choices of sterile mass matrix entries produce surprisingly narrow statistical distributions, large mixing, largest mixing is in μ−τ sector.

If sterile mass matrix power series is expandable in powers of Dirac then mixing angles are identical to quarks'.

(Pseudo-)Dirac cases for reduced rank sterile mass matrix noted but not followed up.



T. Goldman and G. J. Stephenson, Jr. *Phys. Rev.* **D24** (1981) 236

See-Saw

Structure of 6×6 Mass Matrix

$$\begin{pmatrix} \mathbf{0} & \mathbf{m_D} \\ \mathbf{m_D} & \mathbf{M} \end{pmatrix} \begin{pmatrix} \frac{(1-\gamma_5)}{2} L^\nu \\ \mathcal{C}_W \left[\frac{(1+\gamma_5)}{2} L^\nu \right] \end{pmatrix}$$

The Wigner conjugation operator

$$\mathcal{C}_W : (\frac{1}{2}, 0) \leftrightarrow (0, \frac{1}{2})$$

on Weyl spinors, is similar to CP.

Standard See-Saw Mass Eigenvalues

$$\mu \sim -\frac{(m_D)^2}{M}; \ \sim M + \frac{(m_D)^2}{M}$$

where $M >> m_D$ is assumed.

Lagrangian for Weyl Spinors

$$\mathcal{L}_L = \frac{1}{2}\, \varphi^\dagger \sigma^\mu \overleftrightarrow{\partial}_\mu \varphi + \frac{1}{2}\, i\, m(\varphi^T \sigma_2 \varphi + \varphi^\dagger \sigma_2 \varphi^*)$$

$$\mathcal{L}_R : \sigma^\mu \Rightarrow \overline{\sigma^\mu} = (\sigma^0, -\vec{\sigma}) \qquad \overleftrightarrow{\partial} = \overrightarrow{\partial} - \overleftarrow{\partial}$$

$$\varphi \equiv \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix}$$

(Grassman variables)

Equations of Motion

$$\partial_t \varphi_1 - \partial_z \varphi_1 - (\partial_x - i\partial_y)\, \varphi_2 = -m\varphi_2^*$$
$$\partial_t \varphi_2 + \partial_z \varphi_2 - (\partial_x + i\partial_y)\, \varphi_1 = +m\varphi_1^*$$

Define

$$\theta = Et - \vec{p} \cdot \vec{x} \; ; \; p_{\pm} = p_x \pm i\, p_y$$

Solutions

$$\varphi_- = \begin{pmatrix} F \exp\{-i\,\theta\} \\ \frac{-p_+}{E - p_z} F \exp\{-i\,\theta\} - \frac{i\,m}{E - p_z} F^* \exp\{+i\,\theta\} \end{pmatrix}$$

$$\varphi_+ = \varphi_-^*$$

Majorana vs. Dirac

$$\Psi_\varphi = \begin{pmatrix} \varphi \\ -\sigma_2 \varphi^* \end{pmatrix} \; ; \quad \Psi_\chi = \begin{pmatrix} +\sigma_2 \chi^* \\ \chi \end{pmatrix}$$

$$\Psi_D = \Psi_\varphi \pm i\, \Psi_\chi$$

$\{PC = \pm T\}$

$$\Psi_D \equiv \begin{pmatrix} a \\ -\sigma_2 a^* \end{pmatrix} + i \begin{pmatrix} s \\ -\sigma_2 s^* \end{pmatrix} = \Psi_a + i\Psi_s$$

Let mass of Ψ_a be m. If Ψ_s also has mass m, then $i\Psi_s$ has mass -m. A 45^0 basis rotation displays m as Dirac! (& flips C)

Then Wigner-Weyl rest states correspond to Pauli-Dirac

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix} \begin{pmatrix} He^{-imt} \\ 0 \\ He^{-imt} \\ 0 \end{pmatrix} = \begin{pmatrix} Ae^{-imt} \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Spin Up Particle

6X6

3 Flavor, Rank 1 Sterile Mass Matrix

Diagonal Dirac mass matrix defines "sterile flavors"

0	0	0	m_1	0	0		ν_{af}
0	0	0	0	m_2	0		ν_{ag}
0	0	0	0	0	m_3		ν_{ah}
m_1	0	0	0	0	0		ν_{sf}
0	m_2	0	0	0	0		ν_{sg}
0	0	m_3	0	0	M		ν_{sh}

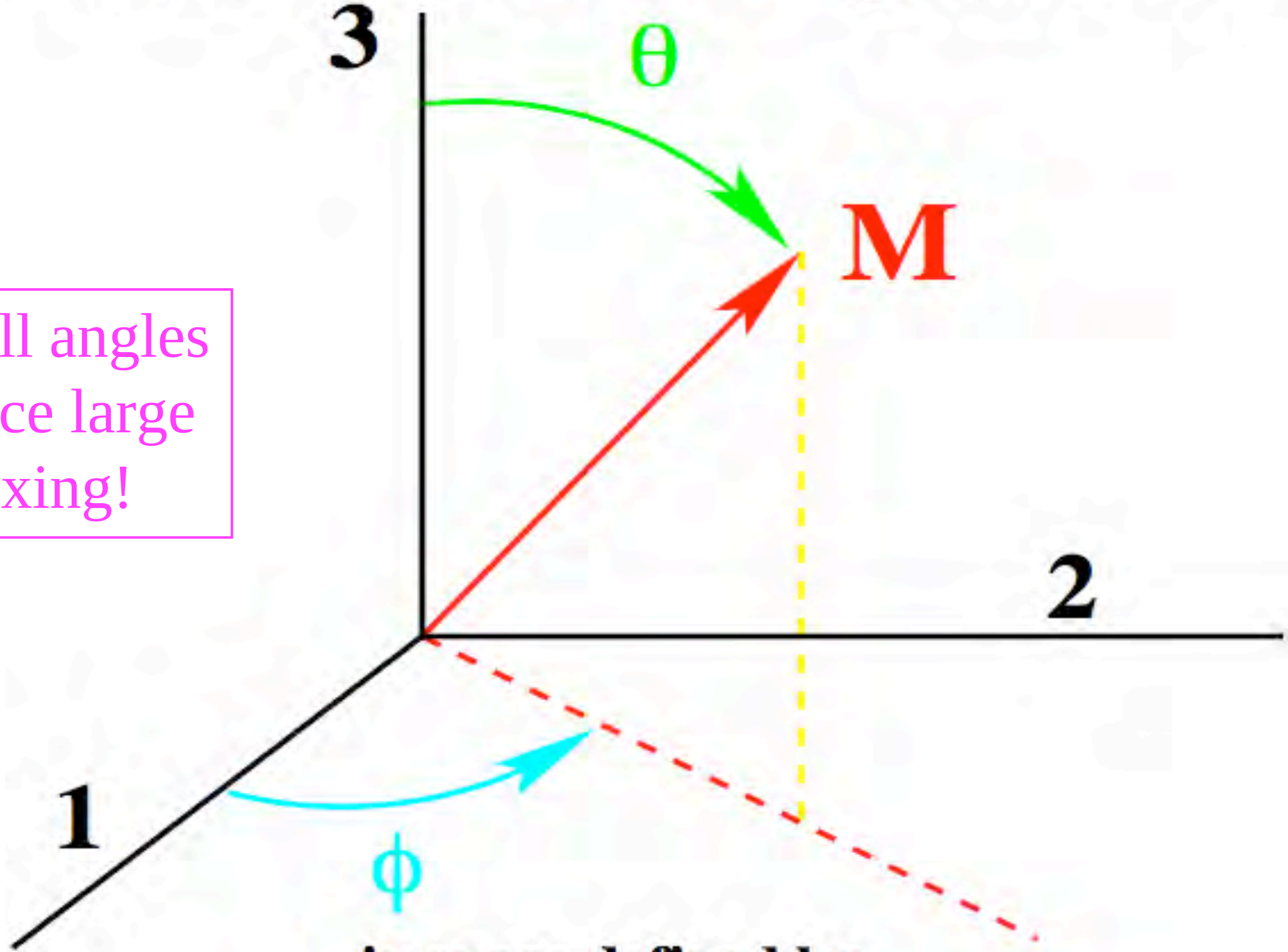
Add CKM (yes, same as quarks) mixing in actives

$$\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

[Ignoring CPV]

$$\begin{pmatrix} U_{11} & U_{21} & U_{31} \\ U_{12} & U_{22} & U_{32} \\ U_{13} & U_{23} & U_{33} \end{pmatrix} \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix} \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{pmatrix}$$

Direction of Single Massive Sterile Eigenstate



Small angles induce large mixing!

in space defined by Dirac mass eigenstates

Structure Illuminated by 4X4 Analytic Example

(simple)

$$\mu = b^2/M$$

$$m_+ = m_0 + a^2/M + O(M^{-2})$$

$$m_- = m_0 - a^2/M + O(M^{-2})$$

} Pseudo-Dirac Pair

$$\mathcal{M} = M + b^2/M$$

where

$$m_0^2 = m_1^2 \cos^2\theta + m_3^2 \sin^2\theta \qquad b = \frac{m_1 m_3}{m_0}$$

$$a = \frac{(m_1^2 - m_3^2)\sin\theta\cos\theta}{m\sqrt{2}}$$

In 6X6 rank-1 case:

