TESTING THE SEE-SAW MECHANISMS WITHIN THE *SU*(5) FRAMEWORK

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INFO 07
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- I.D. and Irina Mocioiu, in preparation.
- I.D. and Pavel Fileviez Pérez, **hep-ph/0612216**, **hep-ph/0606062**.
- I.D., P.F.P. and German Rodrigo **hep-ph/0607208**; I.D., P.F.P. and Ricardo Gonzalez Felipe, **hep-ph/0512068;** I.D. and P.F.P., **hep-ph/0504276**.

OUTLINE OF THE PRESENTATION

- **FERMION MASSES IN *SU*(5)**
- **PROTON DECAY**
- **UNIFICATION**
- **SIMPLE *SU*(5) MODELS AND THEIR PREDICTIONS**
- **CONCLUSIONS**

FERMION MASSES…

NEUTRINO MASSES

The Standard Model (SM) content requires higher-dimensional operators† to accommodate massive neutrinos. The lowest order¤ one is the so-called $d = 5$ operator.

$$\mathcal{L} \sim \frac{\mathcal{O}^d}{\Lambda^{d-4}} \quad \overset{d=5}{\Longrightarrow} \quad Y_{ab}\frac{L_a L_b \Psi_D \Psi_D}{\Lambda} \quad \overset{\nu\ \text{mass}}{\Longrightarrow} \quad (m_\nu)_{ab} = Y_{ab}\frac{\langle\Psi_D\rangle^2}{\Lambda}$$

$$L_a = \underbrace{(\mathbf{1}, \mathbf{2}, -1/2)_a}_{(SU(3), SU(2), U(1))} \qquad \Psi_D = (\mathbf{1}, \mathbf{2}, 1/2) \qquad L = \begin{pmatrix} \nu \\ e \end{pmatrix} \ \Psi_D = \begin{pmatrix} \Psi^1 + \imath\Psi^2 \\ \Psi^3 + \imath\Psi^4 \end{pmatrix}$$

$$\boxed{m_\nu \sim 10^{-1}\,\text{eV}, \qquad Y_{ab} \sim 1, \qquad \langle\Psi_D\rangle \sim 10^2\,\text{GeV} \qquad \Rightarrow \Lambda \leq 10^{14}\,\text{GeV}}$$

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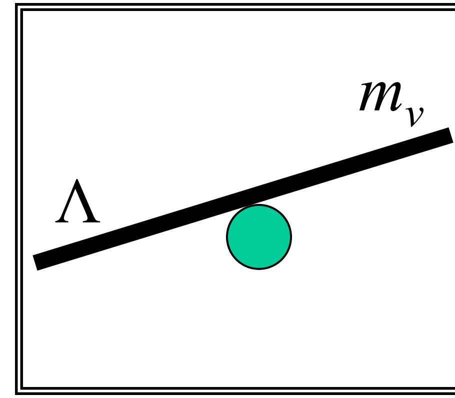
† S. Weinberg (1979).

¤ See the James Jankins talk for a general survey of the relevant higher-dimensional operators. See also talk by Jennifer Kile.



SEESAW MECHANISM

(Why are the m_ν elements so small?)

- TYPE I[■]: fermion(s) — $(\mathbf{1},\mathbf{1},0)$
- TYPE II[†]: scalar — $(\mathbf{1},\mathbf{3},1)$
- TYPE III[‡]: fermion(s) — $(\mathbf{1},\mathbf{3},0)$

The SM transformation properties

■ P. Minkowski (1977); T. Yanagida (1979); M. Gell-Mann, P. Ramond and R. Slansky (1979); R.N. Mohapatra and G. Senjanović (1980).

† G. Lazarides, Q. Shafi and C. Wetterich (1981); R.N. Mohapatra and G. Senjanović (1981).

‡ R. Foot, H. Lew, X.G. He and G.C. Joshi (1989); E. Ma (1998).

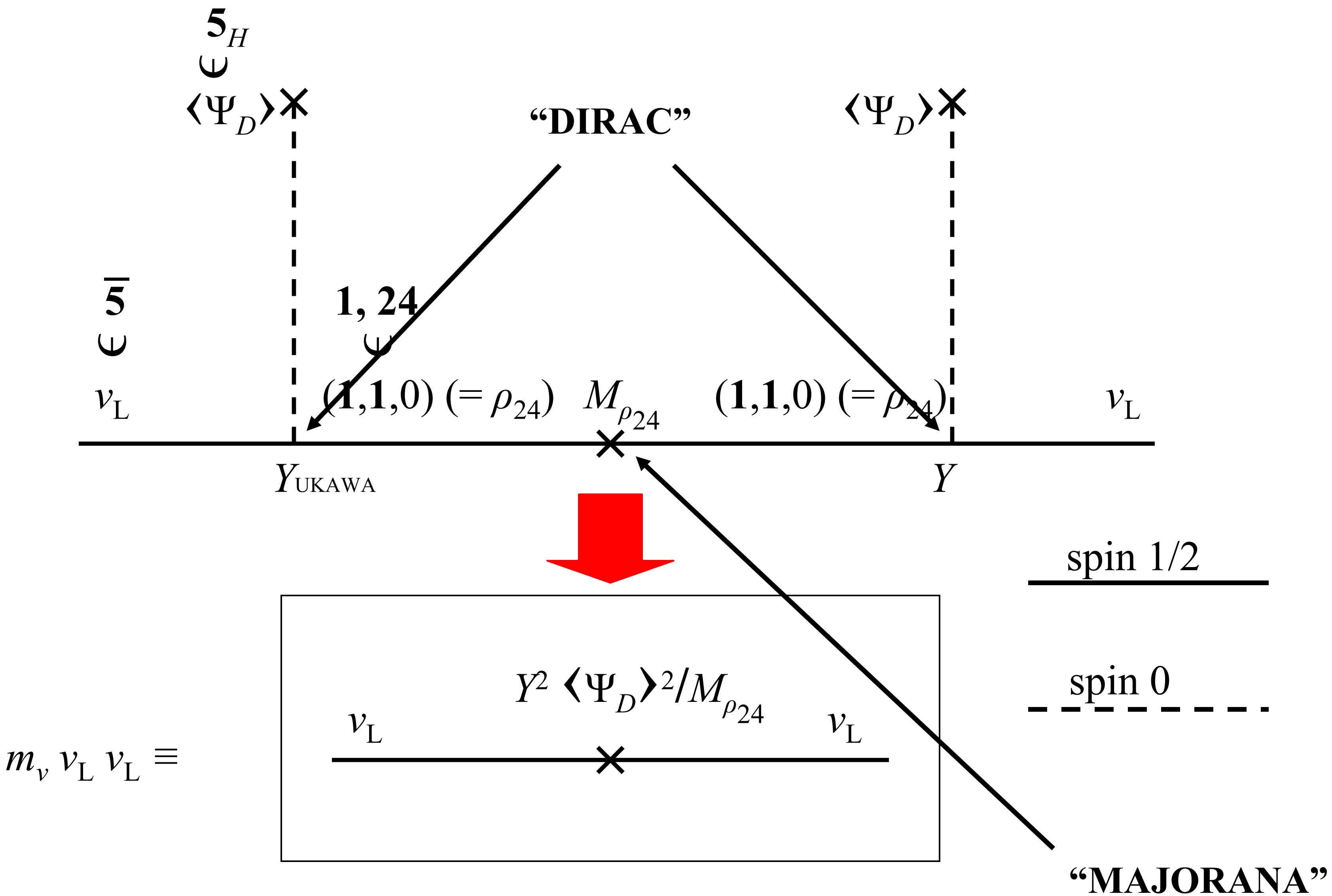
SEESAW MECHANISM IN *SU*(5)

			SM	*SU*(5)
•TYPE I:	fermion(s)	—	(**1**,**1**,0)	**1**, **24**‡
•TYPE II:	scalar	—	(**1**,**3**,1)	**15**†
•TYPE III:	fermion(s)	—	(**1**,**3**,0)	**24**‡

