Relic Neutrinos in the Cosmic Microwave and Gravitational Backgrounds

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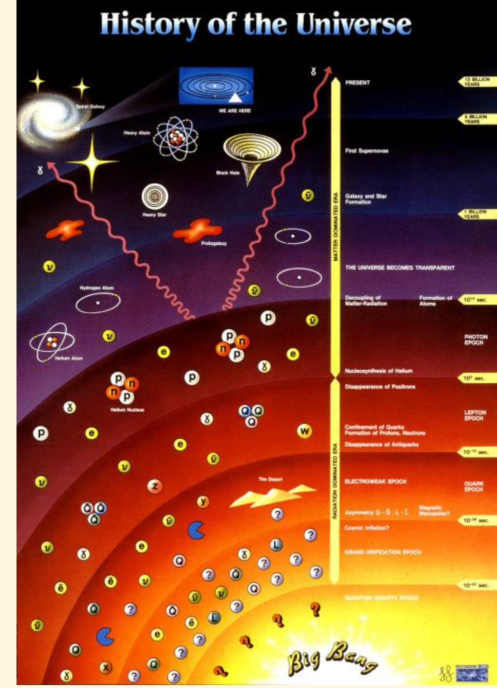
Santa Fe, July, 2005

Motivation

- Neutrinos were among the dominant cosmological species during the radiation era ($\rho_\nu/\rho_{tot} = 41\%$)
  - CMB perturbations interact gravitationally with all the dominant species during the horizon entry
  - The CMB perturbations on the scales of all the CMB acoustic peaks enter the horizon in the radiation era

- During the horizon entry gravity waves interact with free streaming dominant species
- Neutrinos free stream since their decoupling at $z \sim 10^{10}$

Basics: Relic neutrinos



- After neutrino **freeze out** at $T \sim 1$ MeV ($z \sim 10^{10}$, $H^{-1} \sim 0.1$ kpc), **free streaming**

$$f(\tau, x^i, P_i): \qquad \dot{f} + V^i \frac{\partial}{\partial x^i} f + F_i \frac{\partial}{\partial P_i} f = 0$$

- Source of gravitational field

$$T_{\mu\nu} = \int \frac{d^3 P_i}{\sqrt{-g}} \frac{P_\mu P_\nu}{P^0} f$$

- Momentum distribution, set at freeze out, is *almost* thermal Fermi-Dirac.

- While $T >> m_\nu$

$$N_{\nu\,\mathrm{eff}} \equiv \frac{\rho_\nu}{\frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \rho_\gamma} \simeq 3.04$$

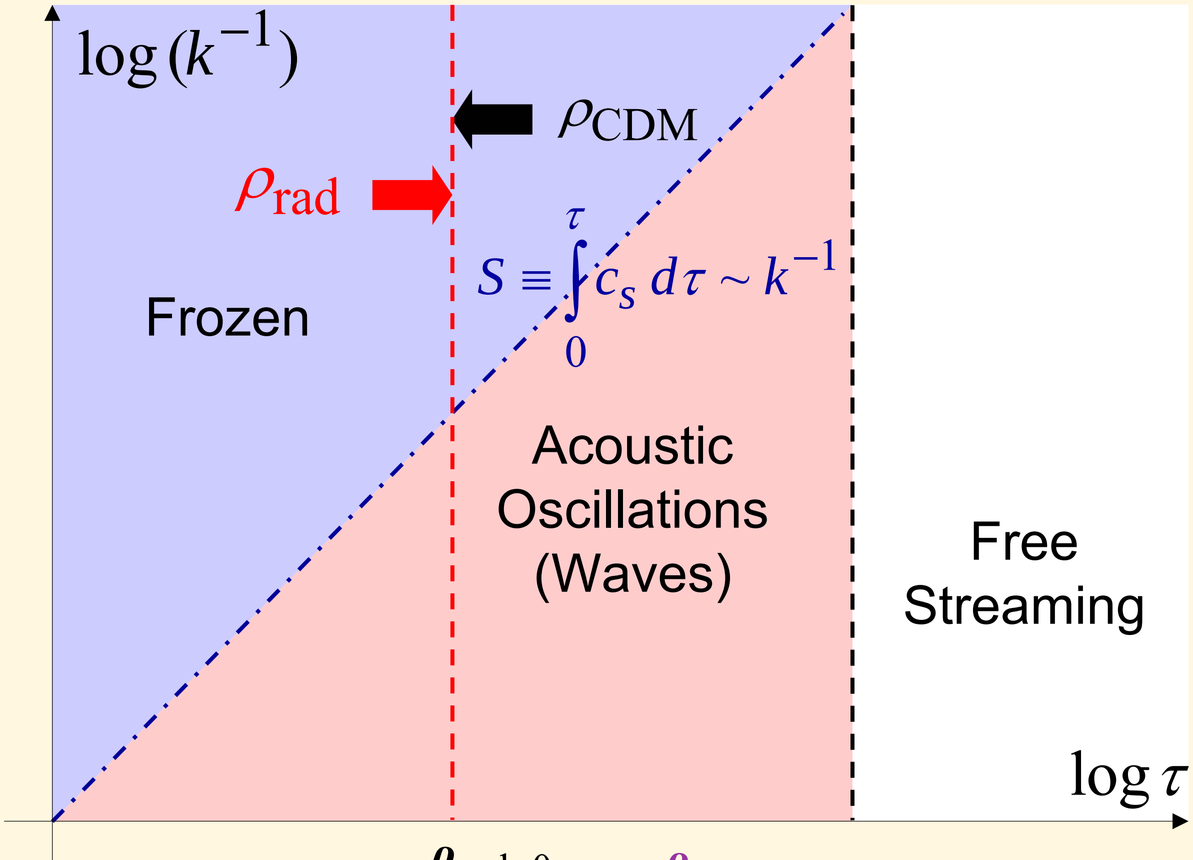
Fermions

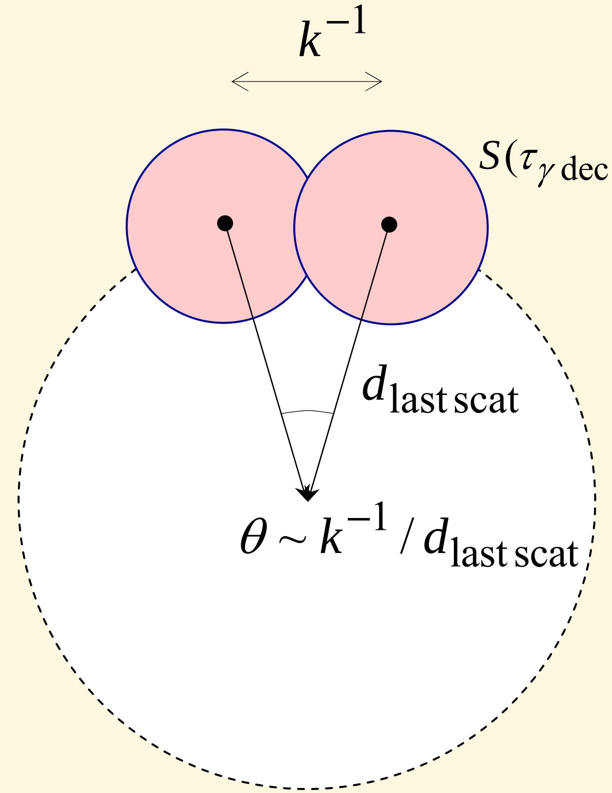
Freeze out before $e^- e^+ \to \gamma\gamma$