2 Pseudo-Dirac Pairs

DETAIL OF 2 FLAVOR ANALYTIC EXAMPLE

Eigenvalue equation:

$$\mu \phi_i = \begin{pmatrix} 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & m_3 \\ m_1 & 0 & Ms^2 & Mcs \\ 0 & m_3 & Mcs & Mc^2 \end{pmatrix} \begin{pmatrix} \alpha_i \\ \beta_i \\ \gamma_i \\ \delta_i \end{pmatrix}$$

$$c = \cos\theta, \; s = \sin\theta$$

Characteristic equation: (McKellar form)

$$\mu\,(m_0^2 - \mu^2)\mu\,(M - \mu) = 2\mu^2 a^2 - (m_0^2 - \mu^2) b^2$$

Eigenvalues:

$$\mu_1 = +\, m_0 - \frac{a^2}{M} - \frac{a^2}{m_0 M^2}\left(m_0^2 - \frac{a^2}{2} - b^2\right)$$

$$\mu_2 = -\, m_0 - \frac{a^2}{M} + \frac{a^2}{m_0 M^2}\left(m_0^2 - \frac{a^2}{2} - b^2\right)$$

$$\mu_3 = -\frac{b^2}{M} + O(M^{-3})$$

$$\mu_4 = M + \frac{2a^2 + b^2}{M} + O(M^{-3})$$

Eigenvector solutions:

$$\frac{\alpha_1}{\beta_1} = \frac{\alpha_2}{\beta_2} = \frac{\beta_3}{\alpha_3} = \frac{m_1}{m_3}\cot\theta$$

$$\frac{\gamma_i}{\alpha_i} = \frac{\mu_i}{m_1} \quad ; \quad \frac{\delta_i}{\beta_i} = \frac{\mu_i}{m_3}$$

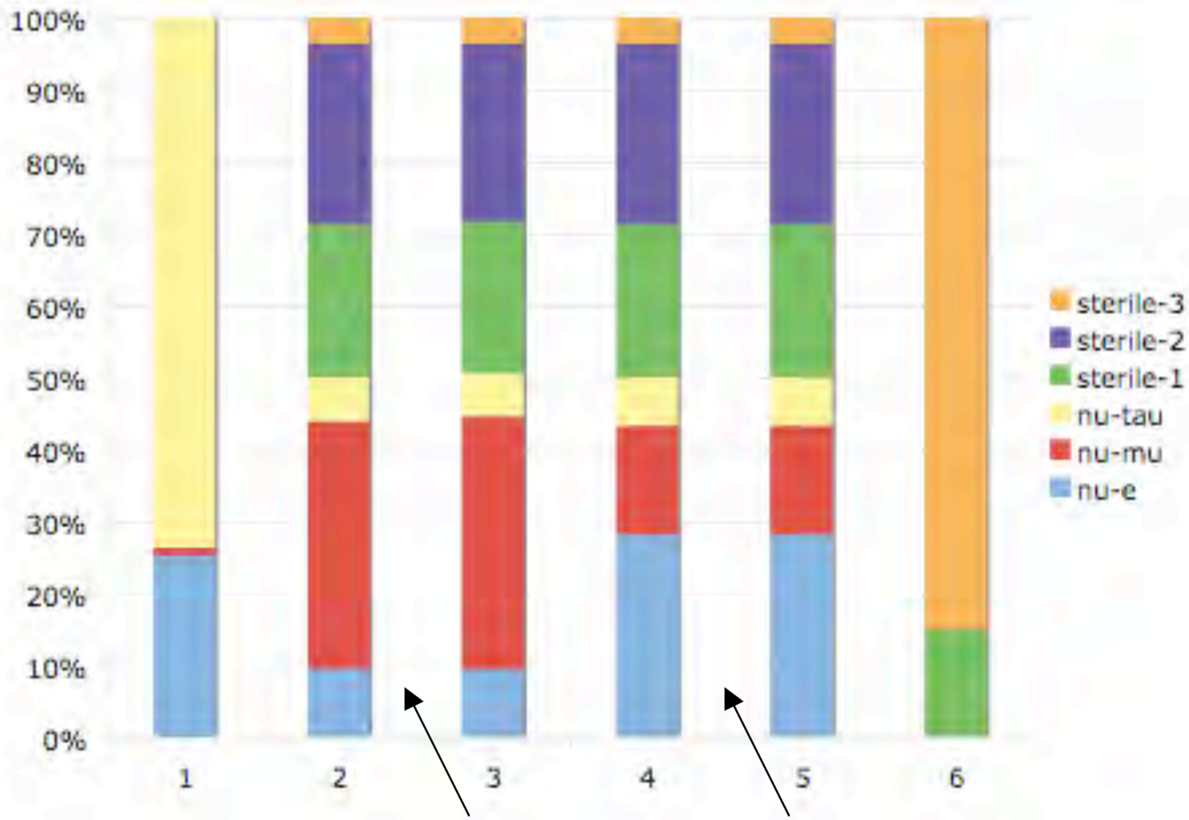
But for large M and small θ, $\mu_1 \sim \mu_2 \sim m_1$
(i.e., small mixing in sterile sector)

so flavor mixing will be large if : $\frac{m_3}{m_1} \sim \tan\theta$
to compensate. Except for