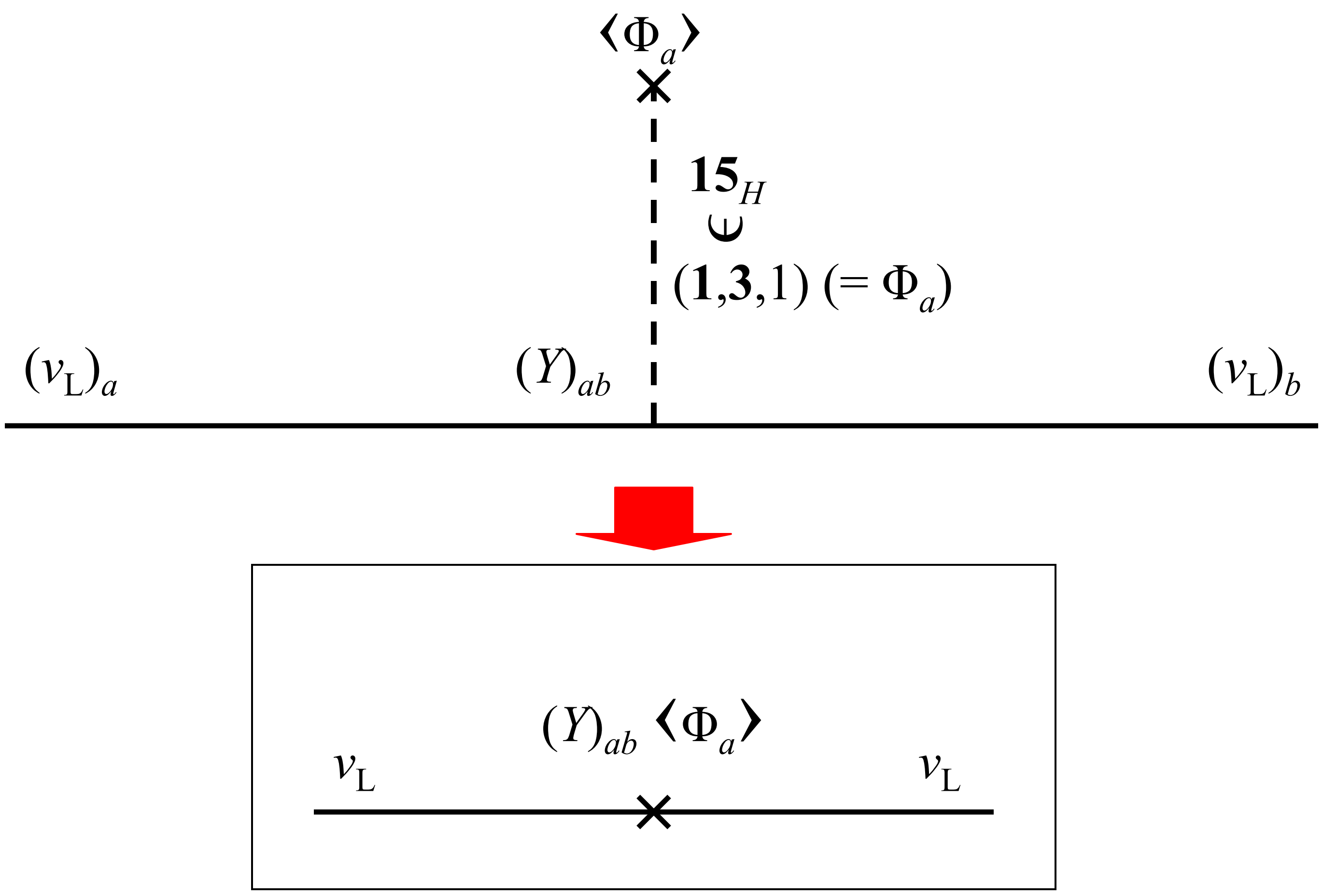
† I.D. and P.F.P. (2005); I.D., P.F.P. and German Rodrigo (2006); I.D., P. F.P. and R. Gonzalez Felipe (2006). I.D. and Irina Mocioiu, in preparation.

‡ B. Bajc and G. Senjanović (2006); I.D. and P.F.P. (2006), P.F.P. (2007).

TYPE I SEESAW IN $SU(5)$

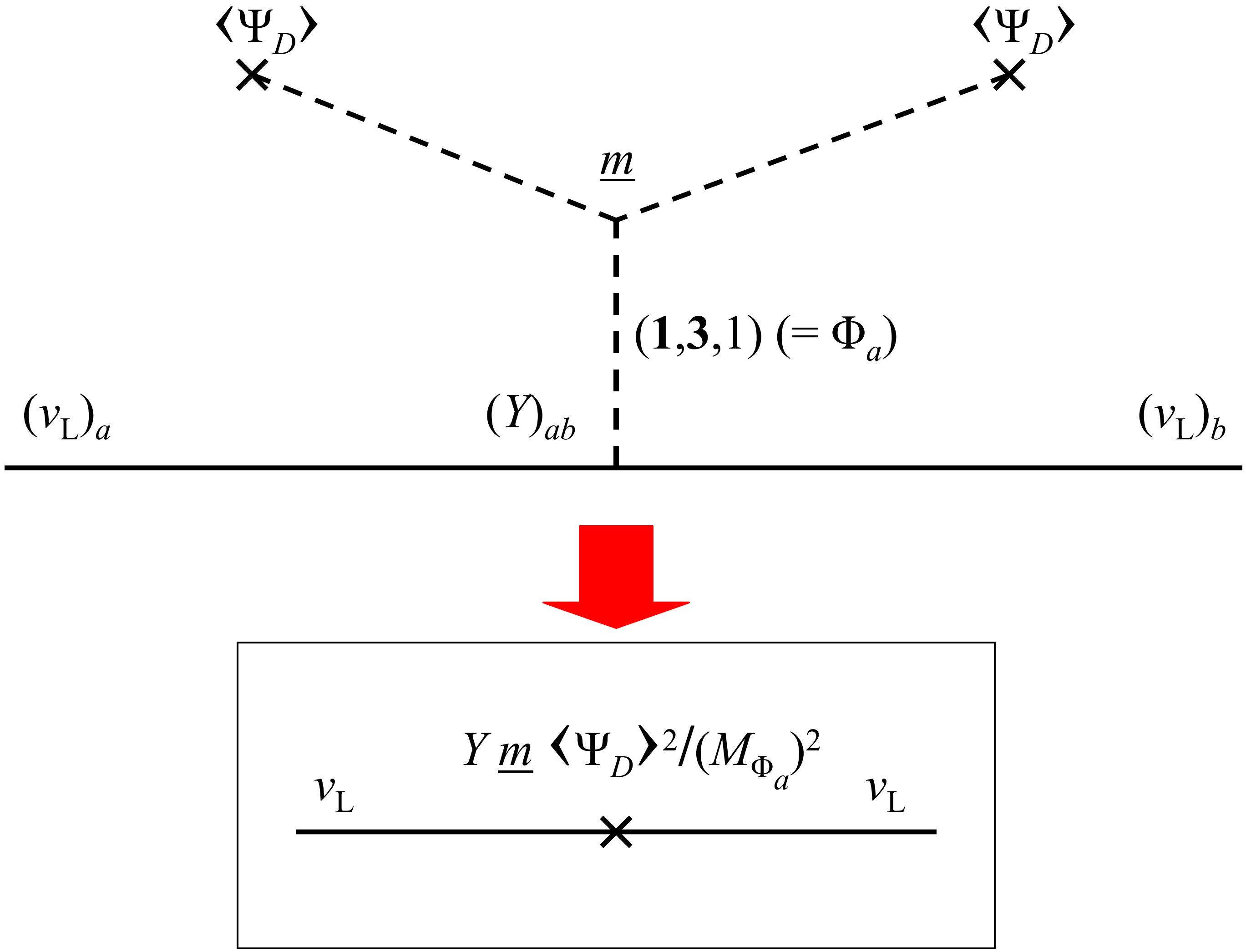


TYPE II SEESAW IN $SU(5)$[†]



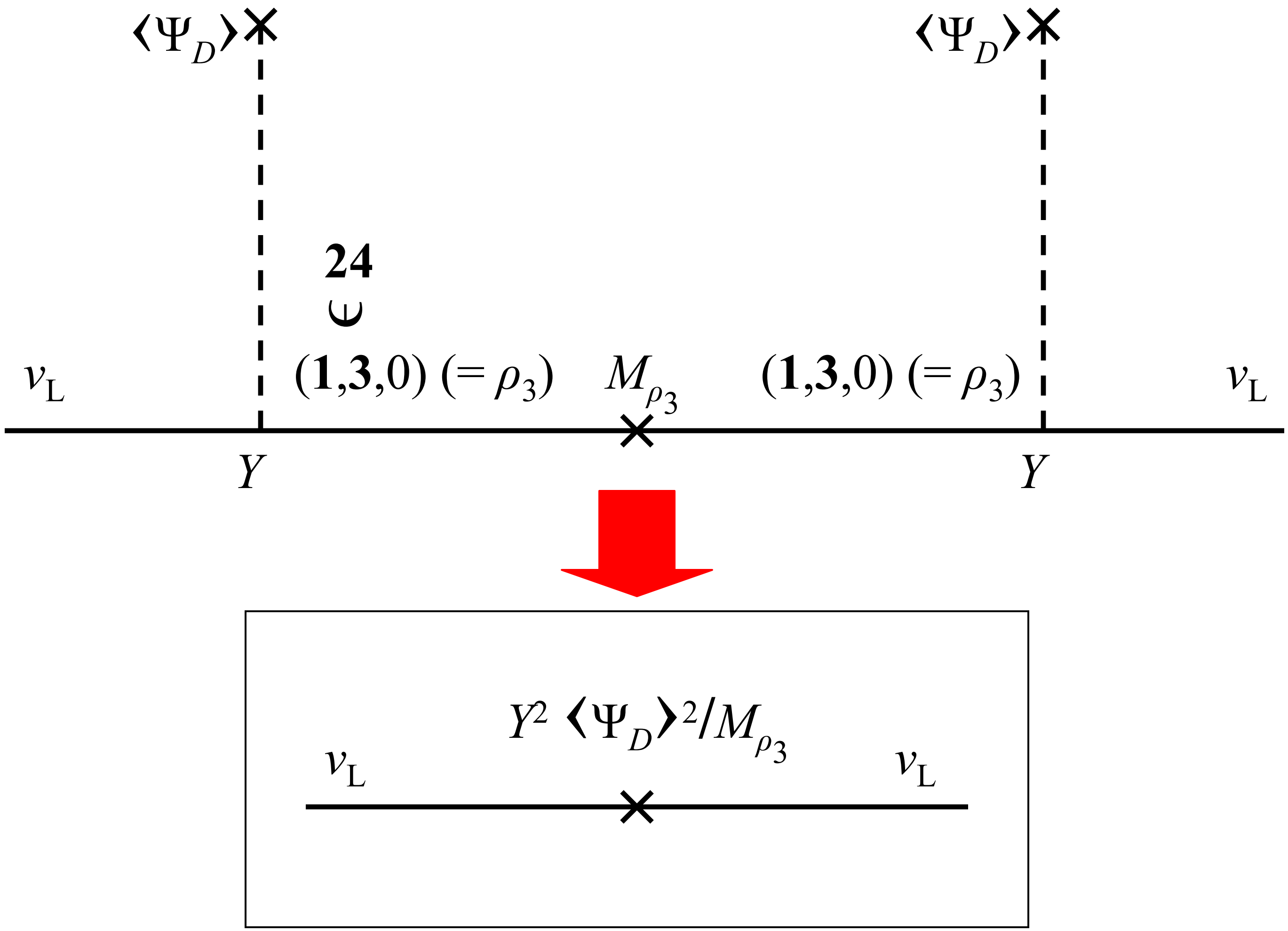
[†]F. Buccella, G.B. Gelmini, A. Masiero and M. Roncadelli (1984).