(Compare for $T >> m_\nu$

$$\rho_\nu = \sum_f n_f\, m_f\,)$$

Basics: Anisotropy of the Cosmic Microwave Background



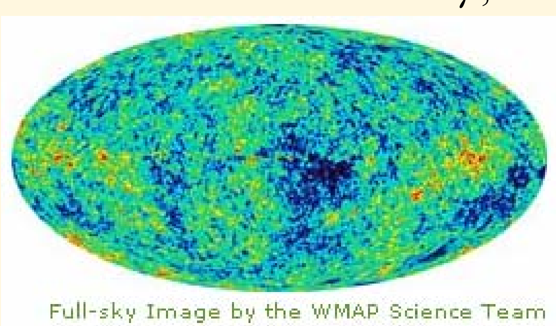


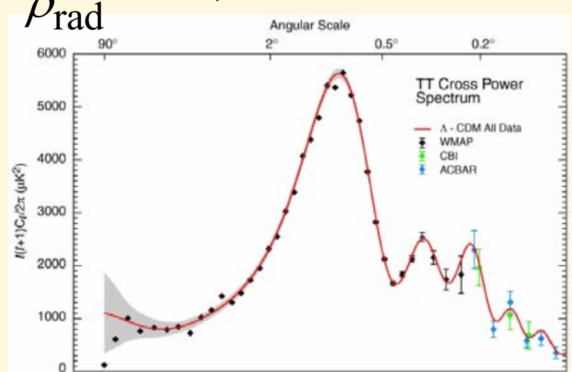
$$z_{\mathrm{eq}} + 1 = \frac{\rho_{c+\mathrm{b},0}}{\rho_{\gamma,0}}\left(1 - \frac{\rho_\nu}{\rho_{\mathrm{rad}}}\right) \qquad z_{\gamma\,\mathrm{dec}} \approx 1090$$

$$C_l^{XY} = \langle \Delta X_l\, \Delta Y_l \rangle$$

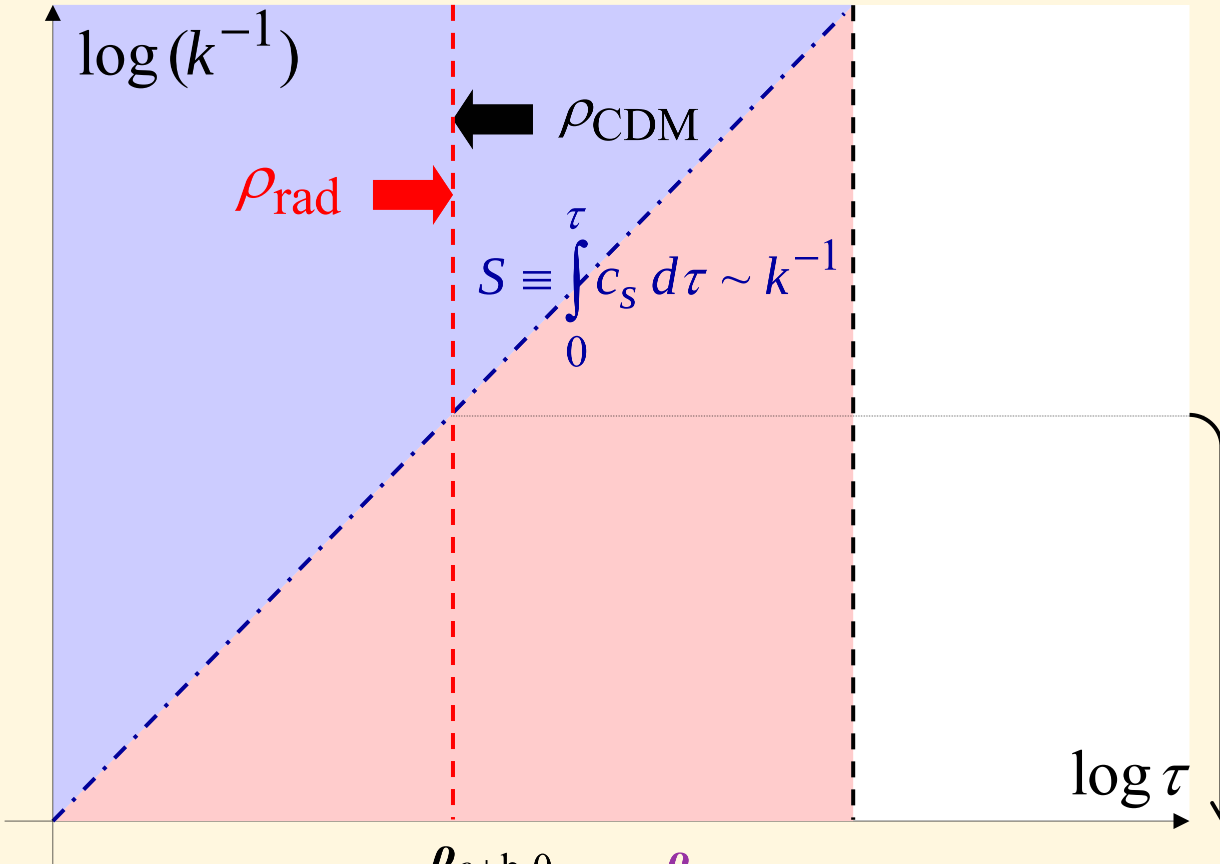
Intensity (Temperature) Polarization

$$X, Y = (T, E, B)$$

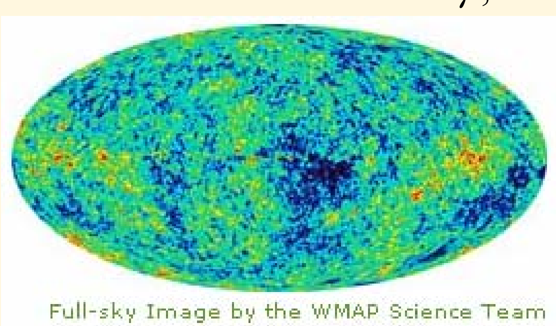


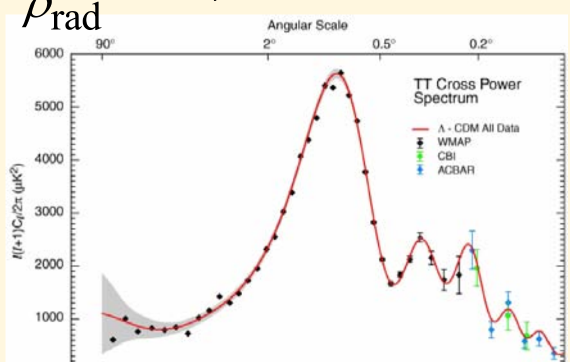


CMB anisotropy as a probe of neutrinos: General notes



$$z_{\rm eq} + 1 = \frac{\rho_{c+{\rm b},0}}{\rho_{\gamma,0}}\left(1 - \frac{\rho_\nu}{\rho_{\rm rad}}\right) \qquad z_{\gamma\,{\rm dec}} \approx 1090$$