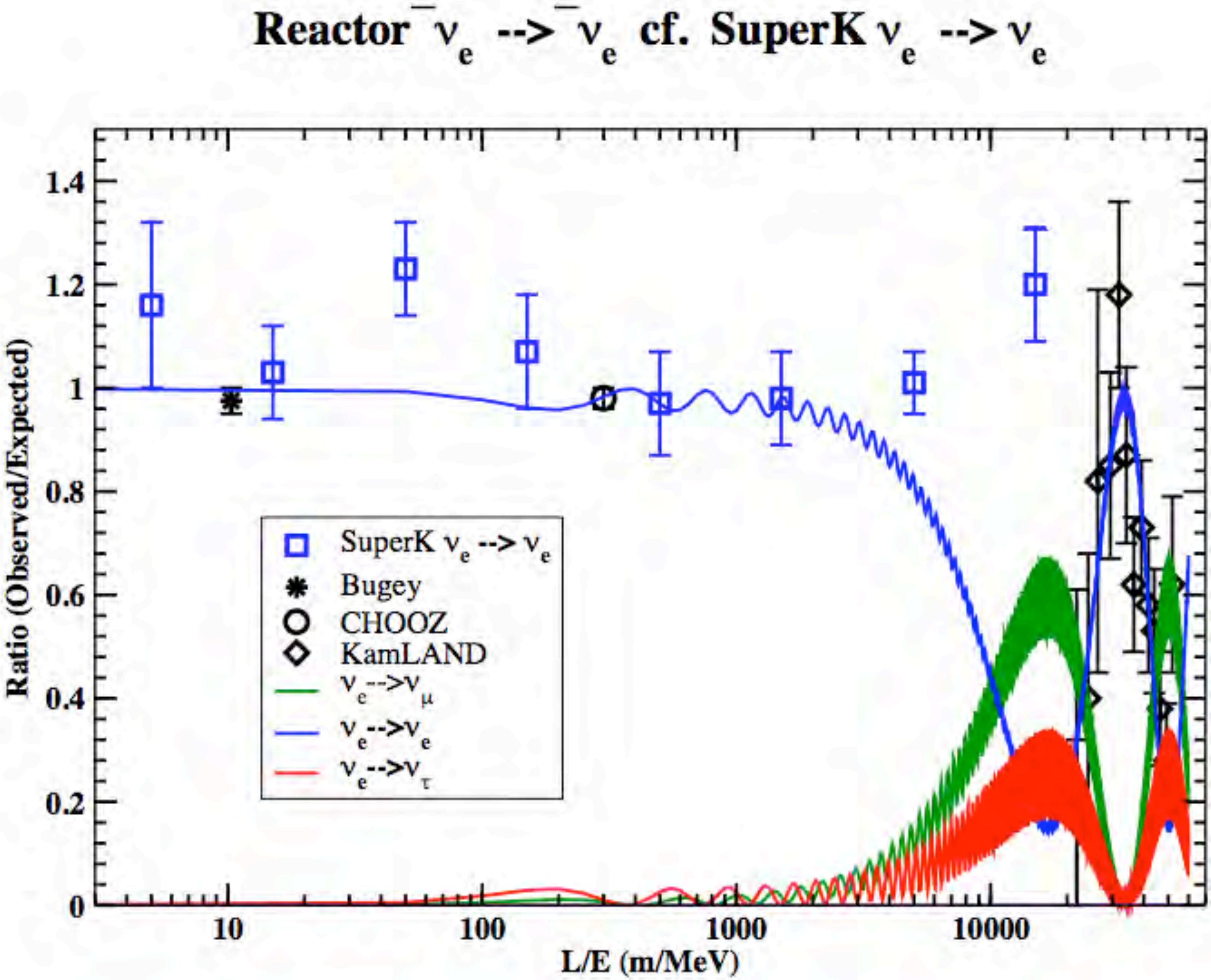
$\mu_3 \sim$ small ⇨ almost pure active

$\mu_4 \sim \mathcal{O}$ (M) ⇨ pure sterile

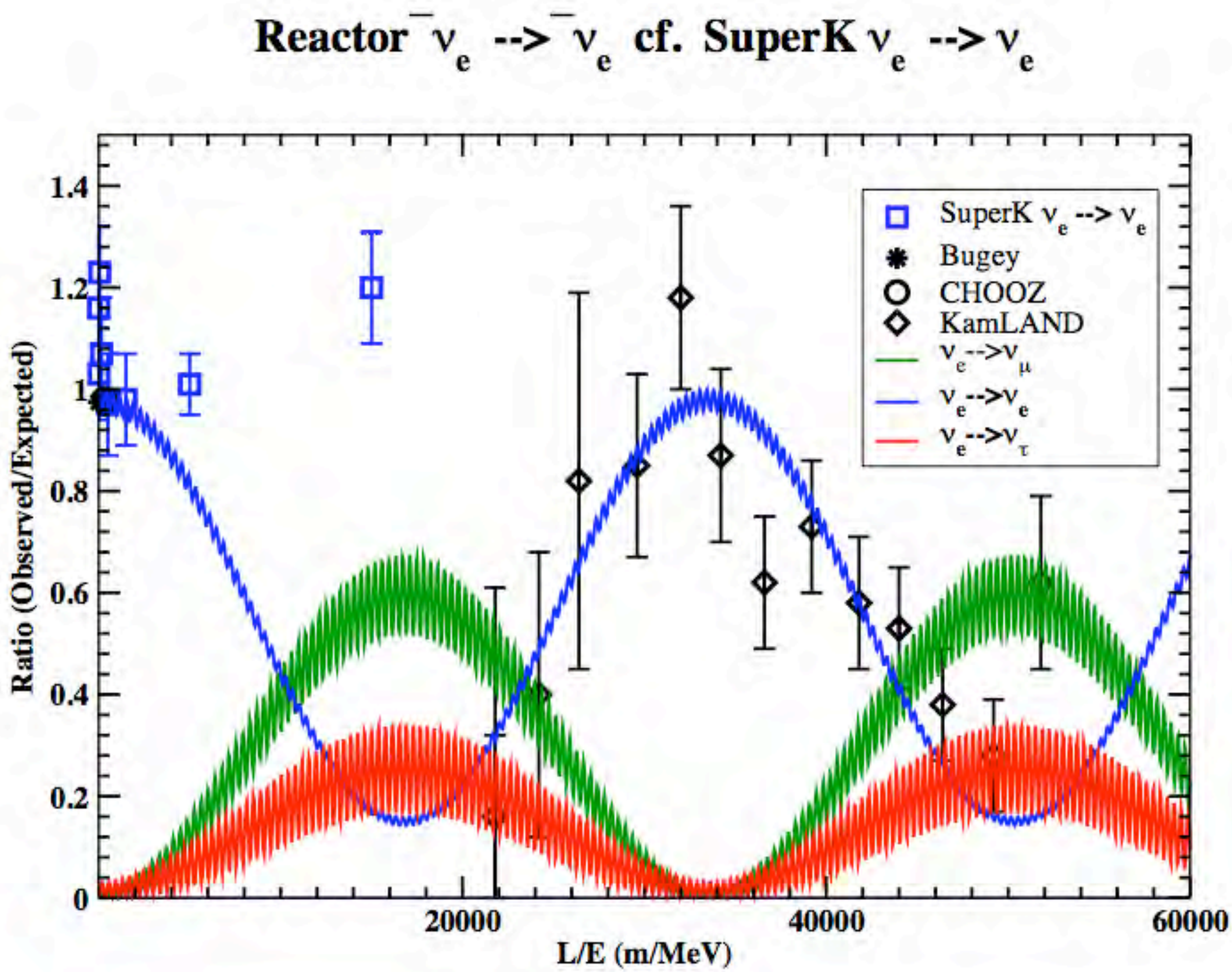
Content of Mass Eigenstates



For 6 x 6: 2 Pseudo-Dirac pairs



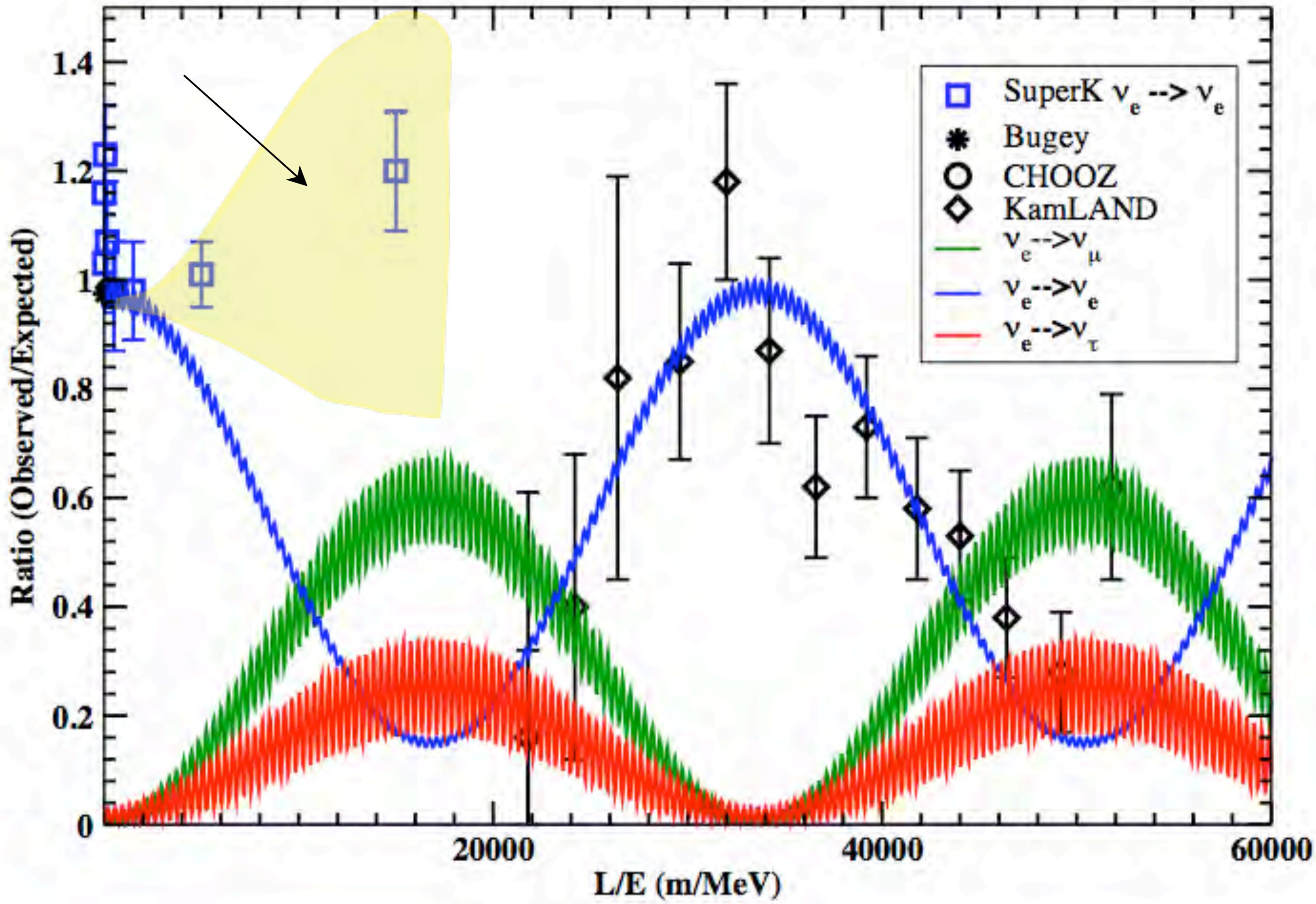
Reactor $\bar{\nu}_e$ --> $\bar{\nu}_e$ cf. SuperK ν_e --> ν_e
Ratio (Observed/Expected)
L/E (m/MeV)
0
0.2
0.4
0.6
0.8
1
1.2
1.4
10
100
1000
10000
SuperK ν_e --> ν_e
Bugey
CHOOZ
KamLAND
ν_e-->ν_μ
ν_e-->ν_e
ν_e-->ν_τ