TYPE II SEESAW IN $SU(5)$[†]



[†] I.D. and P.F.P. (2005).

TYPE III SEESAW IN $SU(5)$



CHARGED FERMION MASSES IN *SU*(5)

$$\overline{\mathbf{5}}_a = (\mathbf{1},\mathbf{2},-1/2)_a + (\overline{\mathbf{3}},\mathbf{1},1/3)_a$$

$$L_a \equiv (\mathbf{1},\mathbf{2},-1/2)_a$$
$$d_a^C \equiv (\overline{\mathbf{3}},\mathbf{1},1/3)_a$$

⇩

$$(\overline{\mathbf{5}}_i)_a = \begin{pmatrix} d_1^C \\ d_2^C \\ d_3^C \\ \nu \\ e \end{pmatrix}_a$$

$$\mathbf{10}_a = (\mathbf{1},\mathbf{1},1)_a + (\mathbf{3},\mathbf{2},1/6)_a + (\overline{\mathbf{3}},\mathbf{1},-2/3)_a$$

$$e_a^C \equiv (\mathbf{1},\mathbf{1},1)_a$$
$$Q_a \equiv (\mathbf{3},\mathbf{2},1/6)_a$$
$$u_a^C \equiv (\overline{\mathbf{3}},\mathbf{1},-2/3)_a$$

⇩

$$(\mathbf{10}^{ij})_a = \begin{pmatrix} 0 & u_3^C & -u_2^C & -u^1 & -d^1 \\ -u_3^C & 0 & u_1^C & -u^2 & -d^2 \\ u_2^C & -u_1^C & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^C \\ d^1 & d^2 & d^3 & e^C & 0 \end{pmatrix}_a$$

$$\mathbf{10} \times \mathbf{10} = \overline{\mathbf{5}} \oplus \overline{\mathbf{45}} \oplus \overline{\mathbf{50}}$$

$$\mathbf{10} \times \overline{\mathbf{5}} = \mathbf{5} \oplus \mathbf{45}$$

H. Georgi and S.L. Glashow (1974).

FERMION MASSES IN $SU(5)$

Up quark masses:

$$\mathbf{10} \times \mathbf{10} = \overline{\mathbf{5}} \oplus \overline{\mathbf{45}} \oplus \overline{\mathbf{50}}$$

Down quark and charged lepton masses:

$$\mathbf{10} \times \overline{\mathbf{5}} = \mathbf{5} \oplus \mathbf{45}$$

$$\mathbf{5}_H = \Psi = (\Psi_D,\ \Psi_T) = \underline{(\mathbf{1}, \mathbf{2}, 1/2)} + (\mathbf{3}, \mathbf{1}, -1/3)$$

$$\mathbf{45}_H = \Delta = (\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7) = (\mathbf{8}, \mathbf{2}, 1/2) + (\overline{\mathbf{6}}, \mathbf{1}, -1/3) + (\mathbf{3}, \mathbf{3}, -1/3) + (\overline{\mathbf{3}}, \mathbf{2}, -7/6) + (\mathbf{3}, \mathbf{1}, -1/3) + (\overline{\mathbf{3}}, \mathbf{1}, 4/3) + \underline{(\mathbf{1}, \mathbf{2}, 1/2)}$$

Neutrino masses:

$$\overline{\mathbf{5}} \times \overline{\mathbf{5}} = \overline{\mathbf{10}} \oplus \overline{\mathbf{15}}$$

$$\mathbf{15}_H = \Phi = (\Phi_a,\ \Phi_b,\ \Phi_c) = \underline{(\mathbf{1}, \mathbf{3}, 1)} + (\mathbf{3}, \mathbf{2}, 1/6) + (\mathbf{6}, \mathbf{1}, -2/3)$$

FERMION MASSES IN $SU(5)$

(type II seesaw)

$$\mathcal{L} = (Y_1)_{ab}\ (\mathbf{10}^{\alpha\beta})_a(\overline{\mathbf{5}}_\alpha)_b\mathbf{5}^*_{H\beta} + (Y_2)_{ab}\ (\mathbf{10}^{\alpha\beta})_a(\overline{\mathbf{5}}_\delta)_b\mathbf{45}^{*\delta}_{H\alpha\beta}$$

$$+\ (Y_3)_{ab}\ (\overline{\mathbf{5}}_\alpha)_a(\overline{\mathbf{5}}_\beta)_b\mathbf{15}^{\alpha\beta}_H$$

$$+\ \epsilon_{\alpha\beta\gamma\delta\epsilon}\left[(Y_4)_{ab}\ (\mathbf{10}^{\alpha\beta})_a(\mathbf{10}^{\gamma\delta})_b\mathbf{5}^\epsilon_H + (Y_5)_{ab}\ (\mathbf{10}^{\alpha\beta})_a(\mathbf{10}^{\zeta\gamma})_b\mathbf{45}^{\delta\epsilon}_{H\zeta}\right] + \cdots$$

$M_D = \left(Y_1^T\ v_5^*\ +\ 2\ Y_2^T\ v_{45}^*\right)/\sqrt{2}$	$D_C^T\ M_D\ D = M_D^{\mathrm{diag}}$
$M_E = \left(Y_1\ v_5^*\ -6\ Y_2\ v_{45}^*\right)/\sqrt{2}$	$E_C^T\ M_E\ E = M_E^{\mathrm{diag}}$
$M_N = Y_3\ v_{15}$	$N^T\ M_N\ N = M_N^{\mathrm{diag}}$
$M_U = \left[4\ (Y_4^T + Y_4)\ v_5\ -\ 8\ (Y_5^T - Y_5)\ v_{45}\right]/\sqrt{2}$	$U_C^T\ M_U\ U = M_U^{\mathrm{diag}}$

$$\langle \mathbf{5}_H \rangle = v_5/\sqrt{2} \qquad \langle \mathbf{15}_H \rangle = v_{15}$$

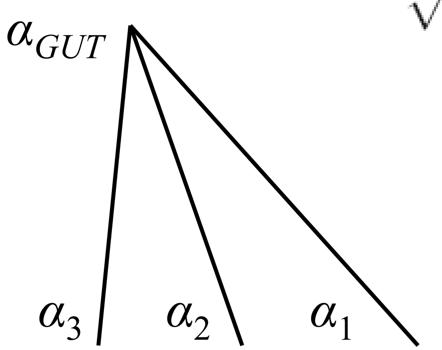
$$\langle \mathbf{45}_H \rangle_1^{15} = \langle \mathbf{45}_H \rangle_2^{25} = \langle \mathbf{45}_H \rangle_3^{35} = v_{45}/\sqrt{2}, \quad \sum_{i=1}^{3} \langle \mathbf{45}_H \rangle_i^{i5} = -\langle \mathbf{45}_H \rangle_4^{45}$$

$SU(5)$ SYMMETRY BREAKING

$$\mathbf{24}_H = \Sigma = (\Sigma_8,\ \Sigma_3,\ \overbrace{\Sigma_{(3,2)},\ \Sigma_{(\bar{3},2)}}^{M_{GUT}=M_{(X,Y)}},\ \Sigma_{24})$$
$$= (\mathbf{8},\mathbf{1},0) + \underline{(\mathbf{1},\mathbf{3},0)} + (\mathbf{3},\mathbf{2},-5/6) + (\bar{\mathbf{3}},\mathbf{2},5/6) + \underline{(\mathbf{1},\mathbf{1},0)}$$

$M_{GUT} \sim 10^{15-16}$ GeV

$SU(5)$: $\langle\Sigma_{24}\rangle = \frac{v}{\sqrt{30}}\begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix}$ ⇒ $SU(3) \times SU(2) \times U(1)$



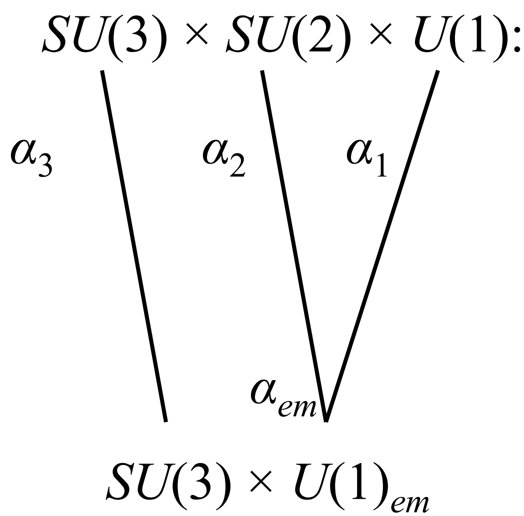
10^2 GeV

$SU(3) \times SU(2) \times U(1)$:

$$\langle\Sigma_3\rangle = v'$$
$$\langle\mathbf{5}_H\rangle = v_5/\sqrt{2}$$
$$\langle\mathbf{15}_H\rangle = v_{15}$$
$$\langle\mathbf{45}_H\rangle_1^{15} = \langle\mathbf{45}_H\rangle_2^{25} = \langle\mathbf{45}_H\rangle_3^{35} = v_{45}/\sqrt{2}$$