- As long as all ν are *ultrarelativistic* and *decoupled*, their impact on the CMB is fully parameterized by $N_{\rm eff}$
- Expect ν signatures for $l > S^{-1}(\tau_{\rm eq})\, d_{\rm last\ scat} \sim 200$
- These signatures are particularly sensitive to $z_{\rm eq}$

* *Mild corrections* around the 1st peak ($l \approx 220$) due *to the early ISW effect*

* Additional late-time signatures of m_ν due to the lensing of *CMB by LSS*

CMB as a probe of neutrinos: Sources of signatures

- Background expansion

$$H^2 = \frac{8\pi G}{3}\left(\rho_{\gamma b} + \rho_\nu + \rho_{CDM} + \rho_{DE} + \rho_K\right)$$

- Primary perturbations

$$d_\gamma \equiv \frac{\Delta n_\gamma}{n_\gamma} \to 3\frac{\Delta T_{\rm CMB}}{T_{\rm CMB}}: \qquad \ddot{d}_\gamma + \mathcal{H}(1-3c_s^2)\,\dot{d}_\gamma - c_s^2\,\nabla^2 d_\gamma = \nabla^2(\Phi + 3c_s^2\Psi)$$

$$ds^2 = a^2\left[-(1+2\Phi)\,d\tau^2 + (1-2\Psi)d\boldsymbol{x}^2\right]$$

- Secondary perturbations
  - lensing of the CMB $\qquad \Delta\tilde{T}_{\rm CMB}(\hat{\boldsymbol{n}}) = \Delta T_{\rm CMB}(\hat{\boldsymbol{n}} + \nabla\phi)$

$$\phi = -\int_0^{r_{\rm ls}} dr\,\frac{d_A(r_{\rm ls}-r)}{d_A(r)\;d_A(r_{\rm ls})}(\Phi + \Psi)$$

CMB as a probe of neutrinos: Background density

ρ_ν causes:

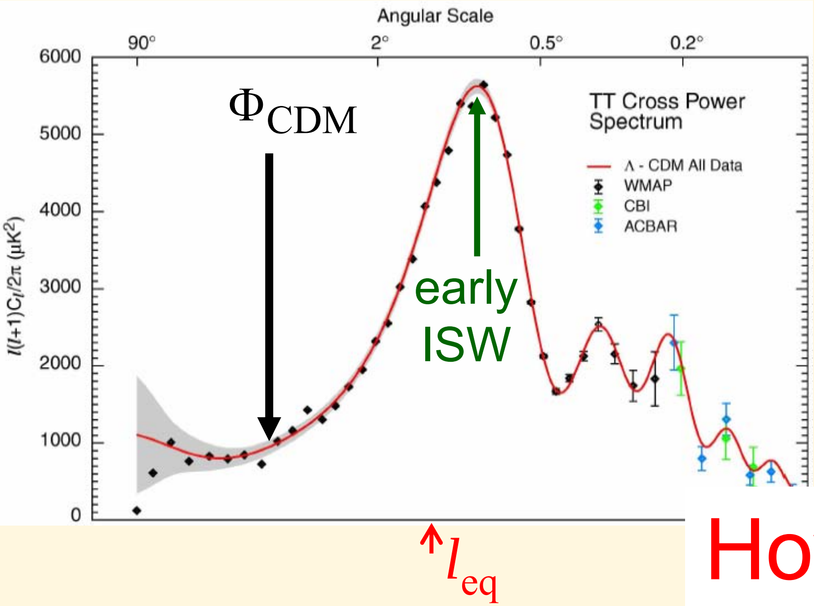
A. Faster Hubble expansion in the radiation era

$$\frac{d\tau}{dz} = -\frac{1}{H} \downarrow$$

– $S_{\text{acoustic}} = \int_0^{\tau_{\gamma\,\text{dec}}} c_s \, d\tau \downarrow$

– $\lambda_{\text{Silk}} = \left(\int A \tau_c \, d\tau\right)^{1/2}, \quad \tau_c \equiv \frac{1}{a n_e \sigma_{\gamma e}} \downarrow$

B. Prolonged radiation era



– shifted $l_{\text{eq}} = S^{-1}(\tau_{\text{eq}})\, d_{\text{last scat}}$:

shifted scale of low-l suppression

– decreased $z_{\text{eq}}/z_{\gamma\,\text{dec}}$:

larger early ISW contribution to the height of the 1st peak

However!

CMB as a probe of neutrinos: Background degeneracies

- The typical CMB (and LSS) observables are
  a) angles $l_{\mathrm{A}} = S^{-1}(\tau_{\gamma\,\mathrm{dec}})\, d_{\mathrm{last\ scat}}$, $l_{\mathrm{Silk}} = \lambda_{\mathrm{Silk}}^{-1}\, d_{\mathrm{last\ scat}}$
  $l_{\mathrm{eq}} = S^{-1}(\tau_{\mathrm{eq}})\, d_{\mathrm{last\ scat}}$
  b) redshifts $z_{\mathrm{eq}} / z_{\gamma\,\mathrm{dec}}$

Bashinsky and Seljak PRD 69, 2004

Bashinsky, astro-ph/0411013

- All of these are preserved if the following ratios are unchanged:

$\rho_b \div \rho_\gamma$ [fixes c_s]

$\rho_{\gamma+\nu} \div \rho_{c+b} \div \rho_{\mathrm{DE}}$ at all z [fixes $H(z)/H_0$]

(achieved by fixing z_{eq}, Ω, Ω_m, w_{DE})

conformal rescaling

$1 - Y_p \div \sqrt{\rho_{\gamma+\nu}}$ [fixes $\tau_c H(z) = \dfrac{H}{a x_e (1-Y_p) \dfrac{\rho_b}{m_H} \sigma_{\gamma e}}$]

[with recombination via $2s \to 1s\,\gamma\gamma$ bottleneck, x_{e} is also degenerate]

- The conformal degeneracy is broken by measuring H_0 or Y_p

CMB as a probe of neutrinos: Primary perturbations

- The gravitationally coupled dynamics of γ fluid and free streaming ν in the radiation era is solvable analytically:

$$d_\gamma \equiv \frac{\Delta n_\gamma}{n_\gamma} \rightarrow 3\frac{\Delta T_{\mathrm{CMB}}}{T_{\mathrm{CMB}}}: \qquad \ddot{d}_\gamma - \frac{1}{3}\nabla^2 d_\gamma = \nabla^2(\Phi + \Psi)$$

In position (real) space

$$d(\tau,k) = \int_{-\infty}^{\infty} e^{-ikx} d(\tau,x) \qquad d(\tau \rightarrow 0,k) = 1 \quad \Leftrightarrow \quad d(\tau \rightarrow 0,x) = \delta(x)$$

eq. (1) becomes

$$\left(\frac{x^2}{\tau^2} - \frac{1}{3}\right) d_\gamma = \Phi + \Psi$$

which solution is

$$\boxed{d_\gamma = \frac{A_\gamma}{2}\delta\left(|x| - \frac{\tau}{\sqrt{3}}\right) + \frac{\Phi + \Psi}{\frac{x^2}{\tau^2} - \frac{1}{3}}}$$