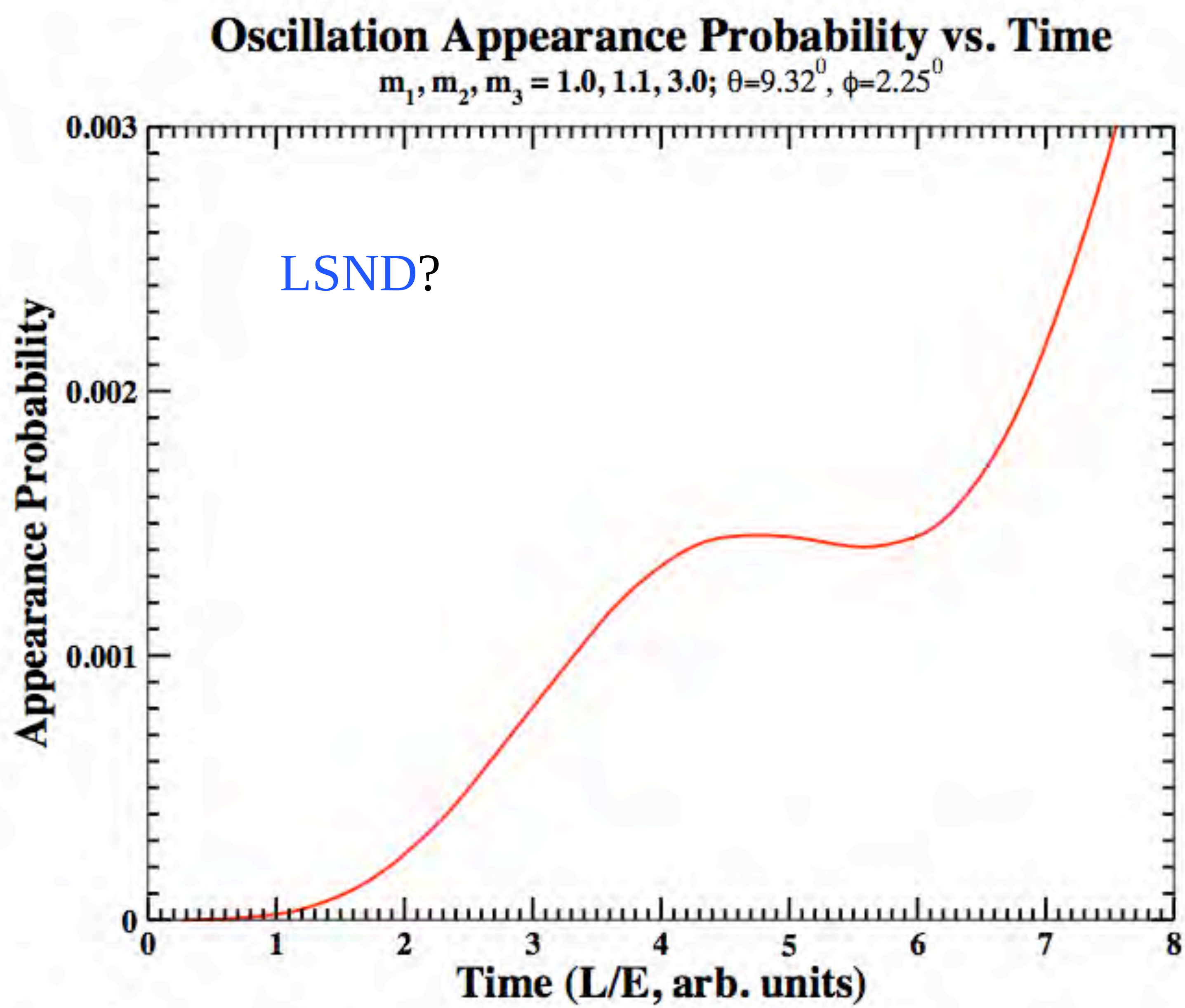
Θ=22.67°, Φ=0.008156,

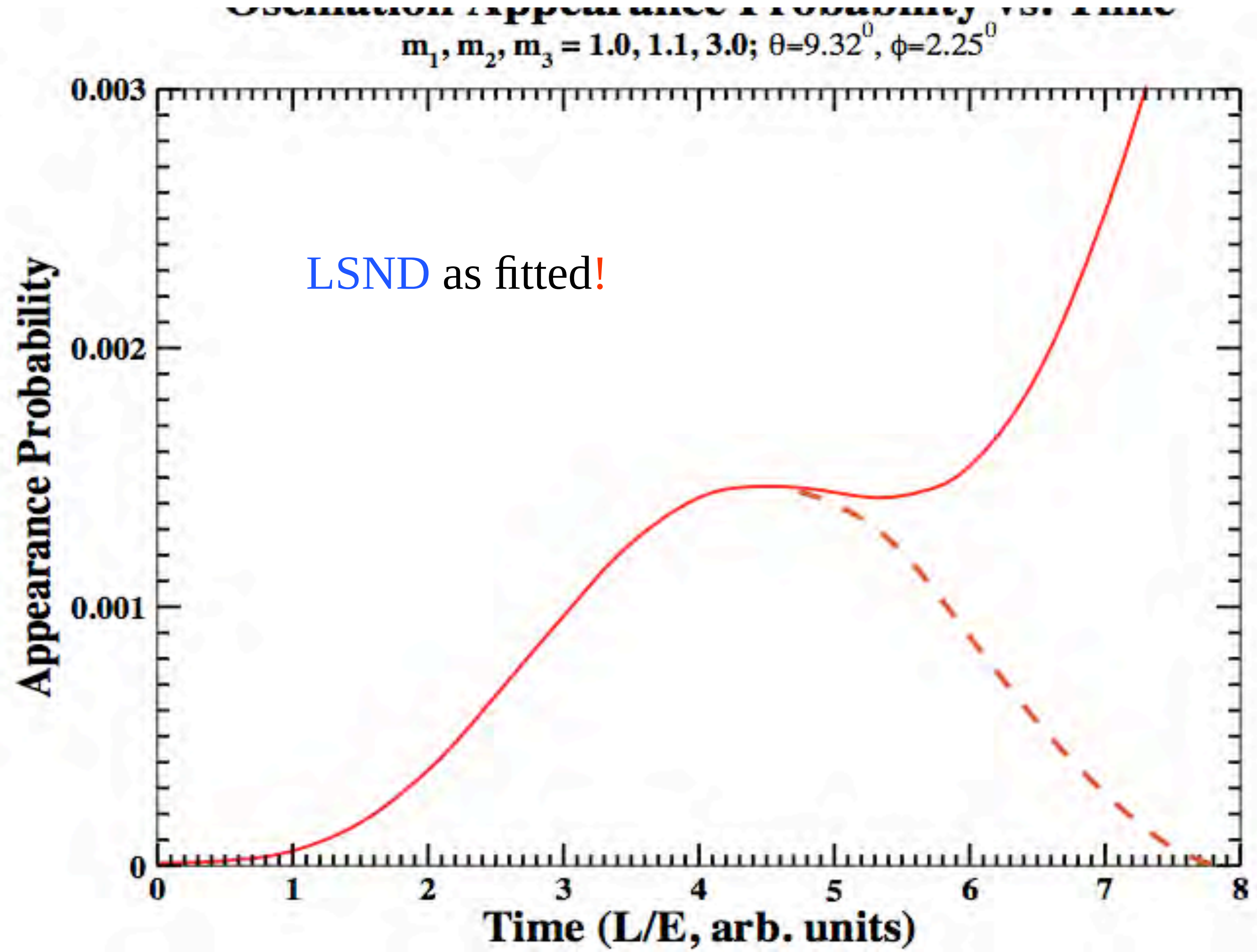
m_1=2.740, m_2=2.877, m_3=3.562 eV/c^2

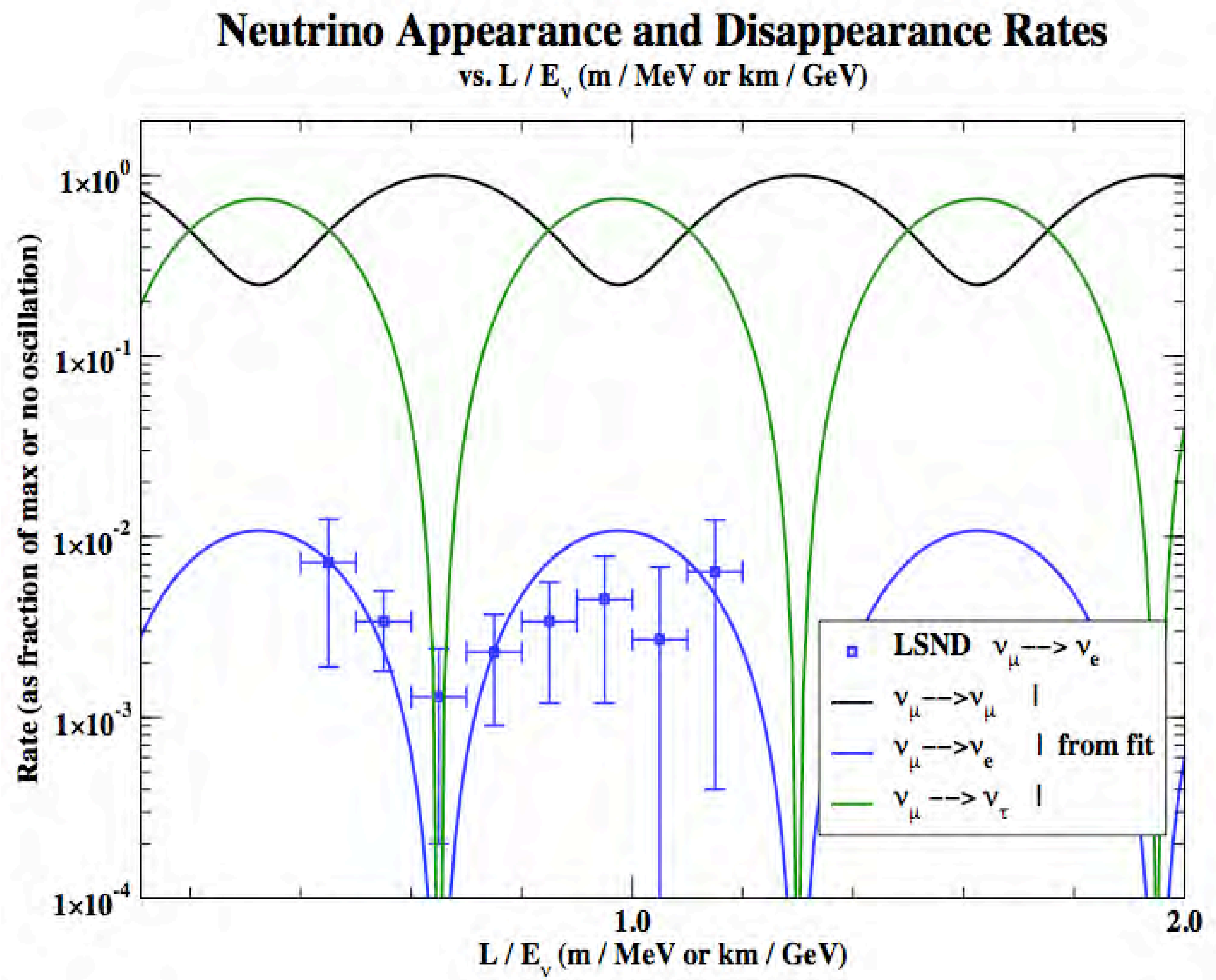
Reactor $\bar{\nu}_e$ --> $\bar{\nu}_e$ cf. SuperK ν_e --> ν_e