⇒ $SU(3) \times U(1)_{em}$

$SU(3) \times SU(2) \times U(1)$ SYMMETRY BREAKING‡



$$\langle \mathbf{5}_H \rangle = v_5/\sqrt{2}$$

$$\langle \Sigma_3 \rangle = v'$$

$$\langle \mathbf{15}_H \rangle = v_{15}$$

$$\langle \mathbf{45}_H \rangle_1^{15} = \langle \mathbf{45}_H \rangle_2^{25} = \langle \mathbf{45}_H \rangle_3^{35} = v_{45}/\sqrt{2}$$

$$\Downarrow$$

$$\rho = \frac{v_5^2 + v_{45}^2 + 4v_{15}^2 + 4v'^2}{v_5^2 + v_{45}^2 + 8v_{15}^2}$$

‡ I.D. and Irina Mocioiu, in preparation.

CHARGED FERMION MASSES

(non-renormalizable terms with $\Psi \equiv 5_H$ and $\Sigma \equiv 24_H$)

$$\mathcal{L} = \underline{\Psi_i^*(10^{ij})_a g_{ab}(\bar{5}_j)_b} + \Psi_i^* \frac{\Sigma^i{}_j}{\Lambda}(10^{jk})_a g_{1ab}(\bar{5}_k)_b + \Psi_i^*(10^{ij})_a g_{2ab}\frac{\Sigma^k{}_j}{\Lambda}(\bar{5}_k)_b$$

$$+ \epsilon_{ijklm}\left(\underline{(10^{ij})_a f_{ab}(10^{kl})_b \Psi^m} + (10^{ij})_a f_{1ab}(10^{kl})_b \frac{\Sigma^m{}_n}{\Lambda}\Psi^n + (10^{ij})_a f_{2ab}(10^{kn})_b \Psi^l \frac{\Sigma^m{}_n}{\Lambda}\right)$$

$$Y_D = -\underline{g^T} + 3\frac{v}{\sqrt{30}\Lambda} g_1^T - 2\frac{v}{\sqrt{30}\Lambda} g_2^T + \cdots$$

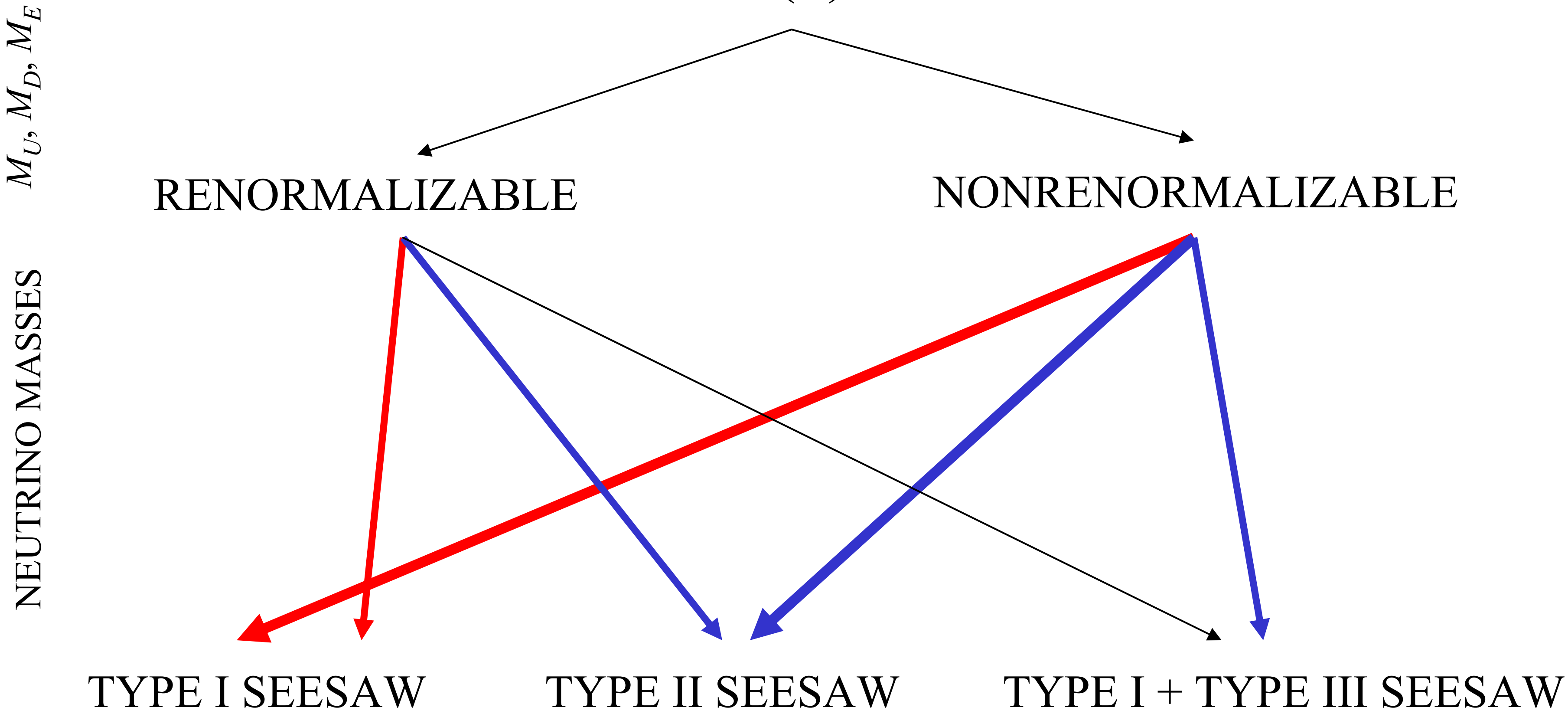
$$Y_E = -\underline{g} + 3\frac{v}{\sqrt{30}\Lambda} g_1 + 3\frac{v}{\sqrt{30}\Lambda} g_2 + \cdots$$

$$Y_U = \underline{4(f^T + f)} - 12\frac{v}{\sqrt{30}\Lambda}(f_1^T + f_1) - 2\frac{v}{\sqrt{30}\Lambda}(4f_2^T - f_2) + \cdots$$

$$\Lambda \leq \sqrt{\frac{2}{\alpha_{GUT}}} \times \frac{M_{GUT}}{Y_\tau - Y_b}, \qquad |(g_2)_{ij}| \leq \sqrt{4\pi}$$

•J.R. Ellis and M.K. Gaillard (1979).

SIMPLE *SU*(5) MODELS



PROTON DECAY IN *SU*(5)

(vector gauge boson *d*=6 contributions†)

$$O_1 = k^2\ \epsilon_{ijk}\ \epsilon_{\alpha\beta}\ \overline{u^C_{ia}}\ \gamma^\mu\ Q_{j\alpha a}\ \overline{e^C_b}\ \gamma_\mu\ Q_{k\beta b}$$
$$O_2 = k^2\ \epsilon_{ijk}\ \epsilon_{\alpha\beta}\ \overline{u^C_{ia}}\ \gamma^\mu\ Q_{j\alpha a}\ \overline{d^C_{kb}}\ \gamma_\mu\ L_{\beta b}$$

$$\Gamma(p \to \pi^0 e^+_\beta) = \frac{C(p,\pi)}{2} A_1^2 \left[A_S^{R\ 2} \left| V_1^{11} V_3^{1\beta} \right|^2 + A_S^{L\ 2} \left| V_1^{11} V_2^{\beta 1} + (V_1 K_1 V_{CKM} K_2)^{11} (V_2 K_2^* V_{CKM}^\dagger K_1^*)^{\beta 1} \right|^2 \right]$$
$$\Gamma(p \to K^0 e^+_\beta) = C(p,K) A_2^2 \left[A_S^{R\ 2} \left| V_1^{11} V_3^{2\beta} \right|^2 + A_S^{L\ 2} \left| V_1^{11} V_2^{\beta 2} + (V_1 K_1 V_{CKM} K_2)^{12} (V_2 K_2^* V_{CKM}^\dagger K_1^*)^{\beta 1} \right|^2 \right]$$

$$V_1 = U_C^\dagger U \qquad V_2 = E_C^\dagger D \qquad V_3 = D_C^\dagger E \qquad V_{UD} = U^\dagger D = K_1 V_{CKM} K_2$$

$$C(a,b) = \frac{(m_a^2 - m_b^2)^2}{8\pi m_a^3 f_\pi^2}\ A_L^2\ |\alpha|^2\ k^4, \qquad k^2 = 2\pi\alpha_{GUT} M_{(X,Y)}^{-2}$$

† P.F.P. hep-ph/0403286 ; I.D. and P.F.P. hep-ph/0409095.

