Neutrino Clouds

Neutrinos from a supernova: flavor- energy - spectrum correlation (FESC)

Why are we interested?

1. If ν_e 's and anti-ν_e 's carry a bigger fraction of the energy, then heating of outer layers is greater, and their blow-off is facilitated.

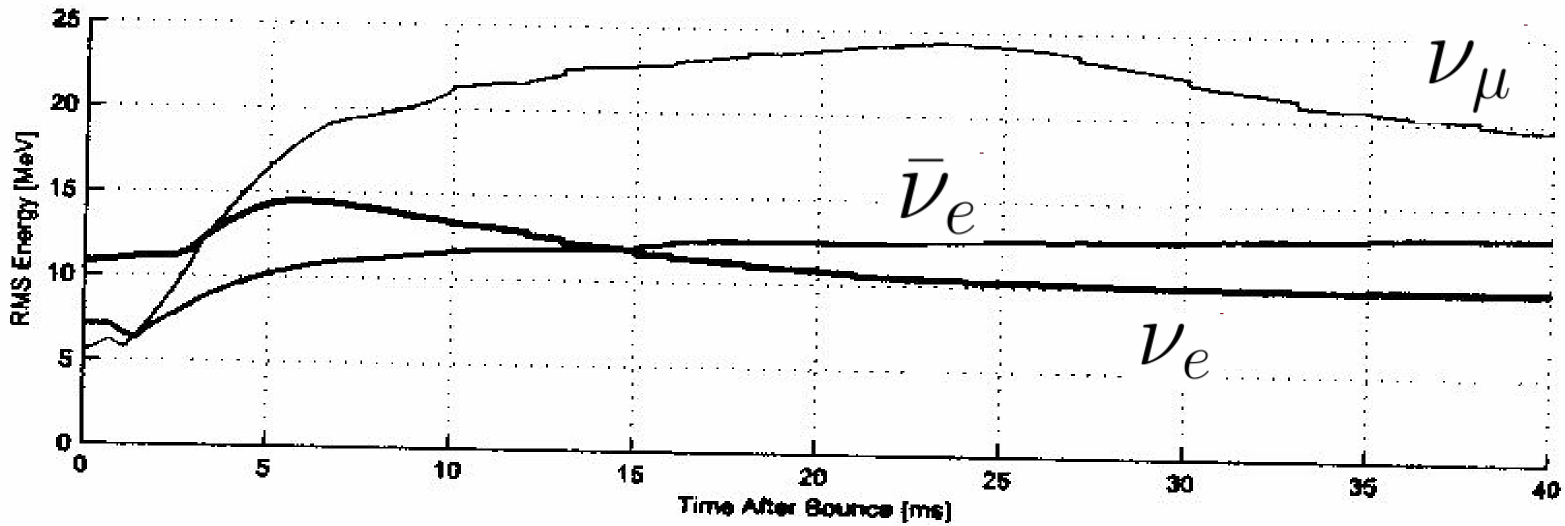
   Absorption cross-sections are greater for ν_e , anti- ν_e , than for $\nu_{\mu,\tau}$, $\nu_{\mu,\tau}$

2. The FESC is critical in determining the n/p ratio in the region in which we could have R-process synthesis of heavy elements.

   It may be required that anti-ν_e have a considerably stiffer spectrum than $\nu_{e,}$ in order to get sufficient neutron richness.

3 Comparison of theory with the observations of ν's from SN 2113b.

FESC



25
20
15
10
5
0
RMS Energy [MeV]
0
5
10
15
20
25
30
35
40
Time After Bounce [ms]
ν_μ
$\bar{\nu}_e$
ν_e

In the supernova core, in the region of the neutrinosphere

- $\rho = 10^{11}$ gm cm^{-3}
- $E_\nu \sim 20$ MeV   Then using present oscillation data for (δm^2),
- we have,

$$(\delta m^2/2E)^{-1} \approx 20\ km.$$

- Very little ν_e oscillation action. (Would be 20 km. osc. dist., but also frozen by electron density, and independently frozen by absorption-emission processes.)

Neutrino interaction time-scales

- From scattering cross-sections:

$$T_{\text{scat}}^{-1} \approx n_s \sigma \approx G_F^2 E^2 n_s$$

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- Medium-fast time scale

$$T_{med} = \sqrt{T_{osc} T_{fast}}$$

(Kostelecky and Samuel ,
and explicated by Pastor, Raffelt and Semikoz,
--all for isotropic distributions, “pathological” initial conditions.)

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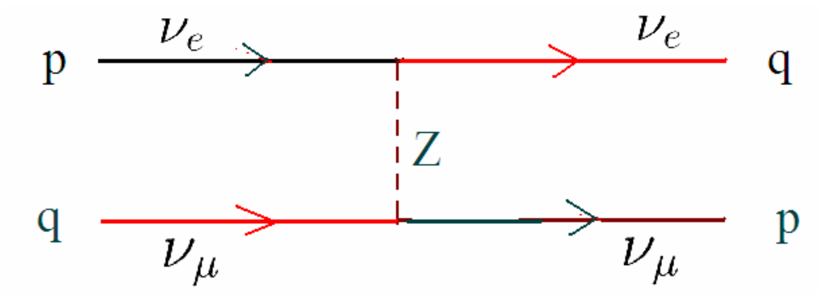
- Medium-fast time scale

$$T_{med} = \sqrt{T_{osc} T_{fast}}$$

Does anything real happen in the short time T_{fast} ?