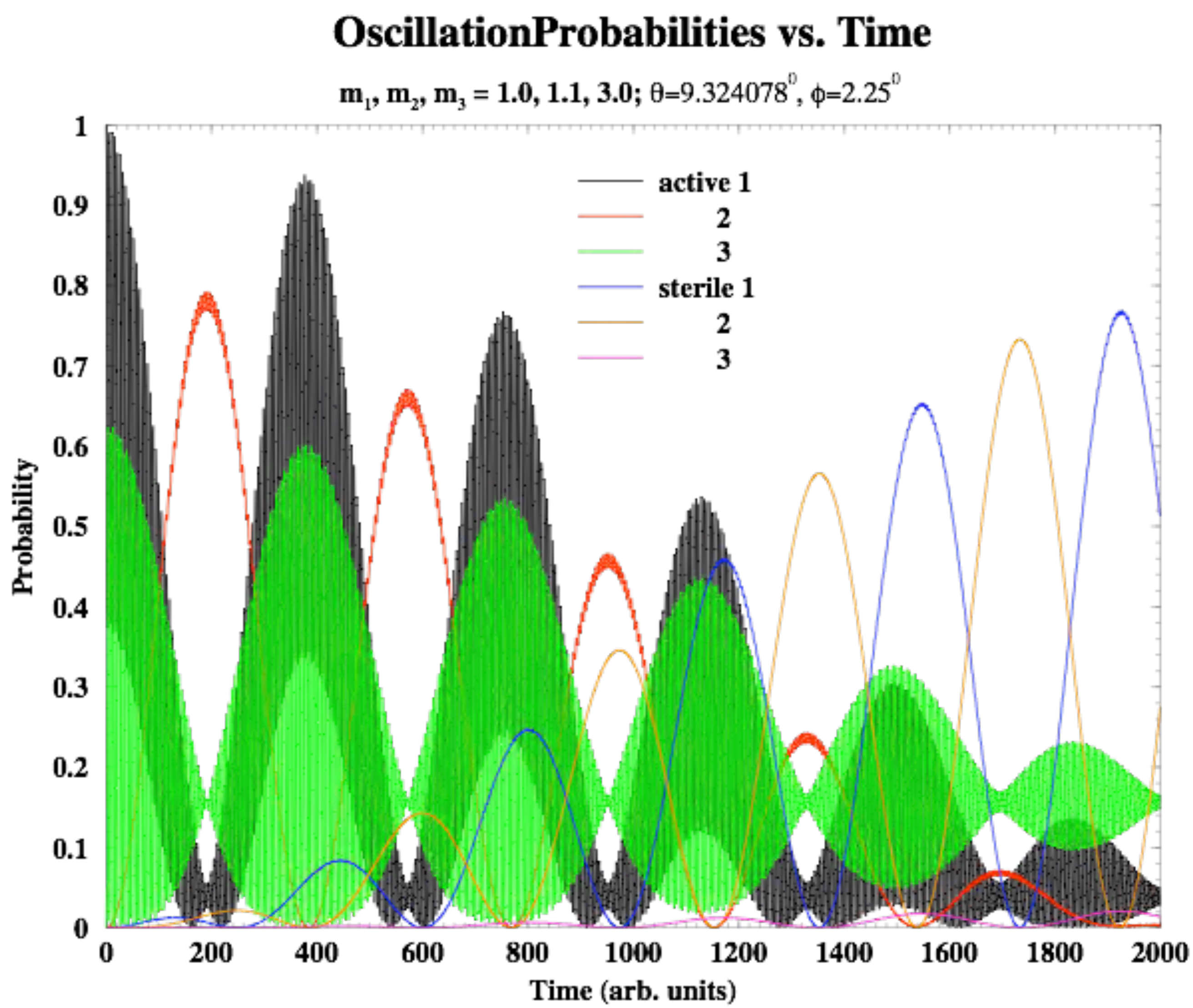
w/add'n (yellow) of ν_e osc. from ν_μ flux to SuperK ν_e signal


Oscillation Appearance Probability vs. Time
m1, m2, m3 = 1.0, 1.1, 3.0; θ=9.32⁰, ϕ=2.25⁰
LSND?
Appearance Probability
0.003
0.002
0.001
0
0
1
2
3
4
5
6
7
8
Time (L/E, arb. units)