PROTON DECAY IN *SU*(5)

(vector gauge boson *d*=6 contributions)

$$U_C = U \qquad D_C = E \qquad E_C = D$$

$$\Gamma(p \to \pi^0 e^+) = \frac{C(p,\pi)}{2} A_1^2 \left[A_S^{R\ 2} + A_S^{L\ 2} (1 + V_{ud}^2)^2 \right]$$

$$\tau^{\text{exp.}}(p \to \pi^0 e^+) > 4.4 \times 10^{33}\,\text{years}$$

$$\tau^{\text{tho.}}(p \to \pi^0 e^+) = 1.0 \times 10^{32} \alpha_{GUT}^{-2} \left(\frac{M_{GUT}}{10^{16}\,\text{GeV}} \right)^4 \,\text{years}$$

$$M_{GUT} > 2.6 \times 10^{16} \sqrt{\alpha_{GUT}}\,\text{GeV}$$

$A_1 = 1 + D + F,\ A_2 = 1 + \frac{m_p}{m_B}(D - F),\ m_p = 938.3\,\text{MeV},\ D = 0.81,\ F = 0.44,\ m_B = 1150\,\text{MeV},$
$f_\pi = 139\,\text{MeV},\ A_L = 1.25,\ \alpha = 0.015\,\text{GeV}^3,\ |V_{ud}| = 0.97377,\ |V_{ub}| = 3.96 \times 10^{-3},\ A_S^L = A_S^R = 2.5$

PROTON DECAY IN GUTS

(vector gauge boson d=6 contributions‡)

$$(V_1 V_{UD})^{1\alpha} = 0; \qquad V_2^{\alpha\beta} = 0 \qquad V_3^{\alpha\beta} = 0 \qquad (\alpha = 1 \text{ or } \beta = 1)$$

$$\Gamma(p \to K^0 \mu^+) = C(p, K) A_2^2 \left[A_S^{R\,2} + A_S^{L\,2} \right] |V_{ub}|^2$$

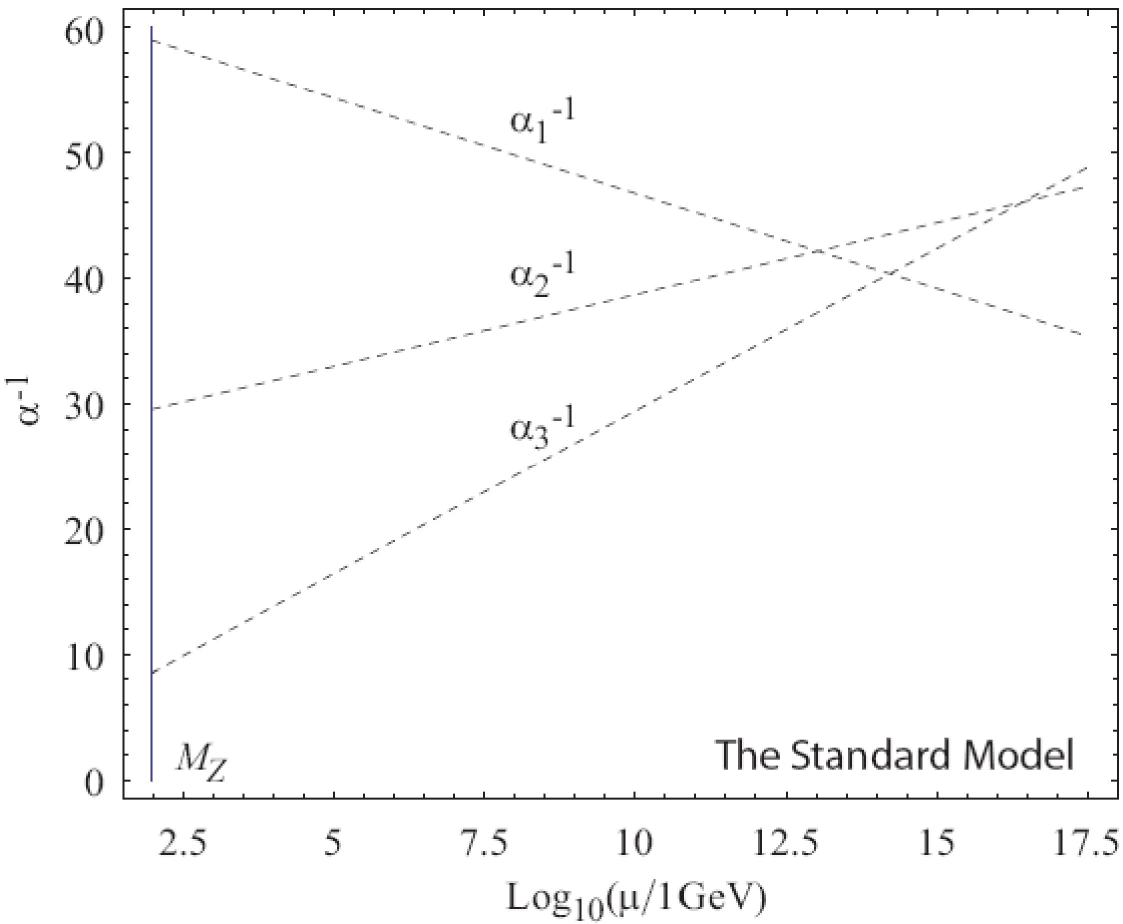
$$\tau^{\text{exp.}} > 4.4 \times 10^{33} \text{ years}$$

$$\tau^{\text{tho.}} = 2.3 \times 10^{38} \alpha_{GUT}^{-2} \left(\frac{M_{GUT}}{10^{16}\,\text{GeV}} \right)^4 \text{ years}$$

$$M_{GUT} > 9.9 \times 10^{14} \sqrt{\alpha_{GUT}}\,\text{GeV}$$

‡ I.D. and P. F.P., "**How long could we live?**" *Phys. Lett. B* **625** (2005), hep-ph/0410198.

UNIFICATION OF GAUGE COUPLINGS IN THE SM



DATA: PDG 2006

UNIFICATION OF GAUGE COUPLINGS

$$\frac{B_{23}}{B_{12}} = \frac{5}{8}\frac{\sin^2\theta_W(M_Z) - \alpha_{em}(M_Z)/\alpha_s(M_Z)}{3/8 - \sin^2\theta_W(M_Z)} = 0.716 \pm 0.005 \qquad (1)$$

B-test

$$\ln\frac{M_{GUT}}{M_Z} = \frac{16\pi}{5}\frac{3/8 - \sin^2\theta_W(M_Z)}{\alpha_{em}(M_Z)\; B_{12}} = \frac{184.9 \pm 0.2}{B_{12}} \qquad (2)$$

The GUT scale relation

$$B_{ij} = B_i - B_j, \qquad B_i = \sum_I b_{iI}\, r_I, \qquad r_I = \frac{\ln M_{GUT}/M_I}{\ln M_{GUT}/M_Z}, \qquad (0 \le r_I \le 1)$$

$$B_1^{\rm SM} = 40/10 + 1/10 \qquad B_2^{\rm SM} = -20/6 + 1/6 \qquad B_3^{\rm SM} = -7$$

$$B_{23}^{\rm SM}/B_{12}^{\rm SM} = 0.53$$

	Higgsless SM	Ψ_D	Ψ_T	Σ_8	Σ_3
ΔB_{23}	$\frac{11}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{3}$
ΔB_{12}	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15}$	0	$-\frac{1}{3}$

Georgi-Glashow $SU(5)$‡

The Georgi-Glashow GUT content

HIGGS SECTOR:

$$\mathbf{24}_H = \Sigma = (\Sigma_8,\ \Sigma_3,\ \overbrace{\Sigma_{(3,2)},\ \Sigma_{(\bar{3},2)}}^{M_V = M_{GUT}},\ \Sigma_{24})$$
$$= (\mathbf{8},\mathbf{1},0) + (\mathbf{1},\mathbf{3},0) + (\mathbf{3},\mathbf{2},-5/6) + (\overline{\mathbf{3}},\mathbf{2},5/6) + (\mathbf{1},\mathbf{1},0)$$

$$\mathbf{5}_H = \Psi = (\Psi_D,\ \Psi_T) = (\mathbf{1},\mathbf{2},1/2) + (\mathbf{3},\mathbf{1},-1/3)$$

MATTER SECTOR:

$$\mathbf{10}_a = (\mathbf{1},\mathbf{1},1) + (\mathbf{3},\mathbf{2},1/6) + (\overline{\mathbf{3}},\mathbf{1},-2/3)$$

$$\overline{\mathbf{5}}_a = (\mathbf{1},\mathbf{2},-1/2) + (\overline{\mathbf{3}},\mathbf{1},1/3)$$

GG model is ruled out by experiments.