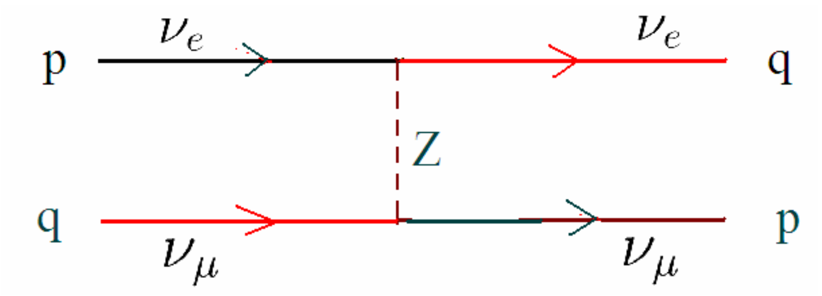
- Forward, flavor exchange



Original idea:

Friedland and Lunardini

- Forward, flavor exchange



Fragment of Hamiltonian related to this graph:

makes all the difference

$$H_{\text{frag}} = \frac{\sqrt{2}G_F}{\text{Vol.}} \int d\Omega \, d\Omega' (1 - \cos\theta_{\Omega,\Omega'}) \rho^+(\Omega) \rho^-(\Omega')$$

where

$$\rho^+(\Omega) = (d\Omega)^{-1} \sum_{q \subset d\Omega} a^\dagger_{\nu_e}(q) a_{\nu_\mu}(q)$$

$$\rho^-(\Omega) = (d\Omega)^{-1} \sum_{p \subset d\Omega} a^\dagger_{\nu_\mu}(p) a_{\nu_e}(p)$$

p, q 's from initially occupied mom. states only

A note on dynamics

- Commutation rules

$$[\rho^+(\Omega), \rho^-(\Omega')] = \rho^{(3)}(\Omega)\delta(\Omega - \Omega')$$ etc.

Heisenberg eqns.:

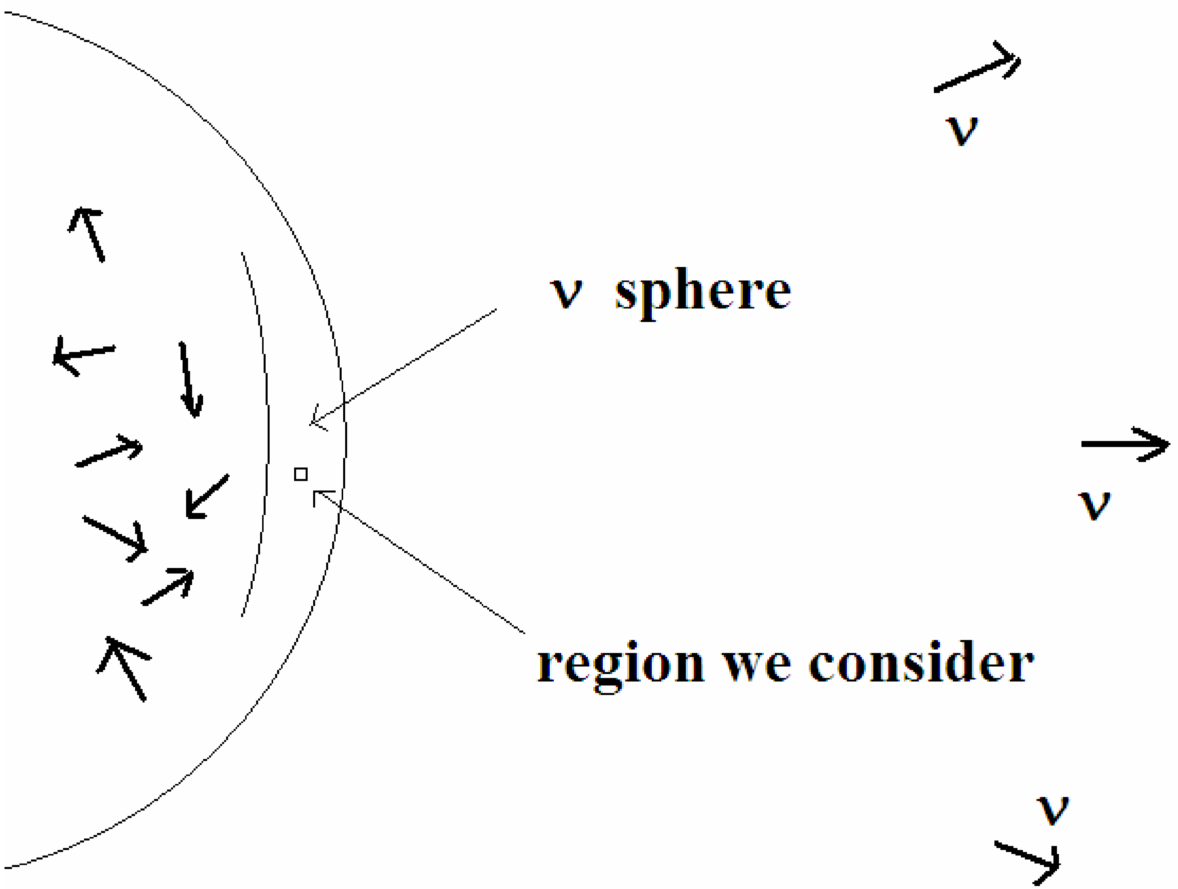
$$i\frac{d}{dt}\rho^+(\Omega) = \rho^{(3)}(\Omega) \int d\Omega' (1 - \cos\theta_{\Omega,\Omega'})\, \rho^+(\Omega')$$