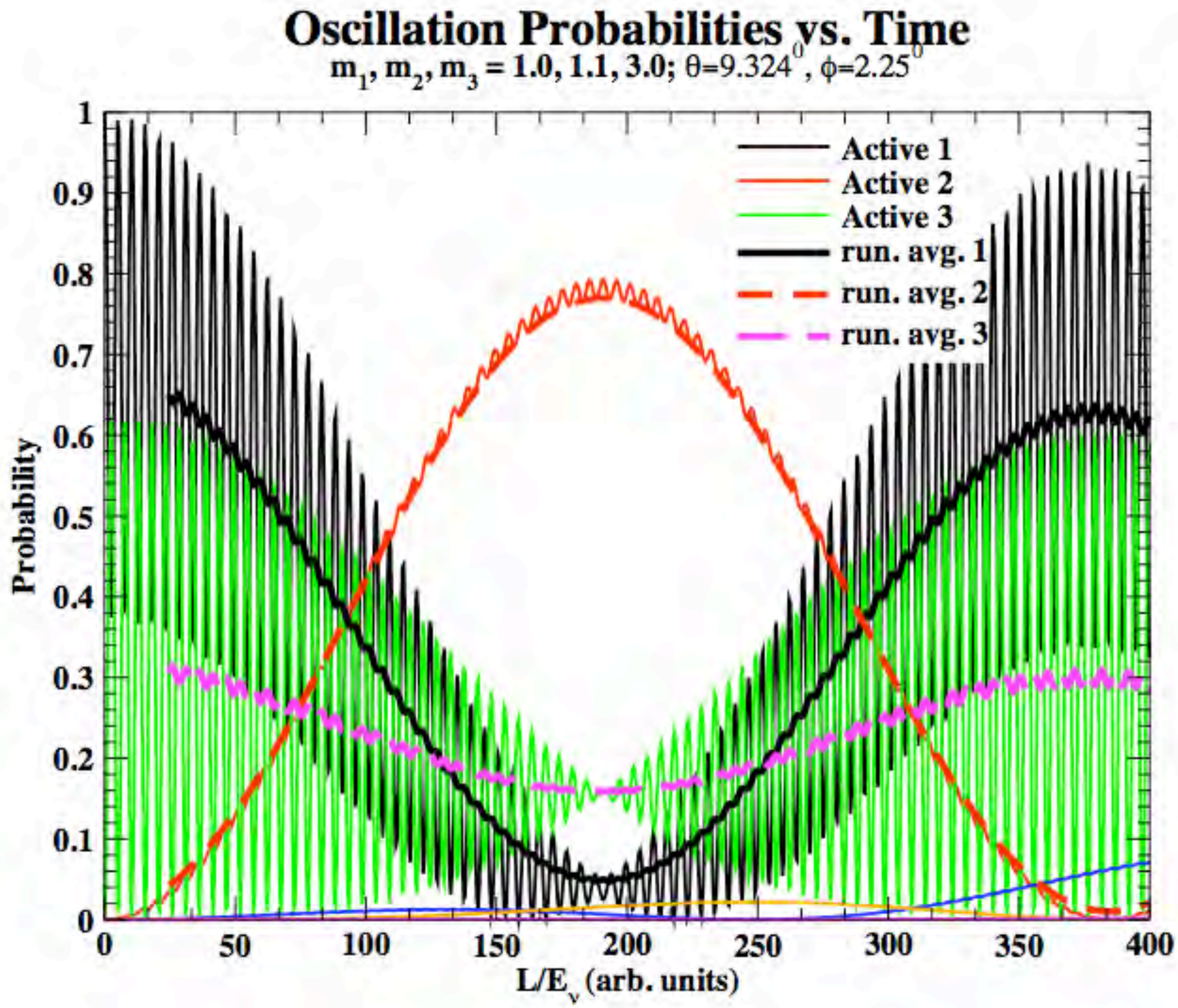

m1, m2, m3 = 1.0, 1.1, 3.0; θ=9.32°, ϕ=2.25°
LSND as fitted!
Appearance Probability
0.003
0.002
0.001
0
0 1 2 3 4 5 6 7 8
Time (L/E, arb. units)



Neutrino Appearance and Disappearance Rates
vs. L / E$_\nu$ (m / MeV or km / GeV)
Rate (as fraction of max or no oscillation)
1x10^0
1x10^{-1}
1x10^{-2}
1x10^{-3}
1x10^{-4}
1.0
2.0
L / E$_\nu$ (m / MeV or km / GeV)
LSND ν_μ --> ν_e
ν_μ-->ν_μ
ν_μ-->ν_e
ν_μ --> ν_τ
from fit