CMB as a probe of neutrinos: CMB perturbations after the horizon reentry

The solution

$$d_\gamma = \frac{A_\gamma}{2}\,\delta\!\left(|x| - \frac{\tau}{\sqrt{3}}\right) + \frac{\Phi+\Psi}{\frac{x^2}{\tau^2} - \frac{1}{3}}$$

determines the subhorizon acoustic modes

Bashinsky and Seljak 04

$$d_\gamma(\tau,k) = \int_{-\infty}^{\infty} e^{-ikx} d_\gamma(\tau,x) \to A_\gamma \cos\frac{k\tau}{\sqrt{3}} - \pi\sqrt{3}\,(\Phi+\Psi)\big|_{x=\frac{\tau}{\sqrt{3}}} \sin\frac{k\tau}{\sqrt{3}}$$

– solves the subhorizon acoustic equation

$$\ddot{d}_\gamma + \frac{1}{3}k^2 d_\gamma = 0$$

– A_γ is fixed by the initial condition

$$d(\tau \to 0, k) = 1 \quad \Leftrightarrow \quad d(\tau \to 0, x) = \delta(x),$$

by which

$$\int_{-\infty}^{\infty} d(\tau,x) = d(\tau, k \to 0) = 1 \qquad \forall\, \tau$$

– $\Phi + \Psi$, hence the phase shift, is determined from linearized Einstein eqs. by all species, including ν

CMB as a probe of neutrinos: Neutrino perturbations

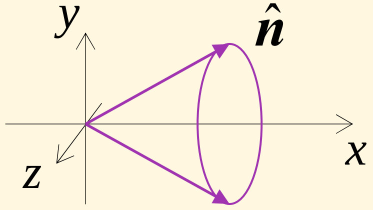
Linear perturbation of ν intensity in a direction n_i

$$\mathcal{I}_\nu \equiv \frac{\Delta I_\nu}{I_\nu}, \qquad I_\nu(x^\mu, \hat{\boldsymbol{n}}) \equiv \int_0^\infty P^3 dP\, f_\nu(\hat{\boldsymbol{n}}P)$$

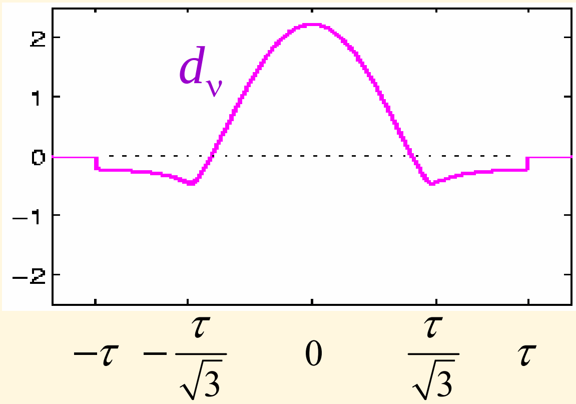
$$\dot{\mathcal{I}}_\nu + n_i \nabla_i \mathcal{I}_\nu = -4\, n_i \nabla_i (\Phi + \Psi)$$

Find scalar modes in the radiation era as self-similar Green's functions:

$$\left(\frac{x}{\tau} - \mu\right) \mathcal{I}_\nu = 4\mu(\Phi + \Psi), \qquad \mu \equiv n_x$$



$$\mathcal{I}_\nu = A_\nu \delta\left(\frac{x}{\tau} - \mu\right) + \frac{4\mu}{\frac{x}{\tau} - \mu}(\Phi + \Psi), \qquad (\rho_\nu/\rho \to 0)$$



Expanding $\mathcal{I}_\nu(\mu)$ over scalar multipoles l $(m=0)$,

$$d_\nu = \frac{3}{4}\int \frac{d\mu}{2}\mathcal{I}_\nu, \qquad \Sigma_\nu = \frac{3}{4}\int \frac{d\mu}{2} P_2(\mu)\, \mathcal{I}_\nu$$

Compare:

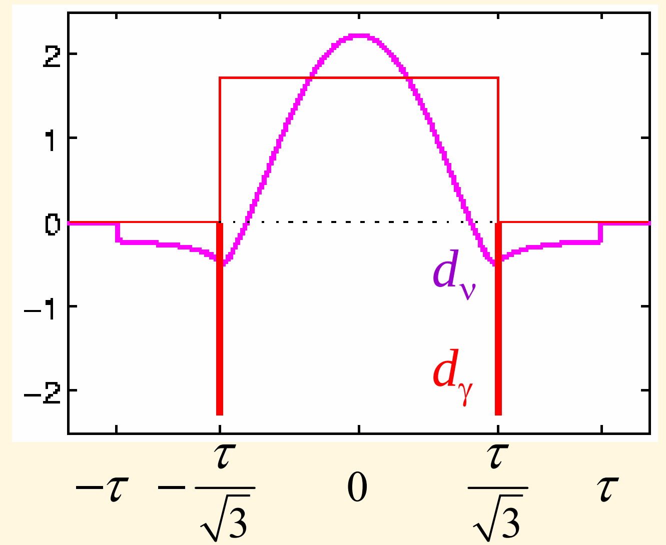
Relativistic free streaming ν ≠ Coupled γ

1. Damping of neutrino inhomogeneities on subhorizon scales due to directional dispersion

$$d_\nu = \frac{\sin(k\tau)}{k\tau} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \qquad d_\gamma = \cos\left(\frac{k\tau}{\sqrt{3}}\right) e^{i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$(\rho_\nu/\rho \to 0)$

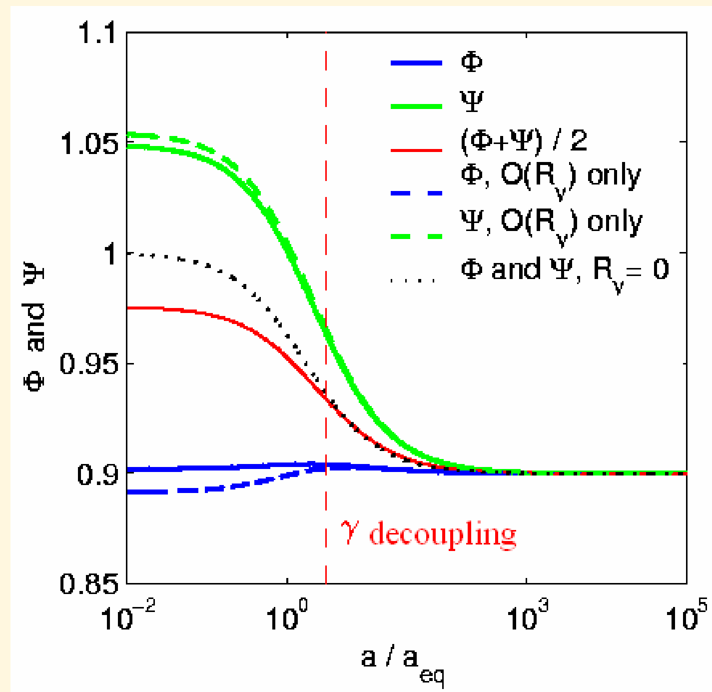
$(\rho_\nu/\rho \to 0)$



2. Propagation with $c = 1$ vs. $c_s < 3^{-1/2}$

3. Different phase of the oscillations

4. Anisotropic stress, which gravitates (hence, $\Phi \neq \Psi$)



CMB as a probe of neutrinos: Neutrino signatures in CMB

- Given d_ν, d_γ and Σ_ν, find $\Phi+\Psi$ by solving the linearized Einstein eqs.