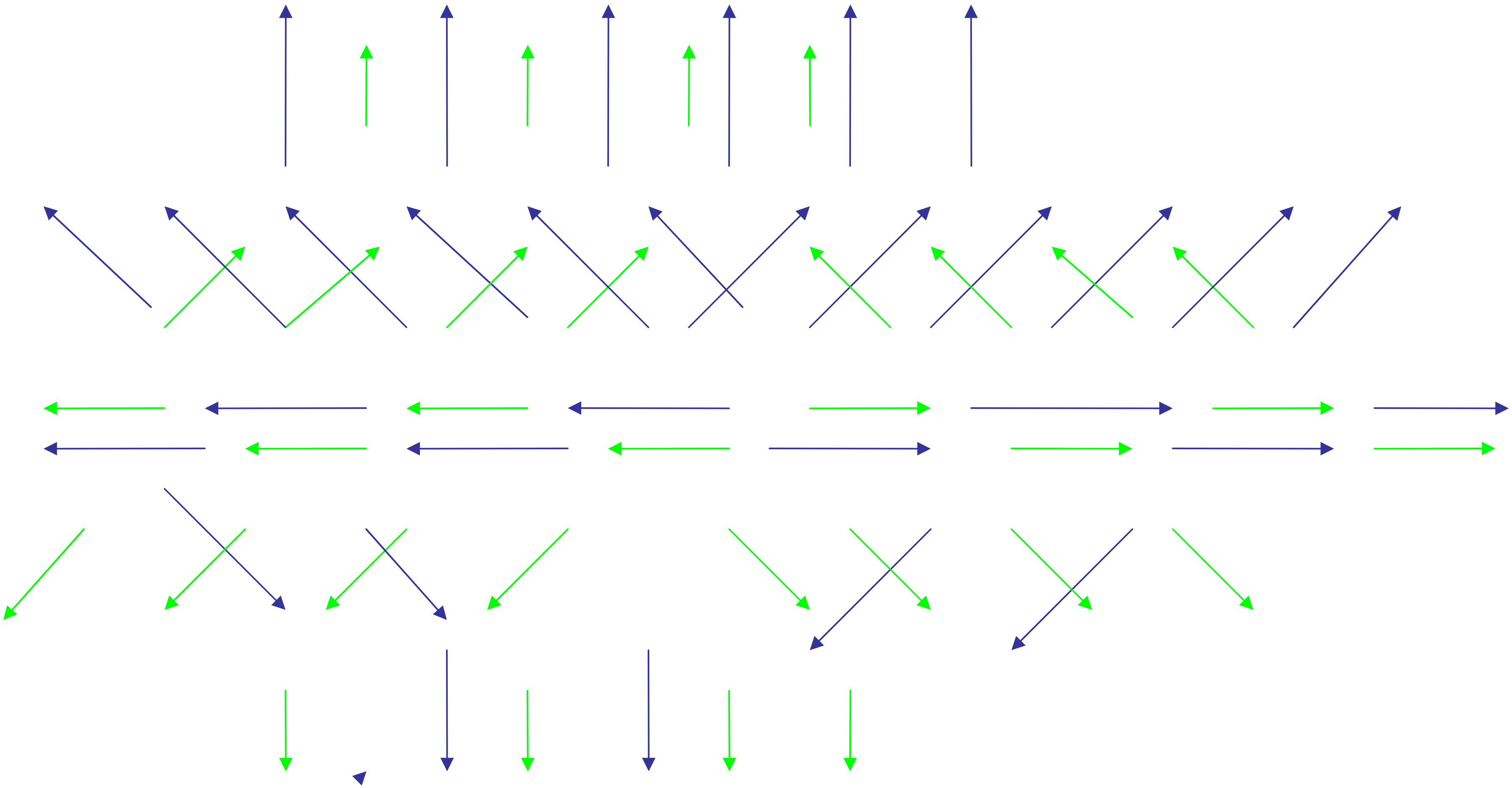
etc.

If we assume that,

$$\langle \rho^i(\Omega, t)\ \rho^j(\Omega', t)\rangle = \langle \rho^i(\Omega, t)\rangle\ \langle \rho^j(\Omega', t)\rangle$$ for all t.

then we have ordinary diff.-int eqns. for the density matrix elements.

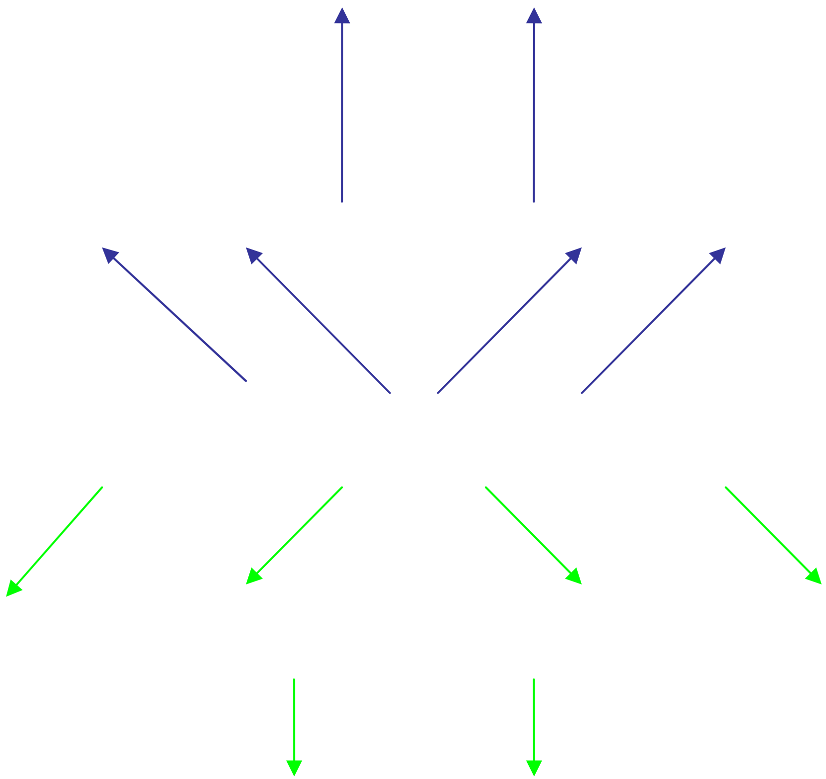

ν
ν sphere
ν
region we consider
ν

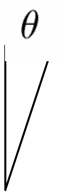


Momentum distribution ν_μ , ν_e near ν -sphere

We can delete ν_μ , ν_e when paired in angle.