‡Georgi and Glashow (1974)

Doršner-Fileviez Pérez $SU(5)$ [‡]

HIGGS SECTOR:

$$\mathbf{24}_H = \Sigma = (\Sigma_8,\ \Sigma_3,\ \overbrace{\Sigma_{(3,2)},\ \Sigma_{(\bar{3},2)}}^{M_V = M_{GUT}\ (= M_{(X,Y)})},\ \Sigma_{24}) = (\mathbf{8},\mathbf{1},0) + (\mathbf{1},\mathbf{3},0) + (\mathbf{3},\mathbf{2},-5/6) + (\overline{\mathbf{3}},\mathbf{2},5/6) + (\mathbf{1},\mathbf{1},0)$$

$$\mathbf{5}_H = \Psi = (\Psi_D,\ \Psi_T) = (\mathbf{1},\mathbf{2},1/2) + (\mathbf{3},\mathbf{1},-1/3)$$

MATTER SECTOR:

$$\mathbf{10}_a = (\mathbf{1},\mathbf{1},1) + (\mathbf{3},\mathbf{2},1/6) + (\overline{\mathbf{3}},\mathbf{1},-2/3)$$

a = 1,2,3

$$\overline{\mathbf{5}}_a = (\mathbf{1},\mathbf{2},-1/2) + (\overline{\mathbf{3}},\mathbf{1},1/3)$$

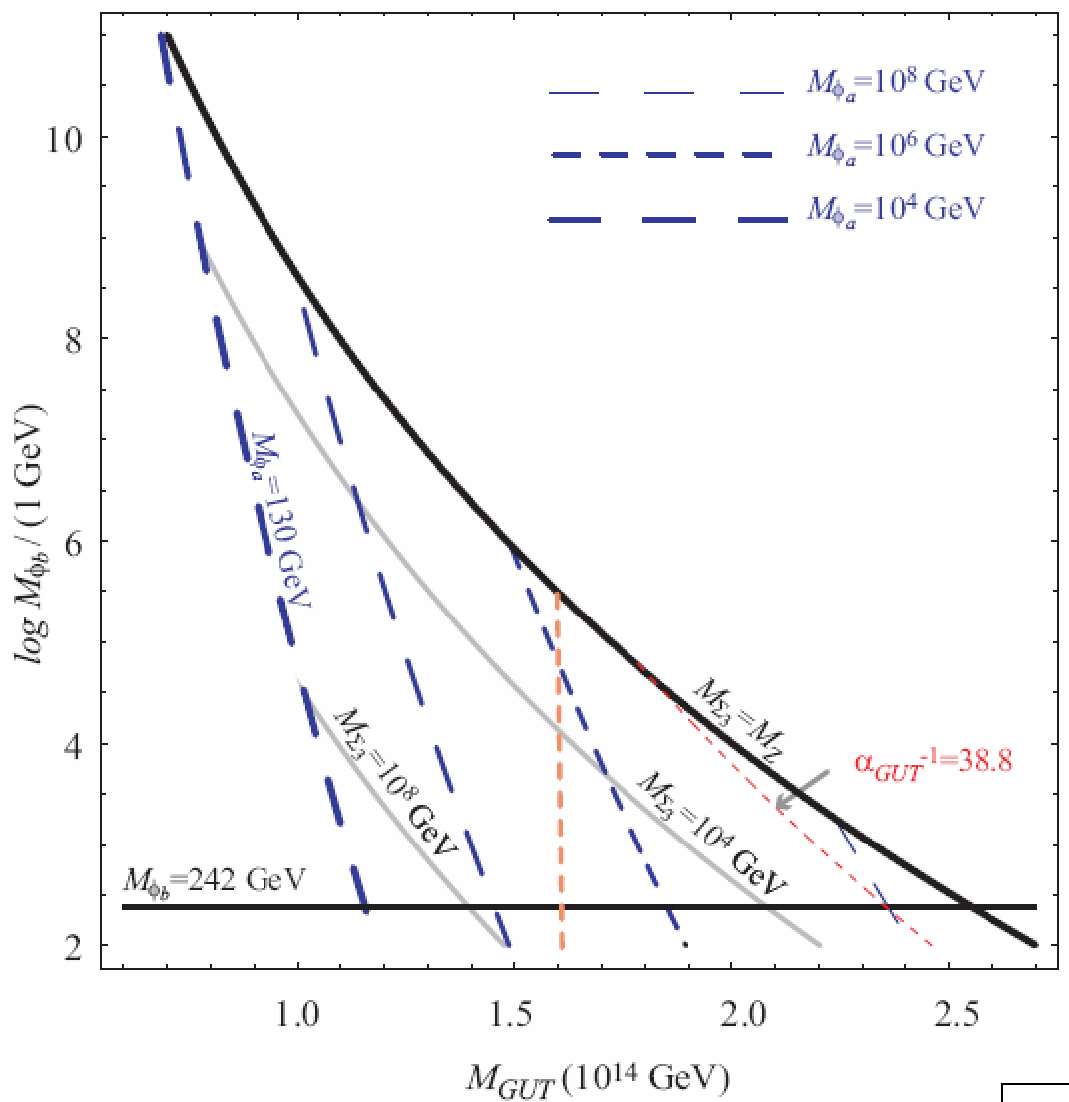
The Georgi-Glashow GUT content

EXTRA SCALAR:

$$\mathbf{15}_H = \Phi = (\underline{\Phi_a},\ \Phi_b,\ \Phi_c) = \underline{(\mathbf{1},\mathbf{3},1)} + (\mathbf{3},\mathbf{2},1/6) + (\mathbf{6},\mathbf{1},-2/3)$$

The Doršner-Fileviez Pérez $SU(5)$[‡] content

‡ I.D. and P.F.P., hep-ph/0504276; I.D., P.F.P. and R.G. Felipe hep-ph/0512068; I.D., P.F.P. and G. Rodrigo hep-ph/0607208.



PREDICTIONS:

i) Light leptoquarks Φ_b.

(See talk by Timur Rashba.)

ii) Light Σ_3 scalar.

iii) $\max(\tau_p^{\text{tho.}}) \sim 10\ \tau_p^{\text{exp.}}$.

	Higgsless SM	Ψ_D	Ψ_T	Σ_8	Σ_3	Φ_a	Φ_b	Φ_c
ΔB_{23}	$\frac{11}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{6}$	$-\frac{5}{6}$
ΔB_{12}	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15}$	0	$-\frac{1}{3}$	$-\frac{1}{15}$	$-\frac{7}{15}$	$\frac{8}{15}$

Georgi-Jarlskog $SU(5)$‡

HIGGS SECTOR:

$$24_H = \Sigma = (\Sigma_8,\ \Sigma_3,\ \overbrace{\Sigma_{(3,2)},\ \Sigma_{(\bar{3},2)}}^{M_V=M_{GUT}\ (=M_{(X,Y)})},\ \Sigma_{24})$$
$$= (\mathbf{8},\mathbf{1},0) + (\mathbf{1},\mathbf{3},0) + (\mathbf{3},\mathbf{2},-5/6) + (\overline{\mathbf{3}},\mathbf{2},5/6) + (\mathbf{1},\mathbf{1},0)$$

$$5_H = \Psi = (\Psi_D,\ \Psi_T) = (\mathbf{1},\mathbf{2},1/2) + (\mathbf{3},\mathbf{1},-1/3)$$

MATTER SECTOR:

$$\mathbf{10}_a = (\mathbf{1},\mathbf{1},1) + (\mathbf{3},\mathbf{2},1/6) + (\overline{\mathbf{3}},\mathbf{1},-2/3)$$

a = 1,2,3

$$\overline{\mathbf{5}}_a = (\mathbf{1},\mathbf{2},-1/2) + (\overline{\mathbf{3}},\mathbf{1},1/3)$$

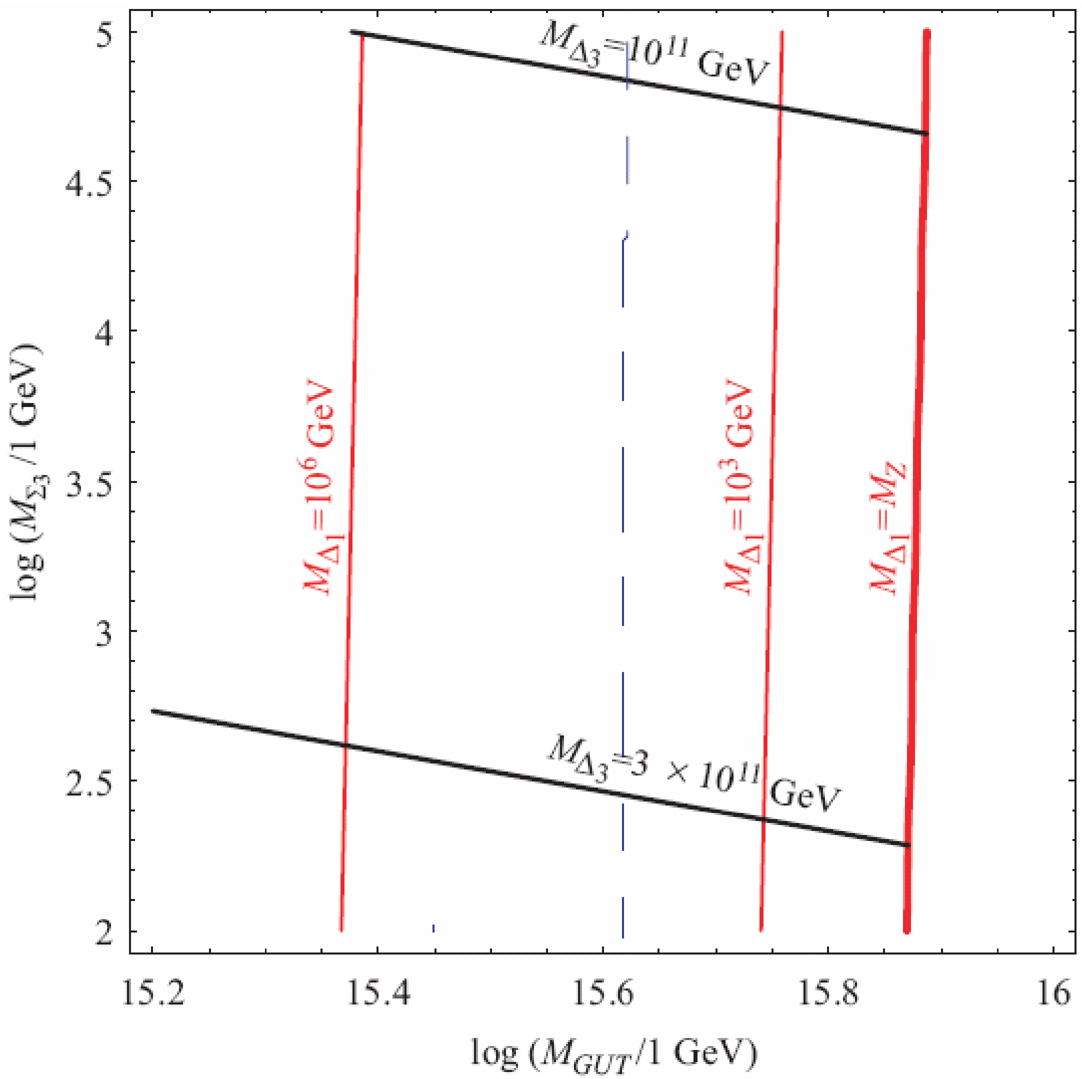
The Georgi-Glashow GUT content

The Georgi-Jarlskog $SU(5)$‡ content

EXTRA SCALAR:

$$45_H = \Delta = (\Delta_1,\Delta_2,\Delta_3,\Delta_4,\Delta_5,\Delta_6,\Delta_7) = (\mathbf{8},\mathbf{2},1/2) + (\overline{\mathbf{6}},\mathbf{1},-1/3)$$
$$+ (\mathbf{3},\mathbf{3},-1/3) + (\overline{\mathbf{3}},\mathbf{2},-7/6) + (\mathbf{3},\mathbf{1},-1/3) + (\overline{\mathbf{3}},\mathbf{1},4/3) + (\mathbf{1},\mathbf{2},1/2)$$

‡ I.D. and Irina Mocioiu, in preparation.



PREDICTIONS:

If the d=6 proton decay operators are not suppressed then the model with the Georgi-Glashow particle content with the type I seesaw is ruled out!

NOTE:

$$M_{GUT} > 2.6 \times 10^{16} \sqrt{\alpha_{GUT}}\ \mathrm{GeV}$$

$$M_{\mathrm{SCALAR}} > 10^{12}\ \mathrm{GeV}$$

	SM	Ψ_D	Ψ_T	Σ_8	Σ_3	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7
ΔB_{23}	$\frac{11}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{5}{6}$	$\frac{3}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{6}$
ΔB_{12}	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15}$	0	$-\frac{1}{3}$	$-\frac{8}{15}$	$\frac{2}{15}$	$-\frac{9}{5}$	$\frac{17}{15}$	$\frac{1}{15}$	$\frac{16}{15}$	$-\frac{1}{15}$

Doršner-Mocioiu $SU(5)$[‡]

HIGGS SECTOR:

$$24_H = \Sigma = (\Sigma_8,\ \Sigma_3,\ \overbrace{\Sigma_{(3,2)},\ \Sigma_{(\bar{3},2)}}^{M_V = M_{GUT}\ (= M_{(X,Y)})},\ \Sigma_{24})$$
$$= (\mathbf{8}, \mathbf{1}, 0) + (\mathbf{1}, \mathbf{3}, 0) + (\mathbf{3}, \mathbf{2}, -5/6) + (\overline{\mathbf{3}}, \mathbf{2}, 5/6) + (\mathbf{1}, \mathbf{1}, 0)$$

$$5_H = \Psi = (\Psi_D,\ \Psi_T) = (\mathbf{1}, \mathbf{2}, 1/2) + (\mathbf{3}, \mathbf{1}, -1/3)$$

MATTER SECTOR:

$$10_a = (\mathbf{1}, \mathbf{1}, 1) + (\mathbf{3}, \mathbf{2}, 1/6) + (\overline{\mathbf{3}}, \mathbf{1}, -2/3)$$

$a = 1,2,3$

$$\overline{5}_a = (\mathbf{1}, \mathbf{2}, -1/2) + (\overline{\mathbf{3}}, \mathbf{1}, 1/3)$$

The Georgi-Glashow GUT content

EXTRA SCALARS:

$$45_H = \Delta = (\Delta_1, \Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7) = (\mathbf{8}, \mathbf{2}, 1/2) + (\overline{\mathbf{6}}, \mathbf{1}, -1/3)$$
$$+ (\mathbf{3}, \mathbf{3}, -1/3) + (\overline{\mathbf{3}}, \mathbf{2}, -7/6) + (\mathbf{3}, \mathbf{1}, -1/3) + (\overline{\mathbf{3}}, \mathbf{1}, 4/3) + (\mathbf{1}, \mathbf{2}, 1/2)$$

$$15_H = \Phi = (\Phi_a,\ \Phi_b,\ \Phi_c) = (\mathbf{1}, \mathbf{3}, 1) + (\mathbf{3}, \mathbf{2}, 1/6) + (\mathbf{6}, \mathbf{1}, -2/3)$$

The Doršner-Mocioiu $SU(5)$[‡] content

‡ I.D. and Irina Mocioiu, in preparation.

	$\overbrace{\text{SM}}^{n=0}$	Ψ_D	Ψ_T	Σ_8	Σ_3	Δ_1	Δ_2	Δ_3	Δ_4	Δ_5	Δ_6	Δ_7	Φ_a	Φ_b	Φ_c
ΔB_{23}	$\frac{11}{3}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{2}{3}$	$-\frac{5}{6}$	$\frac{3}{2}$	$\frac{1}{6}$	$-\frac{1}{6}$	$-\frac{1}{6}$	$\frac{1}{6}$	$\frac{2}{3}$	$\frac{1}{6}$	$-\frac{5}{6}$
ΔB_{12}	$\frac{22}{3}$	$-\frac{1}{15}$	$\frac{1}{15}$	0	$-\frac{1}{3}$	$-\frac{8}{15}$	$\frac{2}{15}$	$-\frac{9}{5}$	$\frac{17}{15}$	$\frac{1}{15}$	$\frac{16}{15}$	$-\frac{1}{15}$	$-\frac{1}{15}$	$-\frac{7}{15}$	$\frac{8}{15}$

PREDICTIONS:

We fix the mass of Φ_a at 300 GeV in order to insure its detection at LHC and then find the maximal value of the GUT scale that one can have. This exercise yields the following:

$$M_{\Sigma_3} = M_Z \qquad M_{\Sigma_8} = 10^5\,\text{GeV} \qquad M_{\Delta_1} = M_Z \qquad M_{\Delta_2} = 2\times 10^{10}\,\text{GeV} \qquad M_{\Delta_7} = M_Z$$

$(M_{GUT}/10^{16}\text{ GeV})$	α_{GUT}^{-1}	$\tau^{d=6\text{ gauge}}/\tau^{\text{exp.}}$	$\tau^{d=6\text{ scalar}}/\tau^{\text{exp.}}$
1.4	29.4	40	1

Bajc-Senjanović $SU(5)$[‡]

HIGGS SECTOR:

$$\mathbf{24}_H = \Sigma = (\Sigma_8,\ \Sigma_3,\ \overbrace{\Sigma_{(3,2)},\ \Sigma_{(\bar{3},2)}}^{M_V = M_{GUT}\ (= M_{(X,Y)})},\ \Sigma_{24})$$
$$= (\mathbf{8},\mathbf{1},0) + (\mathbf{1},\mathbf{3},0) + (\mathbf{3},\mathbf{2},-5/6) + (\bar{\mathbf{3}},\mathbf{2},5/6) + \underline{(\mathbf{1},\mathbf{1},0)}$$

$$\mathbf{5}_H = \Psi = (\Psi_D,\ \Psi_T) = \underline{(\mathbf{1},\mathbf{2},1/2)} + (\mathbf{3},\mathbf{1},-1/3)$$

MATTER SECTOR:

$$\mathbf{10}_a = (\mathbf{1},\mathbf{1},1) + (\mathbf{3},\mathbf{2},1/6) + (\bar{\mathbf{3}},\mathbf{1},-2/3)$$

$a = 1,2,3$

$$\bar{\mathbf{5}}_a = (\mathbf{1},\mathbf{2},-1/2) + (\bar{\mathbf{3}},\mathbf{1},1/3)$$

The Georgi-Glashow GUT content

EXTRA MATTER:

$$\mathbf{24} = \rho = (\rho_8,\ \underline{\rho_3},\ \rho_{(3,2)},\ \rho_{(\bar{3},2)},\ \underline{\rho_{24}})$$

$$= (\mathbf{8},\mathbf{1},0) + \underline{(\mathbf{1},\mathbf{3},0)} + (\mathbf{3},\mathbf{2},-5/6) + (\bar{\mathbf{3}},\mathbf{2},5/6) + \underline{(\mathbf{1},\mathbf{1},0)}$$

The Bajc-Senjanović $SU(5)$[‡] content

‡ B. Bajc and G. Senjanović, hep-ph/0612029.