- Thus for subhorizon CMB oscillations

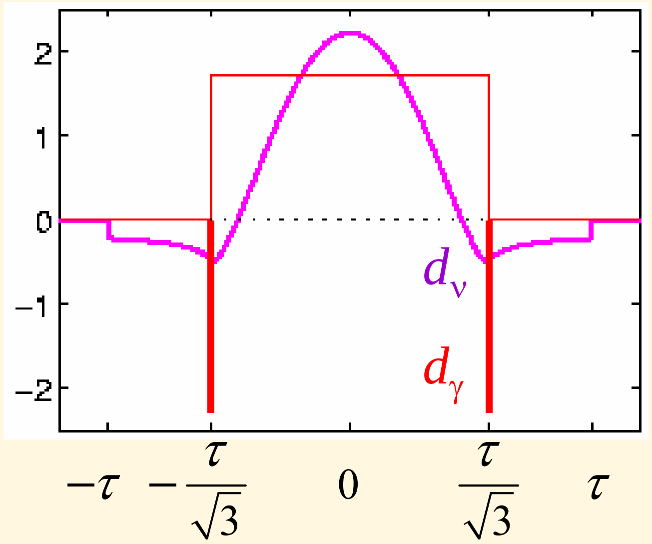
$$d_\gamma(\tau,k) \to A_\gamma \cos\frac{k\tau}{\sqrt{3}} - \pi\sqrt{3}\,(\Phi+\Psi)\big|_{x=\frac{\tau}{\sqrt{3}}} \sin\frac{k\tau}{\sqrt{3}}$$

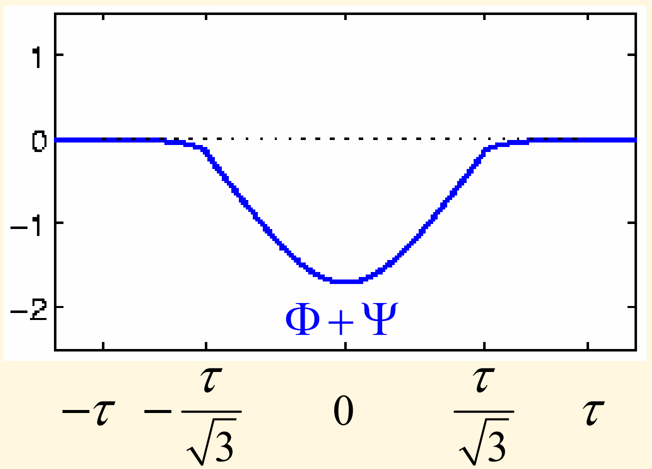
find the amplitude reduction and phase shift

$$\frac{\Delta A_\gamma}{A_\gamma} \approx -0.27\,\frac{\rho_\nu}{\rho}, \qquad \Delta\varphi \approx 0.19\,\pi\,\frac{\rho_\nu}{\rho}$$

- Theorem: For *adiabatic* perturbations, in real space, $\Phi \equiv \Psi \equiv 0$ beyond the particle horizon
- Corollary: $\Delta\varphi = 0$ unless some matter perturbations propagate faster than c_s

$(\rho_\nu/\rho \to 0)$





CMB as a probe of neutrinos: Non-degenerate signatures of neutrinos

- Appear in the CMB temperature and polarization angular power for $l > S^{-1}(\tau_{eq})\, d_{\text{last scat}} \sim 200$

- Introducing

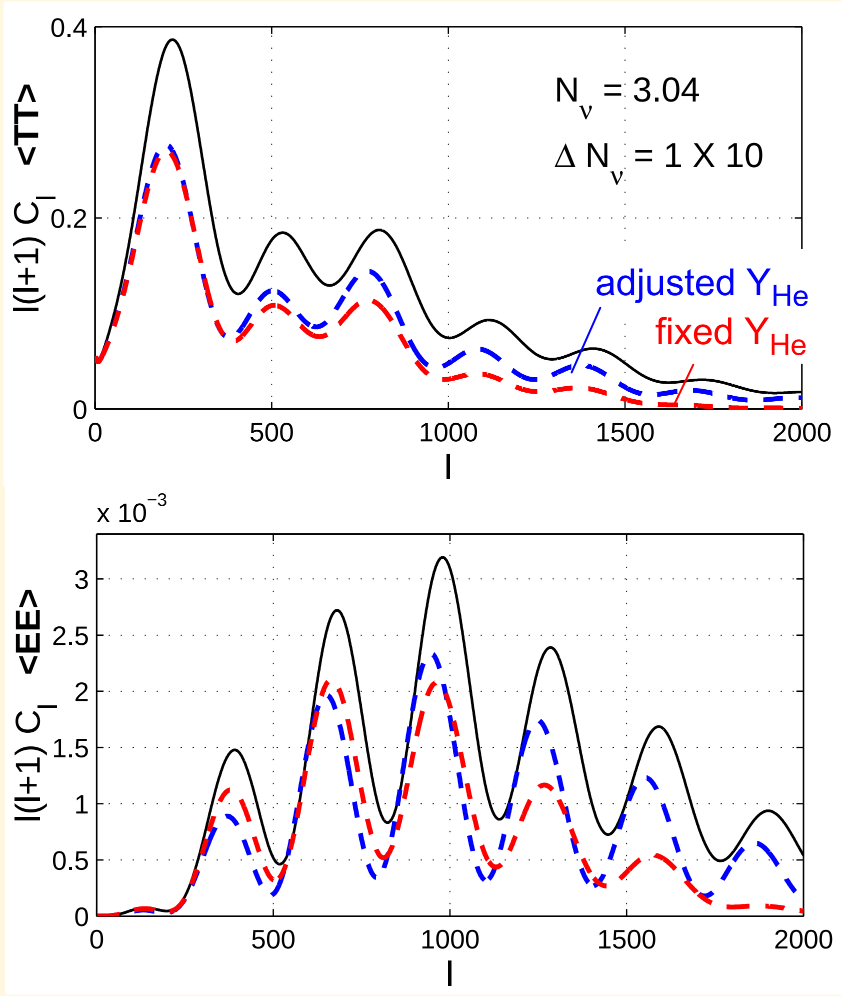
$$R_\nu \equiv \frac{\rho_\nu}{\rho_{\text{rad}}} \qquad (N_\nu \approx 3:\ R_\nu \simeq 0.41)\ ,$$

$$\delta l_{\text{peaks}} \approx -57 R_\nu$$ (all peaks)