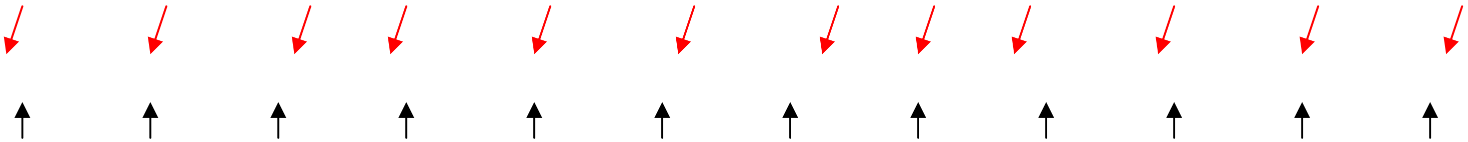
So, in effect,





Spin systems

How long for this:



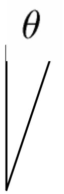
to go into something inverted or scrambled??

(for tiny angle theta, very large N, and under the influence of)

$$H_1 = g \sum_{i,j} [\sigma_i^+ \sigma_j^- + \sigma_i^+ \sigma_j^-]$$

or

$$H_2 = g \sum_{i,j} [\sigma_i^+ + \sigma_i^+][\sigma_j^- + \sigma_j^-]$$



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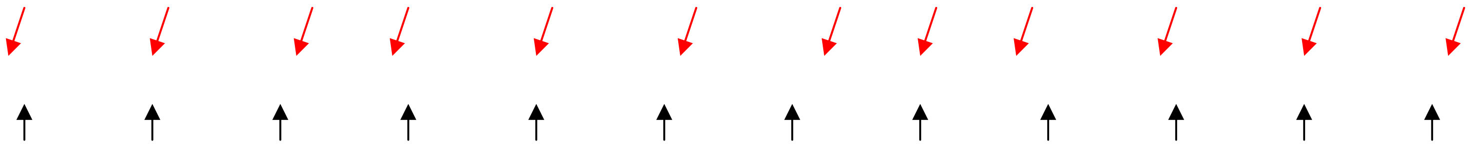

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Answer:

$$\text{For } H_1 \ , \qquad t_{\text{mix}} \approx g^{-1} N^{-1} \,\text{Min}\,[\,|\log\theta|, \ \log N\,]$$

fast, for large N

$$\text{For } H_2 \ , \qquad t_{\text{mix}} \approx g^{-1} N^{-1/2}$$

Back to neutrinos. Up and down beams.

$$H = \frac{\sqrt{2}G_F}{\text{Vol.}} \int d\Omega d\Omega' (1 - \cos\theta_{\Omega,\Omega'}) \rho^+(\Omega)\rho^-(\Omega')$$
$$\rightarrow \frac{\sqrt{2}G_F}{\text{Vol.}} [\rho^+ \, \rho^- + \, \rho^+ \, \rho^-]$$

where

$$\rho^+ = \sum_{q \subset \text{up}} a^\dagger_{\nu_e}(q) a_{\nu_\mu}(q) \quad , \quad \rho^+ = \sum_{q \subset \text{dn}} a^\dagger_{\nu_e}(q) a_{\nu_\mu}(q)$$

$$\rho^- = \sum_{q \subset \text{up}} a^\dagger_{\nu_\mu}(q) a_{\nu_e}(q) \quad , \quad \rho^- = \sum_{q \subset \text{dn}} a^\dagger_{\nu_\mu}(q) a_{\nu_e}(q)$$

Note: no intragroup interactions in the above

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If we take

$$\frac{\sqrt{2}G_F}{\text{Vol.}} \longrightarrow g$$

we get exactly the spin model with the fast evolution

$$t_{\text{mix}} \approx g^{-1} N^{-1} \, \text{Min} \, [\, |\log\theta|, \; \log N \,]$$

or

$$t_{\text{mix}} \approx (\sqrt{2} n_\nu G_F)^{-1} \, \text{Min} \, [\, |\log\theta|, \; \log N \,]$$

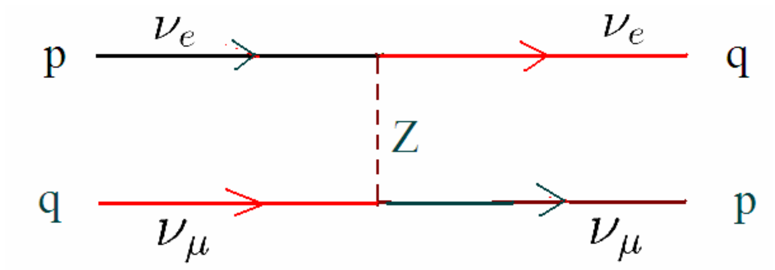
$$t_{\rm mix} \approx (\sqrt{2} n_\nu G_F)^{-1}\, \mathrm{Min}\,[\,|\log\theta|,\ \log N\,]$$

Fast rate, as promised

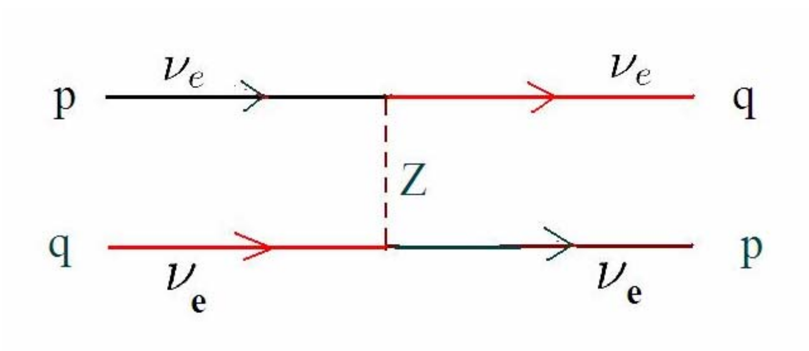
θ is initial mixing of down-moving states