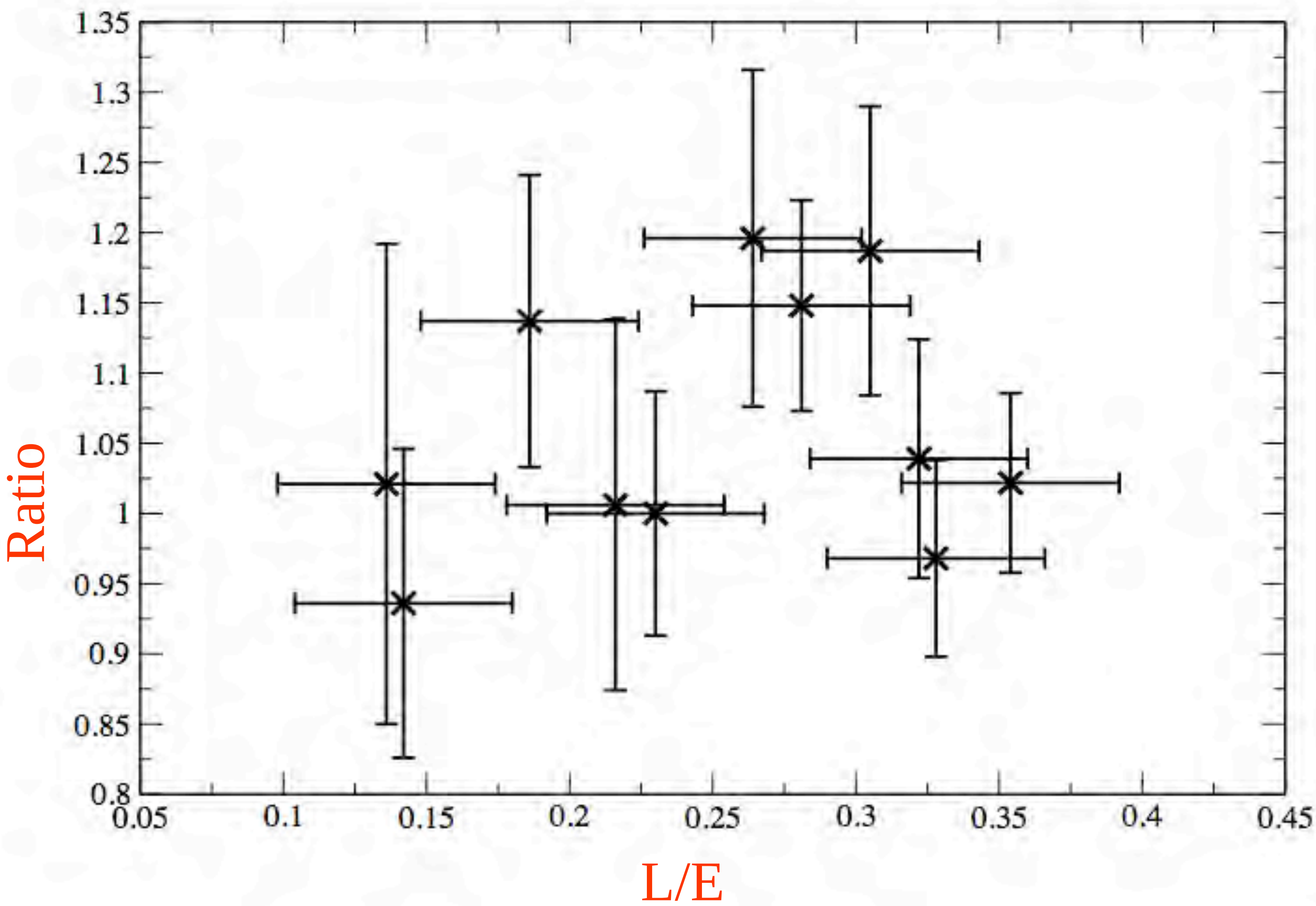


Large mixing amplitudes to all channels on multiple scales

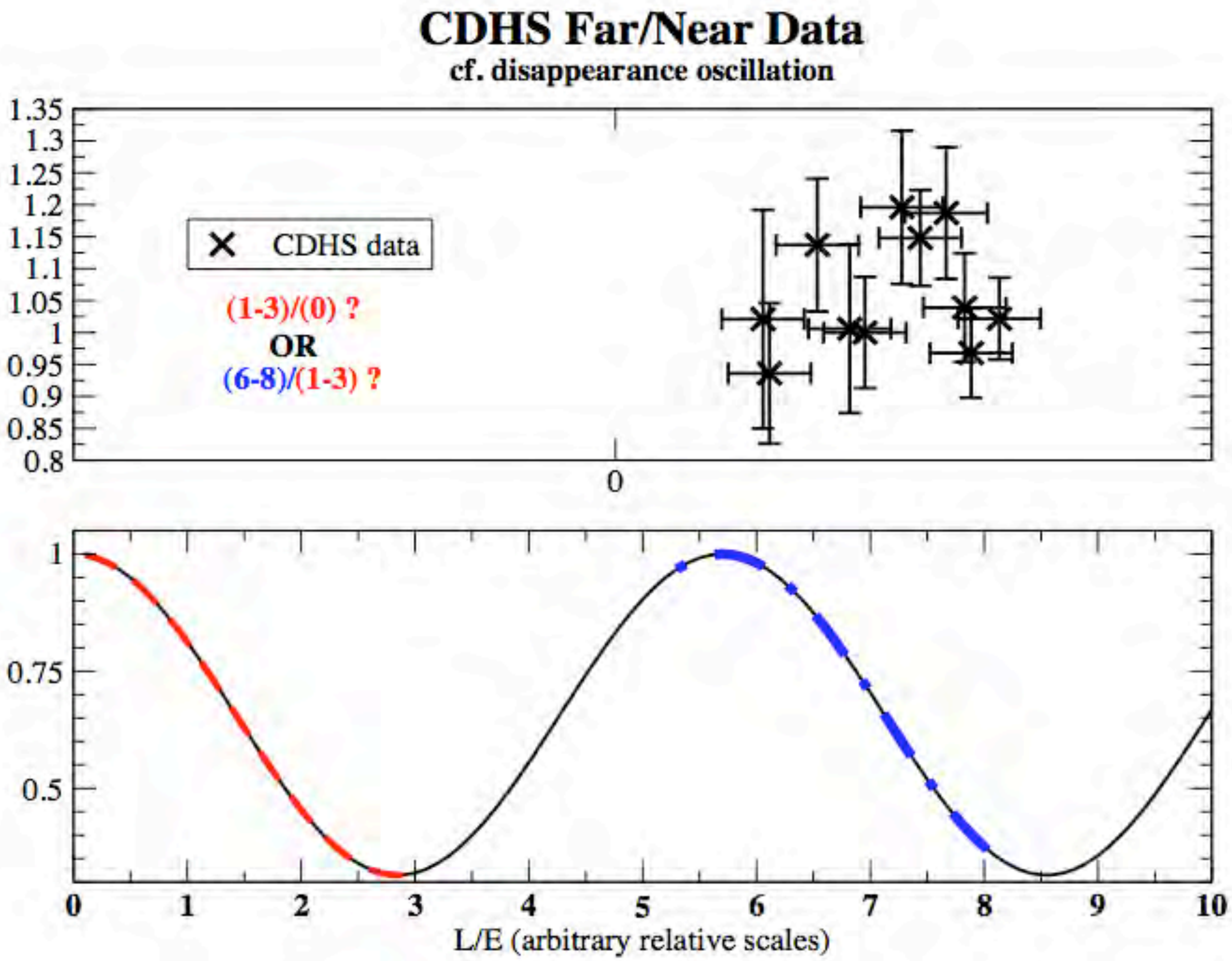


Effects of Finite Resolution

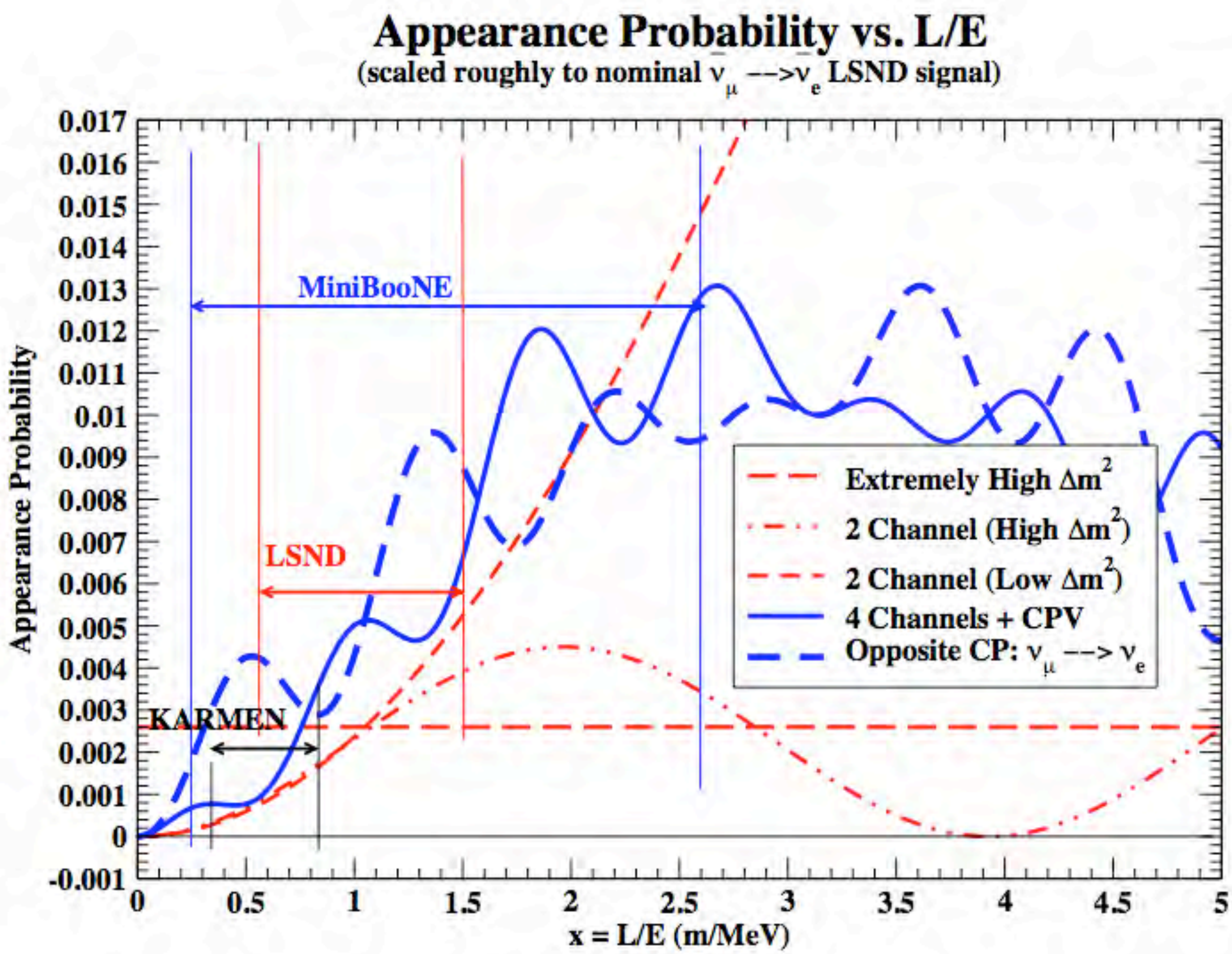
CDHS Far/Near vs. L/E



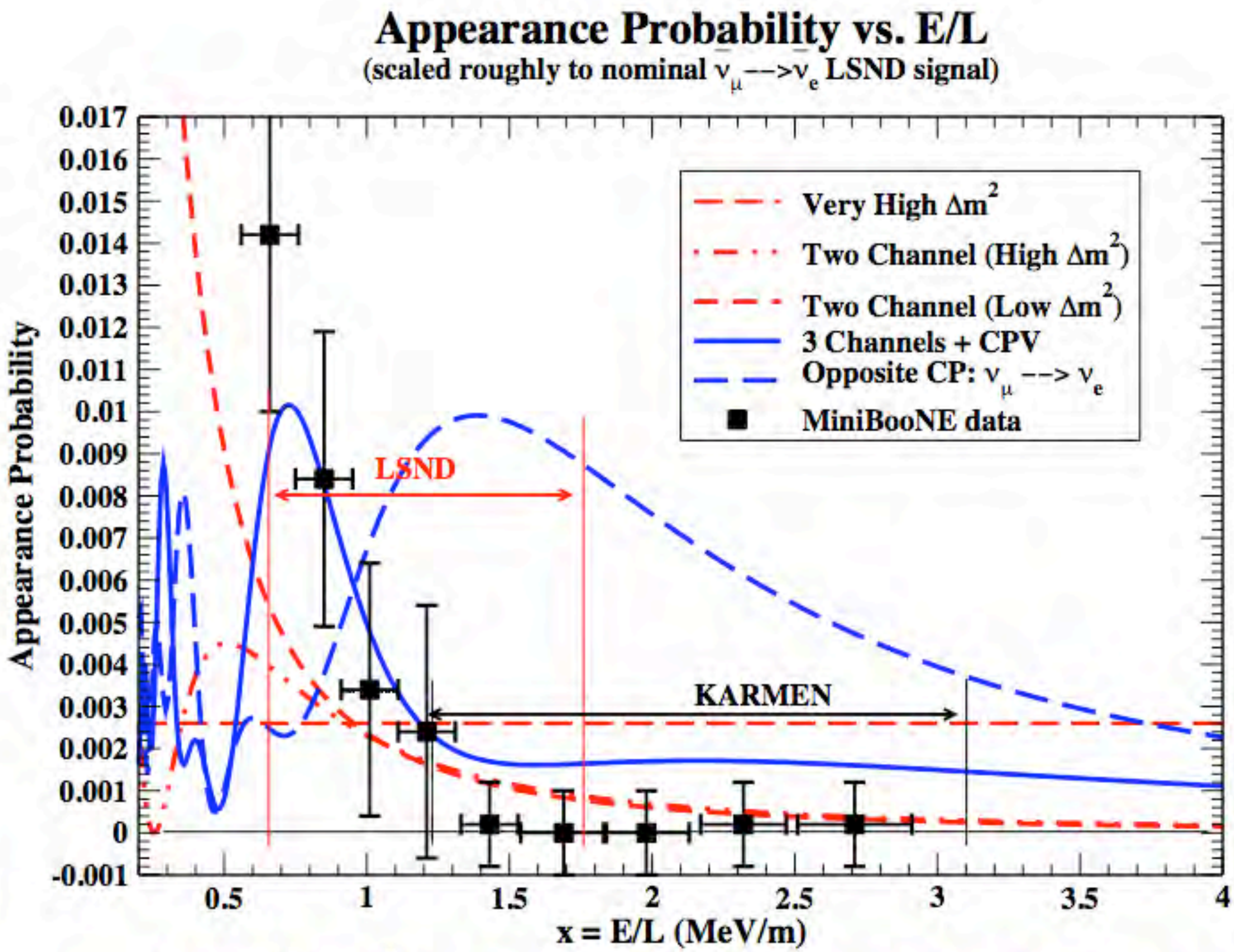
Two Detectors Are Better Than One?