NEUTRINO MASSES

(type I+type III seesaw)

$45_H{}^{il}{}_j$ ‡

The "DIRAC" part:

$$Y_{1a}(\overline{5}_i)_a \rho^i{}_j \Psi^j + \frac{1}{\Lambda} Y_{2a}(\overline{5}_i)_a \rho^i{}_j \Sigma^j{}_k \Psi^k + \frac{1}{\Lambda} Y_{3a}(\overline{5}_i)_a \Sigma^i{}_j \rho^j{}_k \Psi^k + \frac{1}{\Lambda} Y_{4a}(\overline{5}_i)_a \Psi^i \Sigma^l{}_j \rho^j{}_l$$

rank 1

rank 2

$$m_\nu = \begin{bmatrix} Y_{n1} & Y_{n2} & Y_{n3} \\ \xi\, Y_{n1} & \xi\, Y_{n2} & \xi\, Y_{n3} \end{bmatrix}^T \begin{bmatrix} M_{\rho_3} & 0 \\ 0 & M_{\rho_{24}} \end{bmatrix}^{-1} \begin{bmatrix} Y_{n1} & Y_{n2} & Y_{n3} \\ \xi\, Y_{n1} & \xi\, Y_{n2} & \xi\, Y_{n3} \end{bmatrix}$$

$$m_\nu = \begin{bmatrix} Y_{n1} & Y_{n2} & Y_{n3} \\ Y_{m1} & Y_{m2} & Y_{m3} \end{bmatrix}^T \begin{bmatrix} M_{\rho_3} & 0 \\ 0 & M_{\rho_{24}} \end{bmatrix}^{-1} \begin{bmatrix} Y_{n1} & Y_{n2} & Y_{n3} \\ Y_{m1} & Y_{m2} & Y_{m3} \end{bmatrix}$$

‡ P.F.P. hep-ph/0702287.

The "MAJORANA" part:

$$\underline{m_F \rho^i{}_j \rho^j{}_i} + \lambda_F \rho^i{}_j \rho^j{}_k \Sigma^k{}_i + \frac{1}{\Lambda}\left[a_1 \rho^i{}_j \rho^j{}_i \Sigma^k{}_l \Sigma^l{}_k + a_2 (\rho^i{}_j \Sigma^j{}_i)^2 + a_3 \rho^i{}_j \rho^j{}_k \Sigma^k{}_l \Sigma^l{}_i + a_4 \rho^i{}_j \Sigma^j{}_k \rho^k{}_l \Sigma^l{}_i\right]$$

$$M_{\rho_3} = M_{\rho_{24}} = \ldots$$

$$\begin{bmatrix} M_{\rho_3} & 0 \\ 0 & M_{\rho_{24}} \end{bmatrix}$$

PERTURBATIVITY: $|a_i| \leq \sqrt{4\pi}, \qquad i = 1, 2, 3, 4$

Bajc-Senjanović $SU(5)$

ADJOINT FERMION MASSES (continued):

Seesaw relevant

RGE relevant

$$M_{\rho_{24}} = m_F - \frac{\lambda_F v}{\sqrt{30}} + \frac{v^2}{\Lambda}\left[a_1 + a_2 + \frac{7}{30}(a_3 + a_4)\right]$$

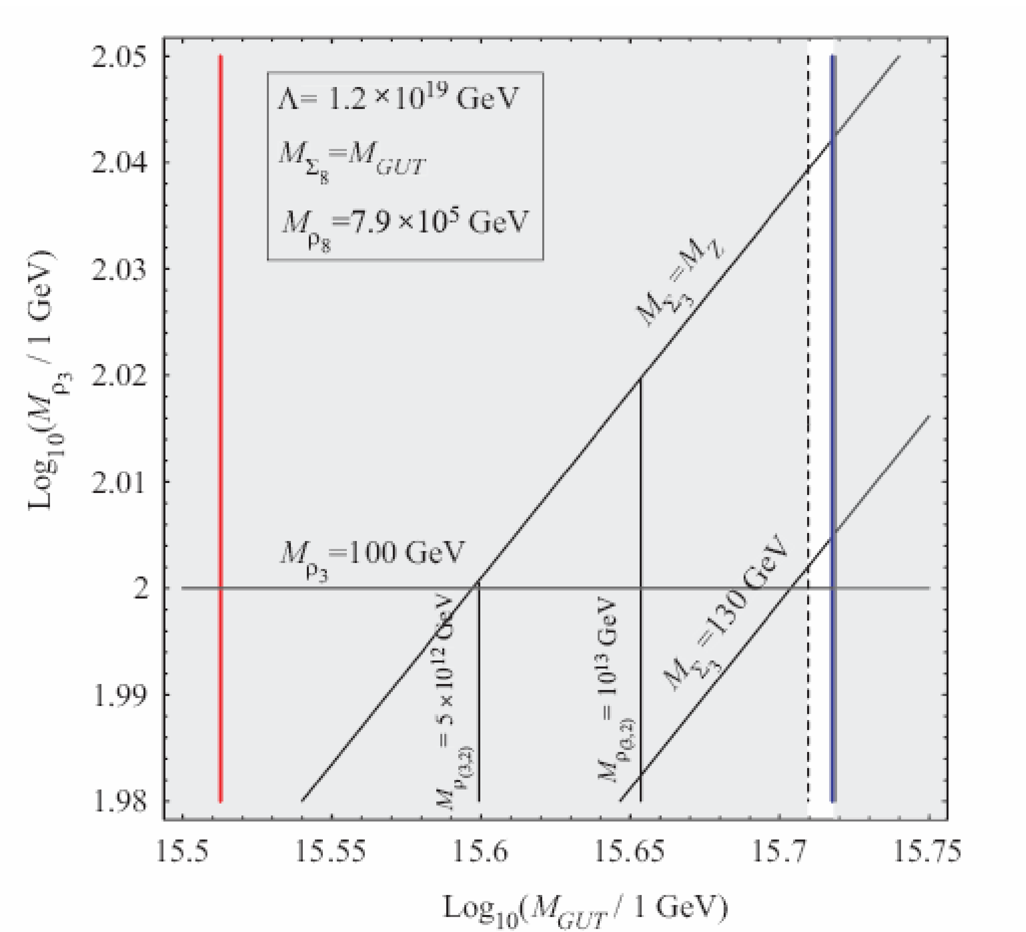
$$M_{\rho_3} = m_F - \frac{3\lambda_F v}{\sqrt{30}} + \frac{v^2}{\Lambda}\left[a_1 + \frac{18}{60}(a_3 + a_4)\right]$$

$$M_{\rho_8} = m_F + \frac{2\lambda_F v}{\sqrt{30}} + \frac{v^2}{\Lambda}\left[a_1 + \frac{8}{60}(a_3 + a_4)\right]$$

$$M_{\rho_{(3,2)}} = m_F - \frac{\lambda_F v}{2\sqrt{30}} + \frac{v^2}{\Lambda}\left[a_1 + \frac{1}{60}(13a_3 - 12a_4)\right]$$

$$a_4 = \frac{2\ \Lambda\ \pi\ \alpha_{GUT}\left((M_{\rho_8} - 2M_{\rho_{(3,2)}}) + M_{\rho_3}\right)}{M_{GUT}^2}, \qquad M_{(X,Y)} = \sqrt{5\pi\alpha_{GUT}/3}v = M_{GUT}$$

‡ I.D. and P.F.P. **hep-ph/0612216**.



PREDICTIONS‡:

The predictions depend strongly on the cutoff Λ of the theory.

$$\Lambda = M_{\text{Planck}}$$

$$M_{\rho_{(3,2)}} \sim \frac{M_{GUT}^2}{\Lambda}$$

$$a_4 = \frac{2\ \Lambda\ \pi\ \alpha_{GUT}\left(\left(M_{\rho_8} - 2M_{\rho_{(3,2)}}\right) + M_{\rho_3}\right)}{M_{GUT}^2}$$

⇓

LIGHT FERMIONIC TRIPLET!

LIGHT BOSONIC TRIPLET!

‡ B. Bajc and G. Senjanović, hep-ph/0612029; I.D. and P.F.P. hep-ph/0612216; B. Bajc M. Nemevšek and G. Senjanović, hep-ph/0703080.

Let us fix the mass of fermionic triplet ρ_3 at 300 GeV in order to insure its detection at LHC and then find the maximal value of the GUT scale that one can have. This exercise yields the following:

$$M_{\Sigma_3} = M_Z \qquad M_{\rho_8} = 1.5 \times 10^6 \,\mathrm{GeV}$$

ν MODEL	$(M_{GUT}/10^{16}\,\mathrm{GeV})$	α_{GUT}^{-1}	$\tau^{d=6 \text{ gauge}}/\tau^{\text{exp.}}$	$\tau^{d=6 \text{ scalar}}/\tau^{\text{exp.}}$
D-M	1.4	29.4	40	1
B-S	1.5	37.6	115	15000

CONCLUSIONS

1) The simplest $SU(5)$ **models the type I seesaw mechanism are ruled out under the *usual* assumptions.**

2) Other realistic nonSUSY $SU(5)$ **GUT theories could be tested at LHC.**

Three specific $SU(5)$ **GUT models that aim in that direction are: (i) nonrenormalizable** $SU(5)$ **with the type II seesaw, (ii) renormalizable** $SU(5)$ **with the type II seesaw, and** (iii) nonrenormalizable $SU(5)$ with the hybrid–type I + type III–seesaw scenario.

CONCLUSIONS (CONTINUED)

In all three cases interesting signatures include (in)direct observation of exotic light particles, enhanced rates for specific rare processes, observable proton decay and, most importantly, correlation between these processes and the parameters in the neutrino sector. Latter could also allow us to experimentally probe the underlying seesaw mechanism.

Even though these models are extremely fine tuned they are very predictive and hence falsifiable.