One little problem………

Our "forward" Hamiltonian was based on:



We also have :



etc., Hamiltonian should be

$$H_{\text{frag}} = \frac{\sqrt{2}G_F}{\text{Vol.}}[\rho^+ \ \rho^- + \ \rho^+ \ \rho^- + \frac{\lambda}{2} \rho^{(3)} \ \rho^{(3)}]$$

$$+ \ \text{SU}_2 \text{ singlet}$$

where $\lambda = 1$

The trouble with:

$$H_{\text{frag}} = \frac{\sqrt{2}G_F}{\text{Vol.}}[\rho^+ \rho^- + \rho^+ \rho^- + \frac{\lambda}{2} \rho^{(3)} \rho^{(3)}]$$

for λ <1 --- unstable → fast mixing

for λ >1 --- stable

for λ =1 --- stable

→ normal mixing

So for the physical case, λ=1, with SU2 ,
--------------there is no speed-up.

Friedland and Lunardini

All that work for nothing! ????

Generalizations which do show speed-up, even for $\lambda=1$.

1. More complex angular distributions.

2. 3 neutrino flavors , with anti-neutrinos as well.

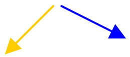
Generalizations which do show speed-up, even for λ=1.

1. More complex angular distributions.

Four bundles at different angles:

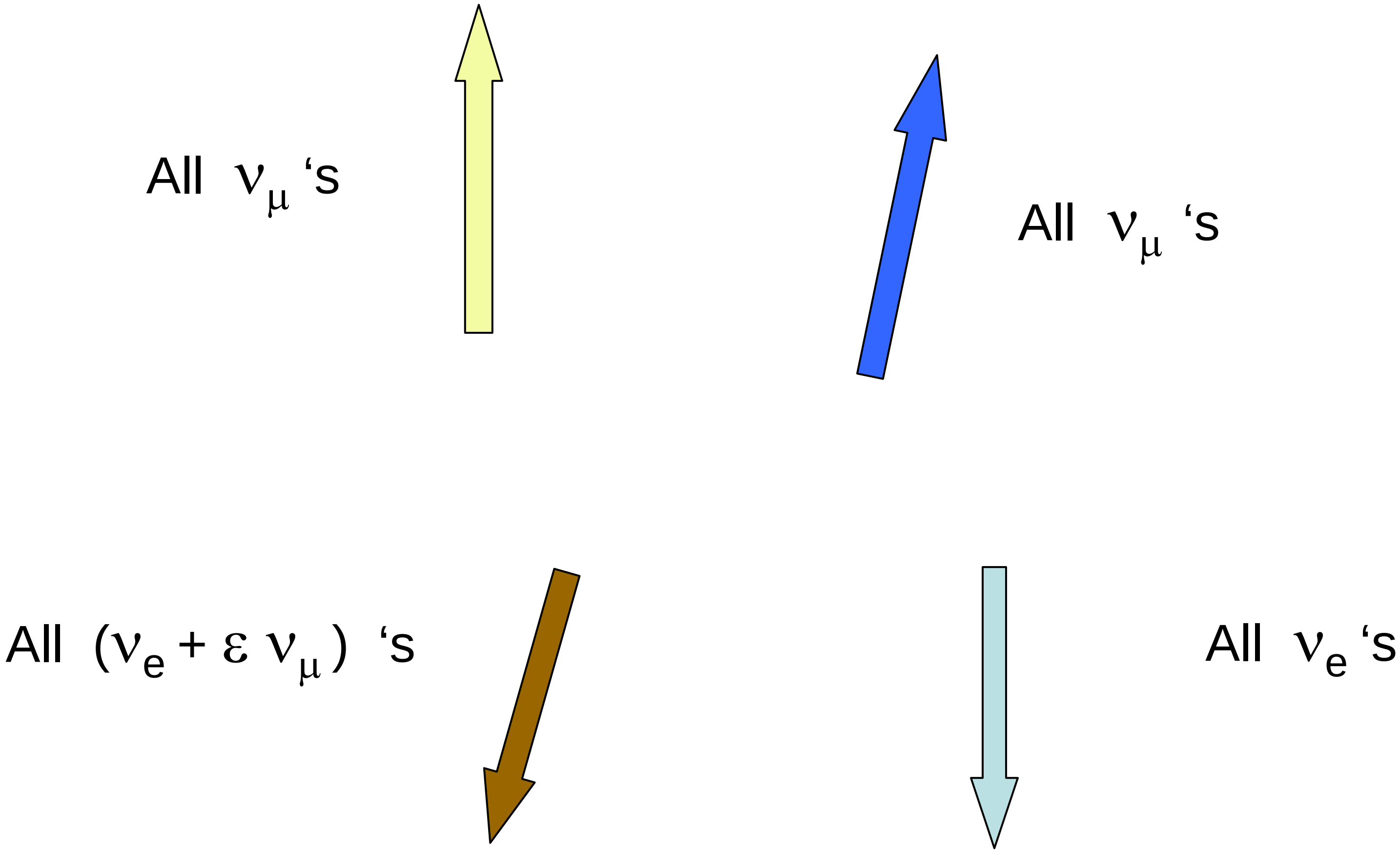
$$\frac{d}{dt}\vec{\rho} = -g_1\,\vec{\rho} \times \vec{\rho} - g_2\,\vec{\rho} \times \vec{\rho} - g_3\,\vec{\rho} \times \vec{\rho}$$

etc.