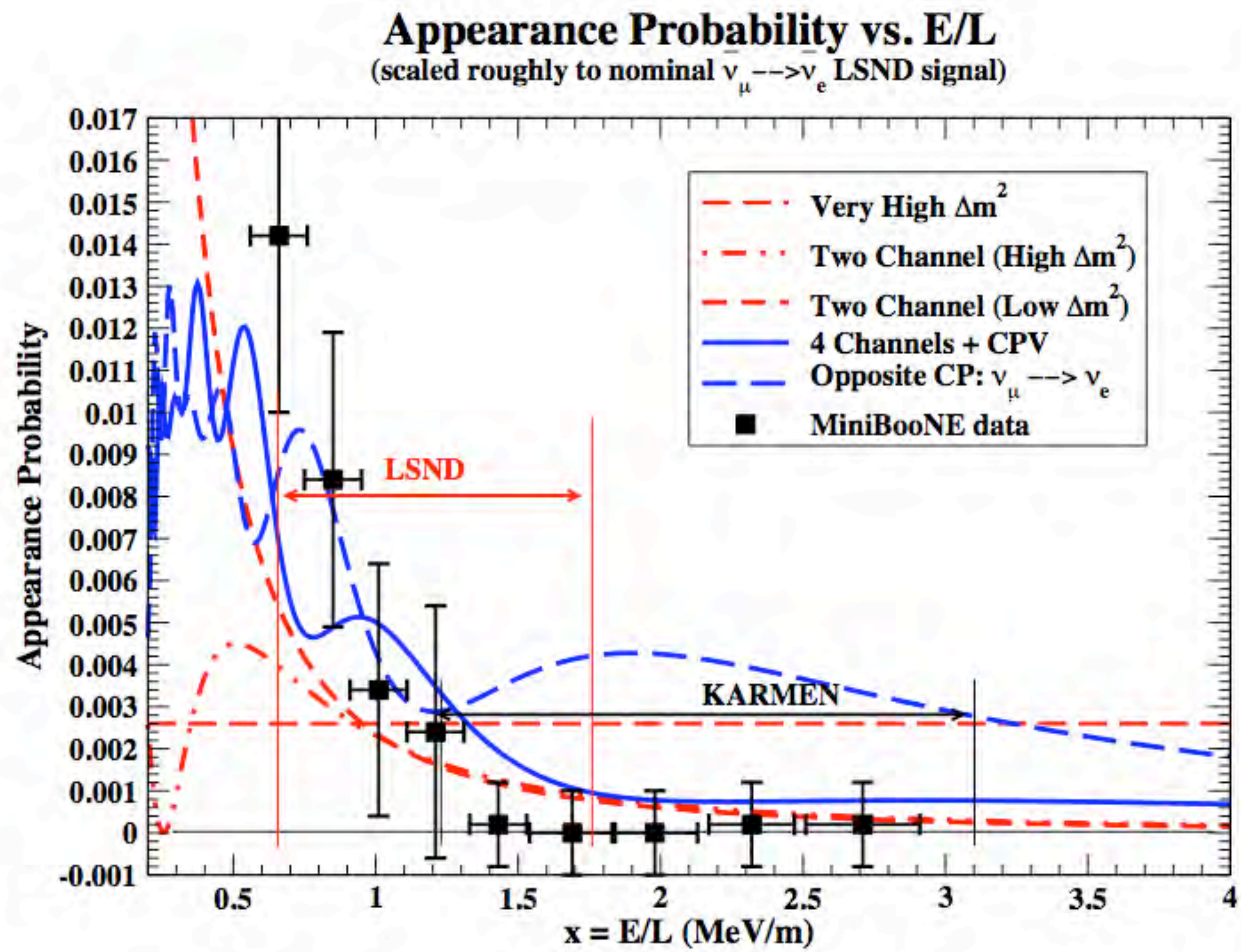
Only if the Near One Is Close Enough!



T. Goldman, G.J. Stephenson, Jr., B.H.J. McKellar, *Phys. Rev.* **D75** (2007) 091301(R)



$0.002*(\sin^2(3x)+\sin^2(1.65x))+0.004*\sin^2(1.35x)$ +/-$0.004*(\sin(6x)-\sin(3.3x)-\sin(2.7x))$


Appearance Probability vs. E/L
(scaled roughly to nominal $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ LSND signal)
Appearance Probability
x = E/L (MeV/m)
Very High Δm^2
Two Channel (High Δm^2)
Two Channel (Low Δm^2)
4 Channels + CPV
Opposite CP: $\nu_\mu \rightarrow \nu_e$
MiniBooNE data
LSND
KARMEN

CONCLUSIONS

- Even within the see-saw framework there could easily be 5 (and even 6) neutrino mass differences and so 4 (or 5) independent oscillation scales. [Use matter effects to find light steriles in exp'ts?]
- Analyses of oscillation data in terms of 2X2 mixing can miss significant physics.
- A global, multichannel analysis with allowance for path-dependent matter effects is essential before firm conclusions can be drawn.

CONCLUSIONS [II]

- No strong constraints on mostly sterile mass eigenstates with small overlap to active states. [Whatever protects hierarchy for gauge bosons also keeps steriles light?]
- Larger mass differences and more independent oscillation scales possible.
- Light pseudo–Dirac pairs appear unlikely to be in agreement with data.

Table 18

E_6 and its subgroups with $U_1^{em} \times SU_3^c$

Group	No. max. subgroups	Satisfactory maximal subgroups	Unsatisfactory maximal subgroups
E_6	8	F_4, $SO_{10} \times U_1$, $SU_2 \times SU_6$, $SU_3 \times SU_3 \times SU_3$	G_2, SU_3, $SU_3 \times G_2$, $[Sp_8]$
F_4	5	SO_9, $SU_3 \times SU_3$	SU_2, $SU_2 \times G_2$, $[SU_2 \times Sp_6]$
SO_9	5	SO_8, $SU_2 \times SU_4$	SU_2, $SU_2 \times SU_2$, $SU_2 \times SU_2 \times Sp_4$, $SO_7 \times U_1$*
SO_8	4	SO_7, $SU_4 \times U_1$	SU_3, $SU_2 \times Sp_4$, $SU_2 \times SU_2 \times SU_2 \times SU_2$
SO_7	3	SU_4	G_2, $SU_2 \times SU_2 \times SU_2$, $Sp_4 \times U_1$
SU_4	3	$SU_3 \times U_1$	Sp_4, $SU_2 \times SU_2$
SO_{10}	6	$SU_5 \times U_1$, $SU_2 \times SU_2 \times SU_4$, SO_9, $SU_2 \times SO_7$, $SO_8 \times U_1$	Sp_4, $Sp_4 \times Sp_4$
SU_6	7	$SU_5 \times U_1$, $SU_2 \times U_1 \times SU_4$, $SU_3 \times SU_3 \times U_1$	SU_3, $SU_2 \times SU_3$, $[SU_4]$, $[Sp_6]$
SU_5	3	$SU_4 \times U_1$, $SU_2 \times U_1 \times SU_3$	Sp_4

* See discussion for table 40.

R. Slansky, *Phys. Rept.* **79** (1981) 1

Table 49
Branchings of E_6 representations

$E_6 \supset F_4$

$(100000) = 27 = 1 + 26$

$(000001) = 78 = 26 + 52$

$(000100) = 351 = 26 + 52 + 273$

$(000020) = 351' = 1 + 26 + 324$

$(100010) = 650' = 1 + 26_1 + 26_2 + 273 + 324$

$(100001) = 1728 = 26 + 52 + 273 + 324 + 1053$

$(000002) = 2430 = 324 + 1053 + 1053'$

$(001000) = 2925 = 52 + 273_1 + 273_2 + 1053 + 1274$

$E_6 \supset SO_{10} \times U_1$ (Value of U_1 generator in parenthesis)

$27 = 1(4) + 10(-2) + 16(1)$

$78 = 1(0) + 45(0) + 16(-3) + \overline{16}(3)$

$351 = 10(-2) + \overline{16}(-5) + 16(1) + 45(4) + 120(-2) + 144(1)$

$351' = 1(-8) + 10(-2) + \overline{16}(-5) + 54(4) + \overline{126}(-2) + 144(1)$

$650 = 1(0) + 10(6) + 10(-6) + 16(-3) + \overline{16}(3) + 45(0) + 54(0) + 144(-3) + \overline{144}(3) + 210(0)$

$1728 = 1(4) + 10(-2) + 16_1(1) + 16_2(1) + \overline{16}(7) + 45(4) + 120(-2) + 126(-2) + 144(1) + \overline{144}(-5) + 210(4) + 320(-2) + 560(1)$

$2430 = 1(0) + 16(-3) + \overline{16}(3) + 45(0) + 126(-6) + \overline{126}(6) + 210(0) + 560(-3) + \overline{560}(3) + 770(0)$

$2925 = 16(-3) + \overline{16}(3) + 45_1(0) + 45_2(0) + 120_1(6) + 120_2(-6) + 144(-3) + \overline{144}(3) + 210(0) + 560(-3) + \overline{560}(3) + 945(0)$

Table 50

Guide to projection matrices for $E_6 \supset \cdots \supset U_1^{em} \times SU_3^c$. The U_1 factors may be found in table 18. The factor X in $P(X \supset Y)$ is chosen to be simple; it is underlined when ambiguity is possible