$$\delta C_l / C_l \approx -0.5\, R_\nu ,$$ adjusted Y_{He} (any l)

$$-1.8\, R_\nu ,$$ fixed Y_{He} (l = 3000)

$$\delta P_{\text{m}} / P_{\text{m}} \approx +1.5\, R_\nu$$



The compared models have equal CMB power on small scales

CMB as a probe of neutrinos: Constraints and Forecasts

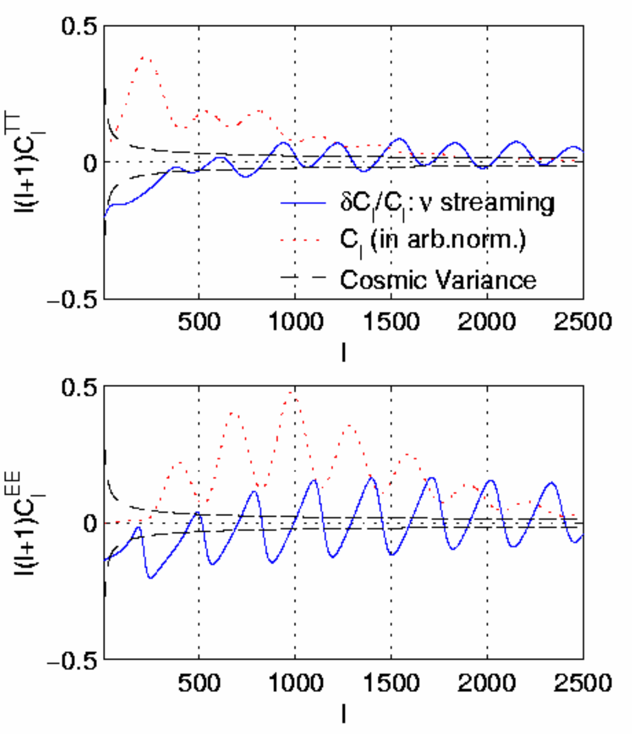
- Current limits

Crotty et al. 03
Pierpaoli 03
Hannestad 03
Barger et al. 03,04

$1.6 < N_\nu < 7\text{-}8$

(CMB+LSS, 95%CL)

The constraints will significantly improve with smaller angular resolution



- Forecasts

Lopez et al. 99

Bowen et al. 01

Bashinsky and Seljak 03

$\Delta N_\nu \sim 0.2$

with Planck (1σ)

Experiment	$f_{\rm sky}$	θ_b	$w_T^{-1/2}$ [μ K']	$w_P^{-1/2}$ [μ K']	ΔN_ν TT	ΔN_ν TT+TE+EE	ΔN_ν (free Y) TT+TE+EE
Planck	0.8	7'	40	56	0.6	0.20	0.24
ACT	0.01	1.7'	3	4	1	0.47	0.9
ACT + Planck					0.4	0.18	0.24
CMBPOL	0.8	4'	1	1.4	0.12	0.05	0.09

CMB anisotropy as a probe of neutrino models

- Discriminate models by
  - ρ_ν / ρ_γ
  - neutrino free streaming / coupling
- Examples of non-standard ρ_ν / ρ_γ
  - sterile states which decouple early (small ΔN_ν, passing BBN)
  - ν (γ) heating / cooling after BBN (changed ΔN_ν, after BBN)

CMB anisotropy as a probe of neutrino models

- Discriminate models by
  - ρ_ν / ρ_γ
  - neutrino free streaming / coupling
- Examples of neutrino recoupling after BBN
  - Low energy symmetry breaking in the neutrino sector
    - alternative to the seesaw for m_ν Chacko et al. 03
    - avoid LSS constraints on m_ν Beacom et al. 04; Chacko et al. 04
    - reconcile LSND with the other data Chacko et al. 04
  - Composite neutrinos Arkani-Hamed and Grossman 98; Okui 04
  - Mass-varying neutrinos Fardon et al. 03
  - Coupling to Lorentz-violating sector Grossman 05

These models are already constrained by the CMB

Hannestad 04; Trotta and Melchiorri 04

Basics: Primordial Gravitational Waves

$$ds^2 = a^2\left[-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j\right]: \qquad \partial_i h_{ij} = 0, \quad \sum_i h_{ii} = 0$$

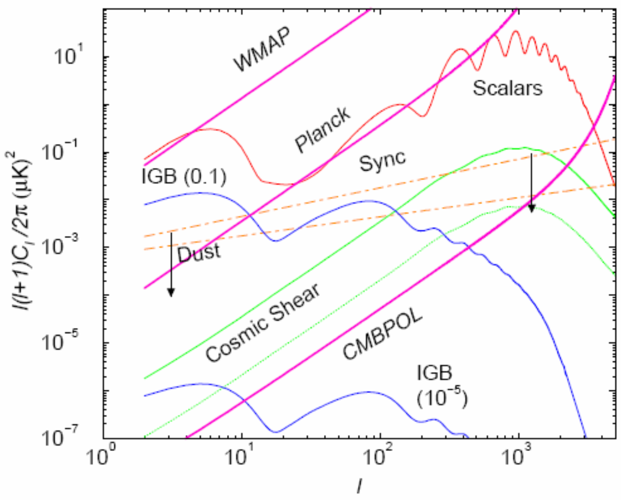
- Produced by inflation

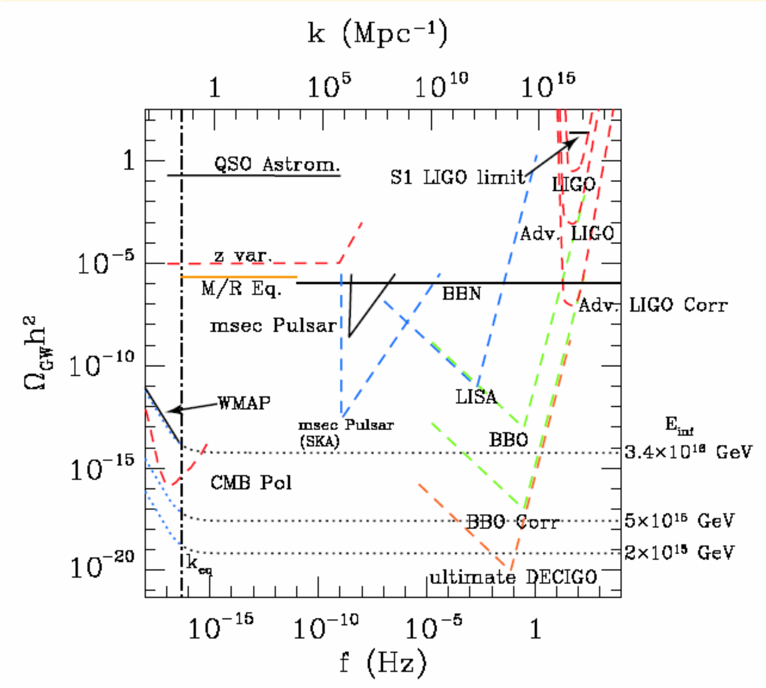
$$\Delta_h^2 \equiv \frac{d < h^2 >}{d \log k} = \frac{H_{\mathrm{infl}}^2}{2\pi^2 M_{\mathrm{Pl}}^2}$$