Where: g_2 = 1- cos()

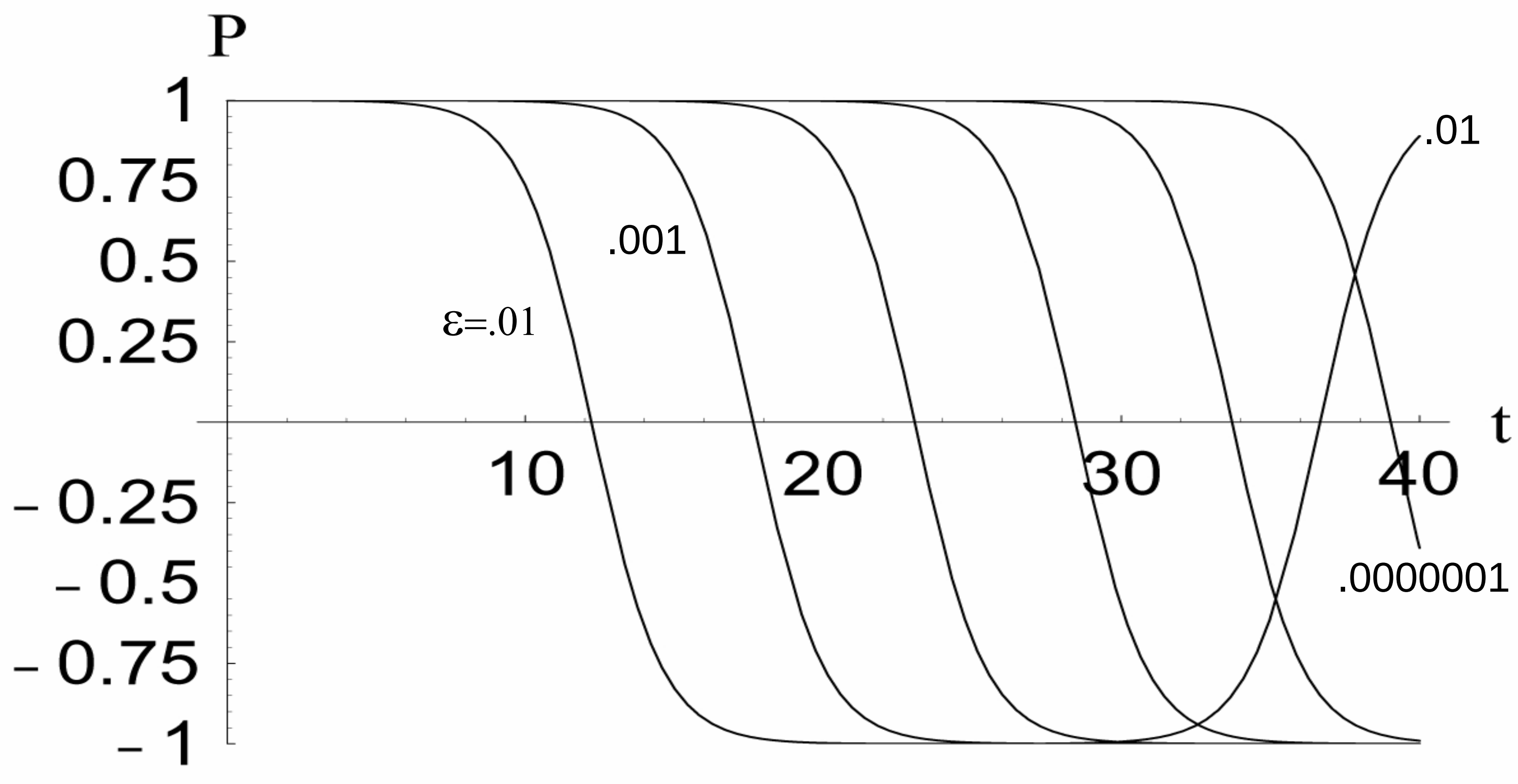
Four bundles in different directions.

- labeled with initial flavors. Neutrino density = n_ν in each group



We take ε = .01 , .001 , .0001, .00001, .000001, .0000001

Time evolution of $P=(N_\mu - N_e)/N$ for the yellow group

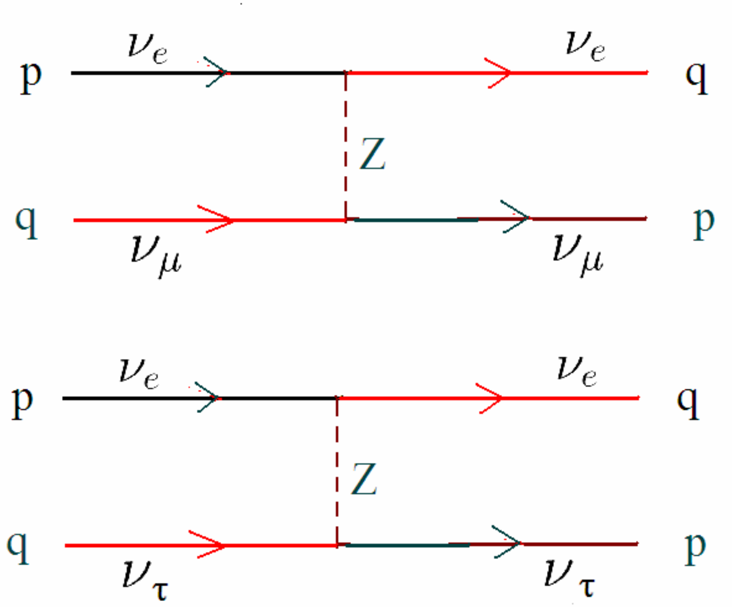


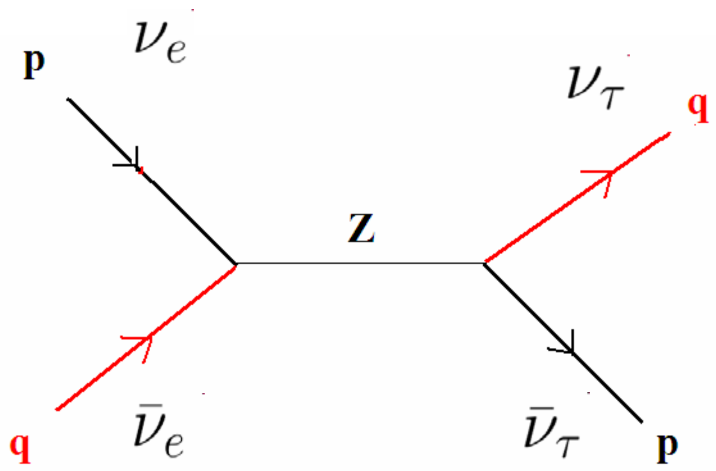
Time is in units $(G_F n_\nu)^{-1}$

To include SU3 and antiparticles:

$$[\rho_i(\Omega), \rho_j(\Omega')] = \delta(\Omega - \Omega') \sum_{k=1}^{9} f_{i,j,k}\rho_k(\Omega) ,$$

with a Hamiltonian including,





A two stream scenario:

- Initial conditions: all ν 's with energies, E=18 MeV

going up:

$$\nu_\mu\ ,\ \nu_\tau\ ,\ \bar{\nu}_\mu\ ,\ \bar{\nu}_\tau$$

going down:

$$\nu_e\ ,\ \bar{\nu}_e$$

And add oscillation terms,

$$\delta m^2_{e,\tau} = 10^{-4}(\mathrm{eV})^2$$

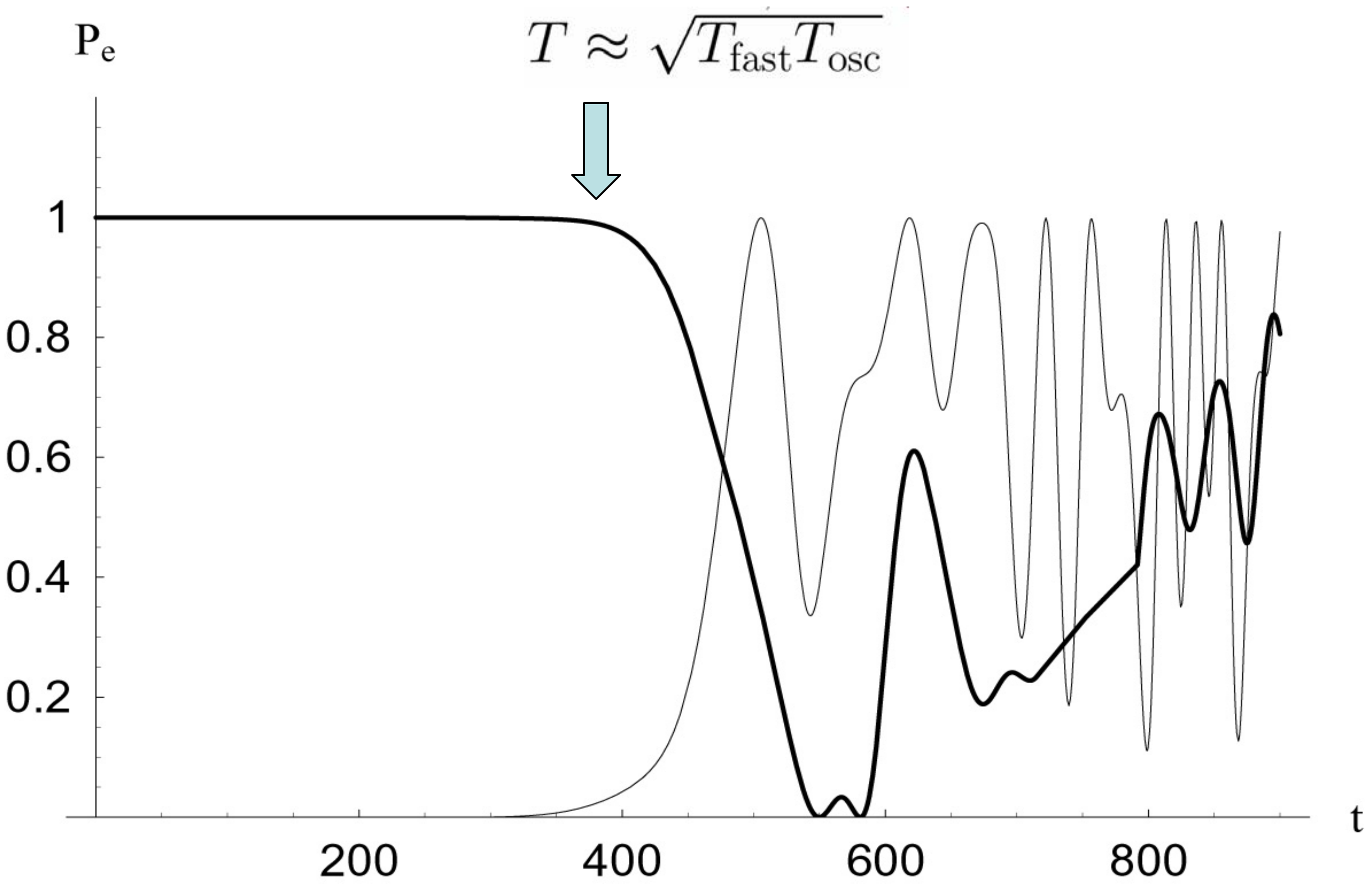
$$\delta m^2_{\mu,\tau} = 4 \times 10^{-3}(\mathrm{eV})^2$$

Define:

$$T_{\mathrm{osc}} = \frac{2E}{\delta m^2_{e,\tau}}$$

Evolution

$P_e = \nu_e$ occupancy. Heavy curve=downward. Light curve=upward



Comments

- We obtained "medium-fast" evolution

$$T \approx \sqrt{T_{\text{fast}} T_{\text{osc}}}$$

where T_{osc} is defined by the oscillation parameter for ν_e .
But where the (40x as large) oscillation parameter for ν_μ
is essential to the "medium-fast" mixing.

Note: we also included an electron density (8 times the ν density) with the usual ν interactions.

This last: surprising?

Conclusions

- Almost none

Conclusions

- One way or another, the non-linear effects will matter.
- Outcome could be complete flavor-spectrum mixing.
- Similar phenomena may take place in other systems.

Other systems with similar physics?

Photon-photon scat:

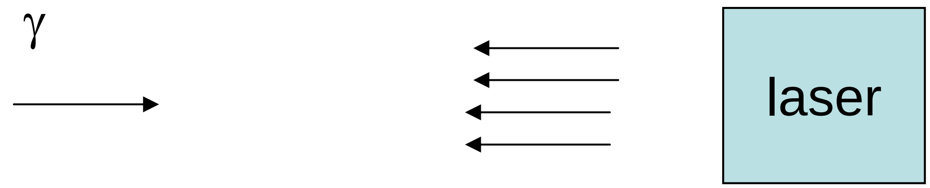
$$L_I = \int d^3x \frac{2\alpha^2}{45m^4}[(\mathbf{E^2} - \mathbf{B^2})^\mathbf{2} + \mathbf{7}(\mathbf{E}\cdot\mathbf{B})^\mathbf{2}]$$

(polarizations now take the place of flavors and Heisenberg-Euler replaces Z-exchange.)

G. L. Kotkin and V. G. Serbo, Phys. Lett. **B413**,122 (1997)

Laser: 2.35 eV, $E/E_{\rm crit} \approx 1.5 \times 10^{-6}$

100 MeV γ



Both beams linearly polarized.

Angle between polarizations not = n$\pi/2$

Mean distance for scattering of the photon –from cross-section and laser beam density -- 10^9 cm.

Question: What is distance for polarization exchange?

Answer: 3 cm. (Kotkin and Serbo)

Colliding photon clouds

Now with one cloud unpolarized and the other polarized:
The polarized cloud loses polarization in distance 3 log[N] cm.

RFS Phys.Rev.Lett. 93 (2004) 133601

Also,

1. $\nu + \nu \longrightarrow 2$ majorons

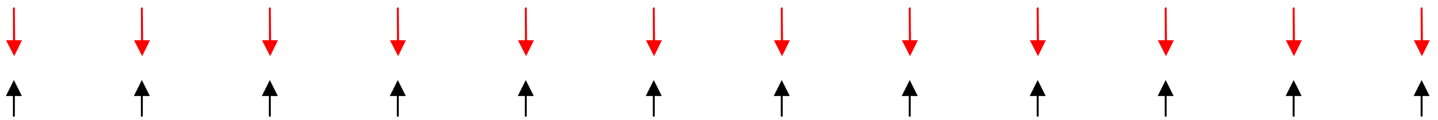
Venues: Supernova core, or

Early U just after freeze-out

2. ν flavor-spectrum equilibration in early U just after freeze-out

3. Various possibilities in the early U at say, 2MeV<T<100MeV, especially in the case of non-infinitesimal neutrino chemical potentials, or in the case of the existence of a sterile neutrino.

Spin systems:



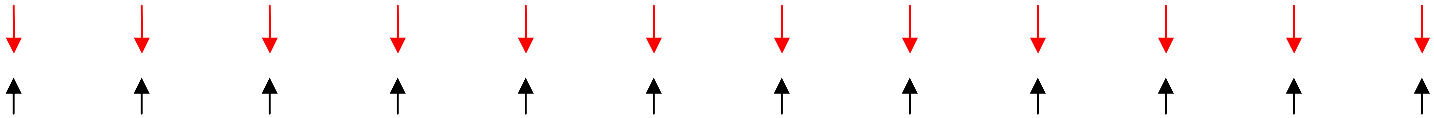
Correspondence to neutrinos:

Upmoving → upper tier

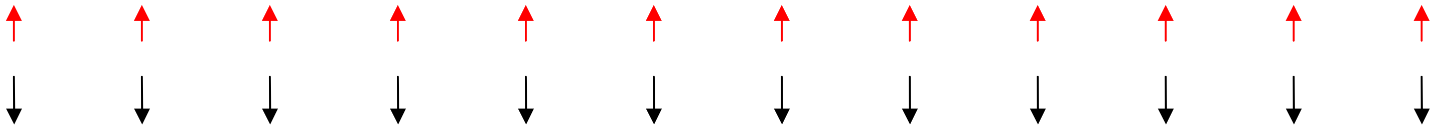
Downmoving → lower tier

Flavor → spin

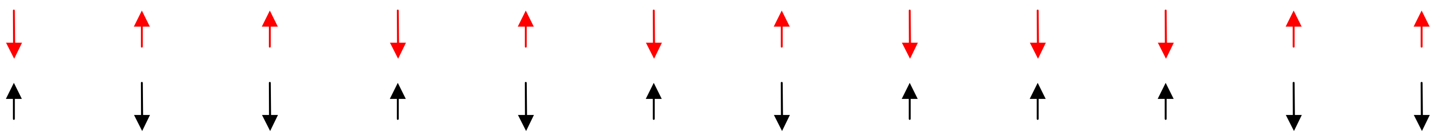
Spin system: How long for this:



to go into this?



or this?



under the influence of

$$H_1 = g \sum_{i,j} [\sigma_i^+ \sigma_j^- + \sigma_i^+ \sigma_j^-]$$

or

$$H_2 = g \sum_{i,j} [\sigma_i^+ + \sigma_i^+][\sigma_j^- + \sigma_j^-]$$

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or

$$H_2 = g \sum_{i,j} [\sigma_i^+ + \sigma_i^+][\sigma_j^- + \sigma_j^-]$$

??

For H_1 , $t_{\text{mix}} \approx g^{-1} N^{-1} \log N$

fast

For H_2 , $t_{\text{mix}} \approx g^{-1} N^{-1/2}$

normal