- Subsequent evolution

$$\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G a^2 \Sigma_{ij}, \qquad \Sigma^i_j \equiv T^i_j - \frac{1}{3}\delta^i_j T^k_k$$

is sourced by anisotropic stress, generated by free streaming





Cooray 05

Smith, Kamionkowski, Cooray 05

- Should be detected if $\rho_{\mathrm{infl}} > 10^{15\text{-}16}$ GeV

Gravitational Waves: Coupled GW-ν evolution

$$\ddot{h}_{ij} + 2\mathcal{H}\dot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G a^2 \Sigma_{ij} \qquad (1)$$

$$\dot{I}_\nu + n_k \nabla_k I_\nu = 2 n_k n_i n_j h_{ij,k}$$

$$\Sigma_{ij}(\tau, x) = -\frac{\rho_\nu}{4} \int_0^\tau d\tau' \int_{-1}^{1} d\mu \, (1-\mu^2)^2 \, \dot{h}_{ij}(\tau', x') \qquad (2)$$

Weinberg 03

- Consider subhorizon scales, where GWs decouple from neutrinos

$$h_{\mathrm{subhor}} \propto \frac{\sin(k\tau - \varphi_0)}{a}$$

The phase shift and amplitude are determined by the coupled evolution through the horizon entry

Gravitational Waves: Radiation era – Phase shift

Fourier space:

$$h_{\rm rad}(k\tau) = A_0 \frac{\sin(k\tau - \varphi_0)}{k\tau} + O\left(\frac{1}{k^2\tau^2}\right)$$

Position space:

$$h_{\rm rad}(\tau, x) = \frac{A_0}{2\tau}\left[\cos\varphi_0\, \theta\left(1 - \left|\frac{x}{\tau}\right|\right) + \frac{\sin\varphi_0}{\pi} \ln\left(1 - \frac{x^2}{\tau^2}\right)\right] + \text{reg. terms}$$

By causality,

$$h_{ij}(\tau, x) = 0 \quad \text{for} \quad |x| > \tau$$

(confirmed by direct calculation)

Hence

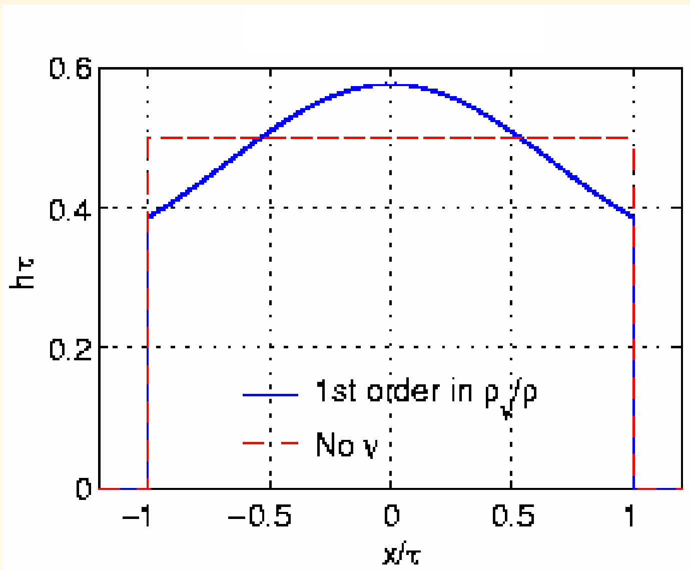
$$\boxed{\sin\varphi_0 = 0}$$

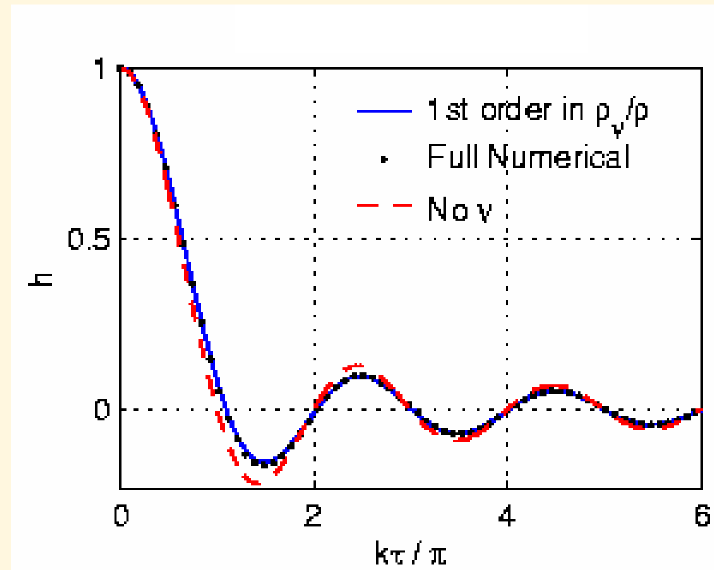
Bashinsky 05

Gravitational Waves: Radiation era – Full evolution

- Calculate analytically in linear order in ρ_ν/ρ

Position space → Fourier space





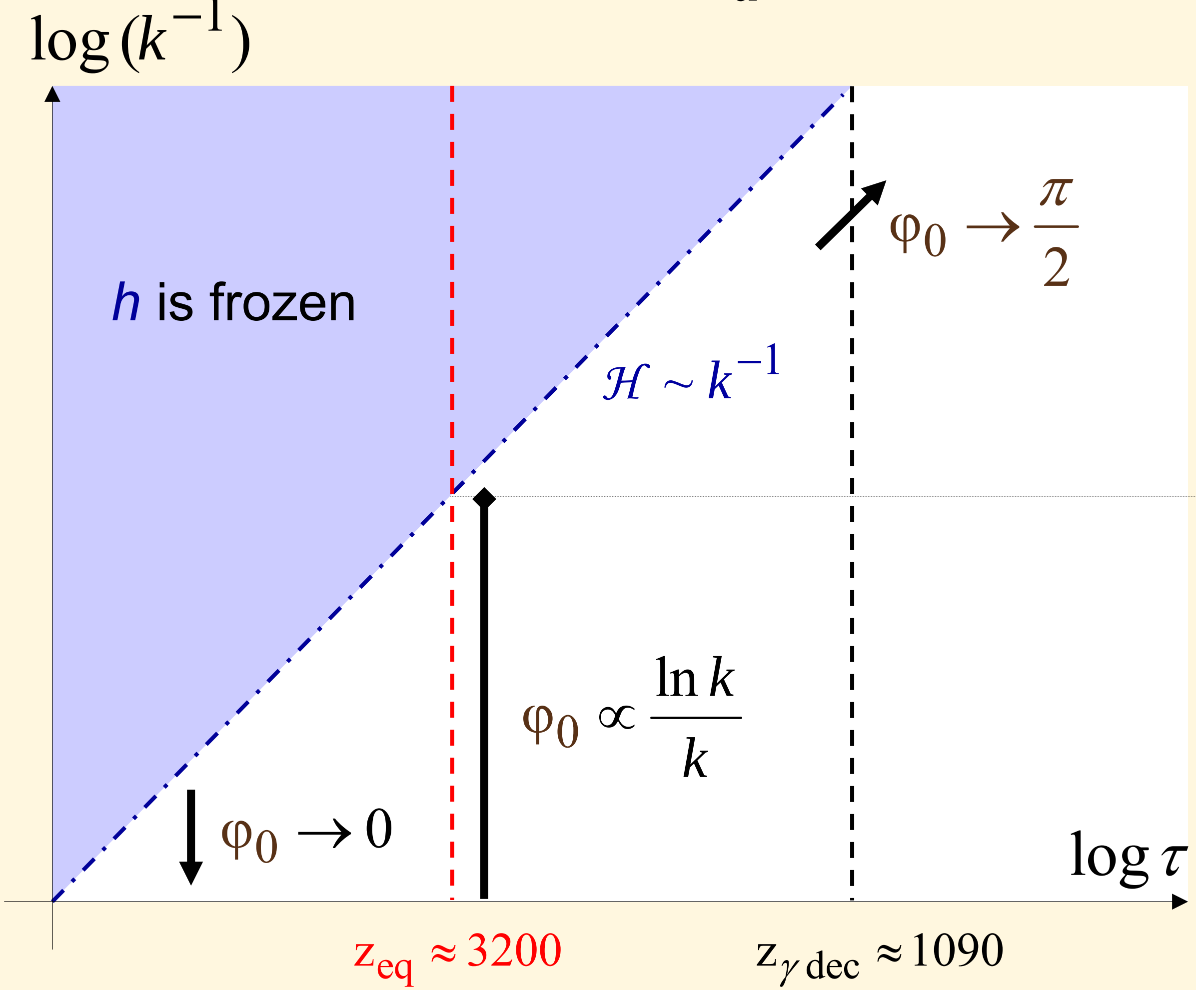
- The solution determines subhorizon GW amplitude as

$$h_{\rm rad}(k\tau) \to h_{\rm prim}\left[1 - \frac{5\rho_\nu}{9\rho} + O\left(\frac{\rho_\nu^2}{\rho^2}\right)\right]\frac{\sin(k\tau)}{k\tau}$$

($N_\nu = 3.04$ gives 23% suppression, double for power spectra)

Gravitational Waves: Phase

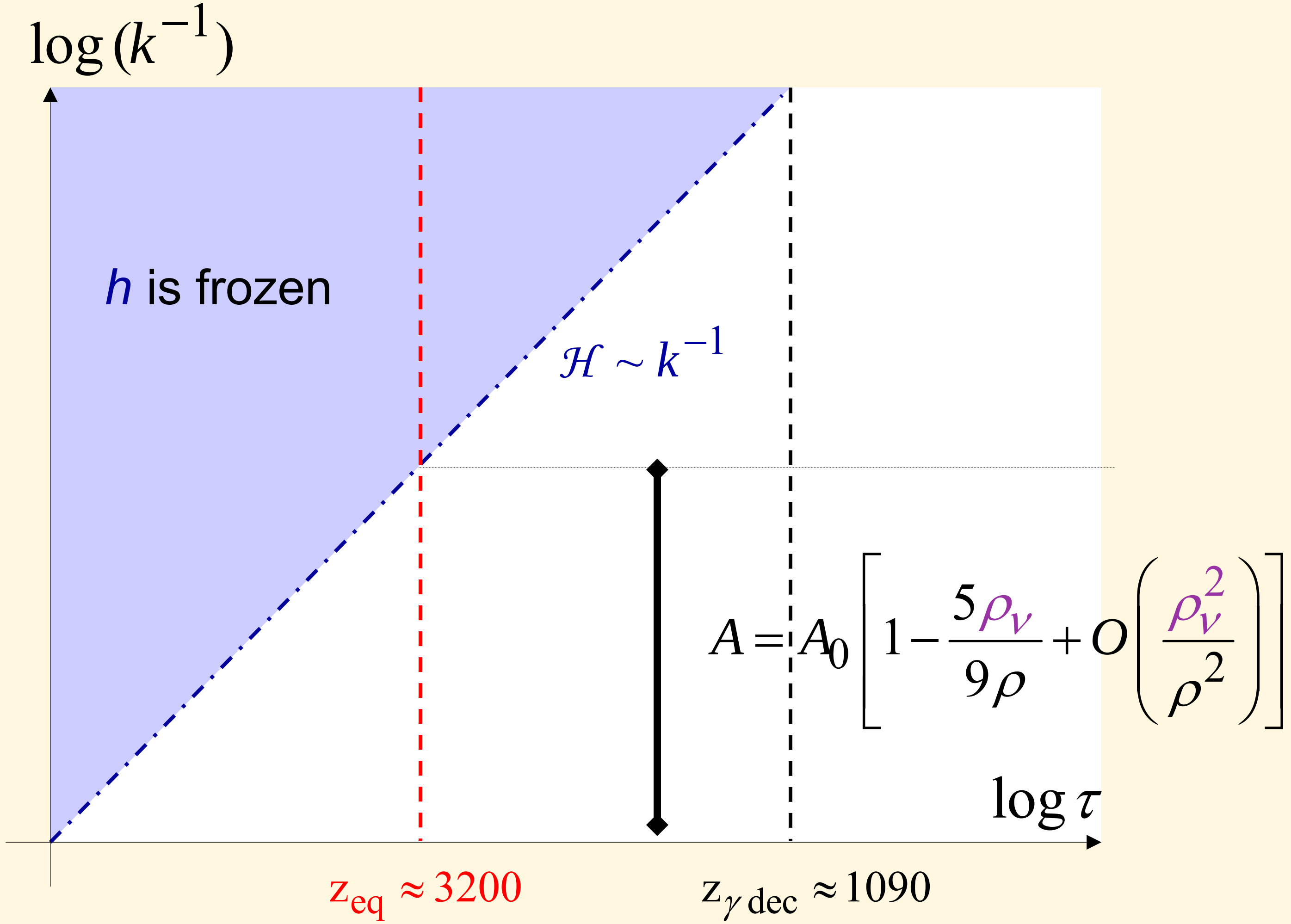
$$h_{\text{subhor}} \propto \frac{\sin(k\tau - \varphi_0)}{a}$$



* Neutrinos have minor effect around $k \sim k_{eq}$

Gravitational Waves: Amplitude

Suppressed by neutrinos for $k_{\nu\,\mathrm{dec}}^{-1}(\sim 0.1\ \mathrm{kpc}) < k^{-1} < k_{\mathrm{eq}}^{-1}(\sim 100\,\mathrm{Mpc})$



Conclusions:

- Relativistic νs were among the dominant species when most of the perturbation modes ($k^{-1}<10^2$ Mpc) entered the horizon

- The present CMB constraints on ν are relatively weak due to
  a) Full degeneracy of the impact of *homogeneous* radiation background on the CMB and LSS
  b) Neutrinos being relativistic up to CMB decoupling

- Nevertheless, the robustness of the signatures of ν perturbations at high l and precision of CMB measurements will lead to competitive constraints on neutrinos since Planck

- The robust CMB signatures of relativistic νs are caused by their free streaming *faster* than the photon-baryon sound speed

- ν free streaming suppresses GW *without* shifting GW phase