Chapter 1

Dynamic Coverage Problems in Sensor Networks

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1.1 Abstract

One of the fundamental issues in sensor networks is related to analyzing the coverage, or how well a network of sensors monitors the physical space for an intrusion. The coverage is a measure of the quality of service (QoS) of the sensor network and has been the subject of various studies. The concept of coverage was introduced by Gage in 1992, who studied it in relation to multi-robot systems. He defined three classes of coverage problems: blanket coverage (also known as area coverage), where the goal is to achieve a static arrangement of sensing elements that maximizes the detection rate of targets appearing in the region, sweep coverage, where the goal is to move a number of sensors across the region as to maximize the probability of detecting a target, and barrier coverage, where the objective is to protect the region from unauthorized penetration. While blanket coverage is relatively well researched, the other two types of coverage have been less studied and offer much greater technical challenges. In this survey we concentrate on the two dynamic types of coverage problems, barrier and sweep coverage. We review recent results, proof techniques, analyses, and open problems and challenges. We focus on methods that allow the performance of the algorithms, i.e.
their efficiencies and accuracies, to be analyzed by theoretical means that would yield worst-case guarantees, rather than using empirical methods.

1.2 Introduction

1.2.1 Historical Picture: Coverage and Tessellation

Coverage may be defined as a task where the objective is to guarantee that a set of entities of interest (e.g. points, objects or events) are completely covered. The covering is broadly defined. For example, it may be physical or using observation points. Coverage is one of the oldest problems in mathematics and physics. For example, in 1619, Johannes Kepler, a famous German mathematician and astronomer, published his seminal book entitled “Harmonices Mundi” that included the first study on tessellation [45]. The task of tessellation is a special coverage case where the goal is to cover infinite two-dimensional space using the repetition of a single or a finite number of geometric shapes. Of course, no overlaps or gaps are allowed. Probably the most celebrated result related to tessellation was discovered by Yevgraf Fyodorov at the end of 19th century. He presented proof that all periodic tilings of the plane feature one of seventeen unique groups of isometrics.

1.2.2 Coverage and Sensor Networks

Although coverage has a long and rich history, it only recently emerged as a premier computer science research topic. This is a confluence of technology push and application pool. The technology push was provided due to creation of sensor network. This rapidly growing area provides means for comprehensive surveillance of both objects and area under reasonable cost and energy constraints.

The second part of the research and development impetus was provided by rapid emergence of security as one of the most important and desired system and application aspects. In a sense, coverage is the fourth wave of information security. The first was created in 1976 by the introduction of public key cryptography. It provided practical and theoretically sound techniques for ensuring privacy of data storage and data communication. The second is related to system security. In a sense, these techniques have longer and richer history than public key cryptography. Recent emphasis has been on hardware-based security and detection of malicious circuitry. The third wave aims at protection of the Internet and the WWW. Although this wave is by far the most diverse and covers issues from phishing to privacy, a significant emphasis has been on denial of service.
1.2. INTRODUCTION

The fourth wave that has been just started is related to physical and social security using large scale sensing, computation, communication, and storage resources. It is often envisioned in the form of multiple sensor network that use (standard) wireless communication infrastructure to enable transfer of data to computational clouds. While the exact system picture has been radically changing (e.g., initially network processing of collected data was a dominant system paradigm), the frontier component (sensor networks) has been constant in all efforts.

Coverage is naturally both a sensor network canonical task and well as the basis for numerous physical and social security tasks. It has extraordinarily broad basis and numerous coverage subtasks can be defined.

The concept of coverage was introduced by Gage, who defined three classes of coverage problems: (i) blanket coverage (also known as area coverage), where the goal is to have each point of the region be within a detection distance from at least one of the sensors (static sensors, static objects coverage); (ii) sweep coverage, where the goal is to move a number of sensors across the region as to maximize the probability of detecting a target (mobile sensors, static objects); and (iii) barrier coverage, where the objective is to optimally protect the region from undetected penetration (static sensors, mobile objects). In addition, one can pose the fourth possible definition: (mobile sensors, mobile objects). The last class of problems is not just practically very important, but also technically very challenging. Its theoretical treatment requires several probabilistic models. Its practical addressing requires sound and realistic statistical models that consider correlations.

One can also envision many other generalizations of dynamic coverage problems. For example, a number of authors considered techniques for maximizing the life-time of the network and, therefore, the length of the pertinent coverage. Also, coverage under multiple objectives and/or multiple constraints, most often related to sensing and communication, has been a popular topic. It is important to note that technological trends may evolve so that communication ranges are much longer than sensing. Nevertheless, multiobjective coverage has tremendous practical importance. For instance, it is a natural way to address common scenarios that detection of an object or an event can be accomplished only by using sensors of different modalities and therefore properties. Another important dimension is providing guarantees of proper functioning of the coverage system in presence of faults or security attacks.

1.2.3 Challenges in Solving Coverage Problems

We place special emphasis on the following four types of challenges.

Algorithmic challenges. Coverage problems are almost always intrinsically multidimensional. Many of them also include time dimension. Interestingly, some of the effective coverage problems can be naturally mapped into equivalent combinatorial and in particular graph formulation. For wide classes of coverage problems and in particular
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Exposure problems very often the most effective techniques involve variational calculus and its discretized realization using dynamic programming.

Finally, in some applications it is important that the algorithms have their localized versions where each sensor node contacts only a small subset of other nodes using high quality communication links in such a way that the overall global optimality is preserved completely or within a certain application bound. These types of coverage problems are most relevant in situations where one of the objectives is low energy operation or preservation of the communication bandwidth. Also, this type of operation may be important when security is one of the important requirements. Our last remark is that probabilistic and statistical analysis of coverage algorithms is increasingly important.

Modeling challenges. There are two main aspects that require careful modeling decisions. The first is modeling of sensitivity of sensors. Of course, for different types of sensors different types of models are more appropriate. Initially many coverage tasks were treated under assumption that the detection is binary, e.g. either an object of interest is observed or not. Consequently, much more comprehensive sensing models are introduced. For example, exposure requires that an object of interest is under surveillance is such a way that an integral of closeness over time is above a user specified threshold. Also, directionality of some type of sensors was recognized. Of course, more and more complex models can be and should be addressed. However, as is often case in statistics, a more complex sensing model does not imply a more realistic problem formulation and may significantly reduce (or enhance) the application domain.

The other important modeling issue is related to targeted objects and terrain. For example, in many applications the mobility models are of prime importance. It is common to start from simple and intuitive models and keep increasing their complexity. It is interesting to mention that mobility models, unfortunately, have a long and painful history of being not just tremendously speculative, but even obviously and deeply completely counterproductive.

System challenges. It is customary that papers in top sensor networks are divided into two groups: theory and system. Not so rarely theory papers are considered elegant and well mathematically founded but of rather low practical relevance. On the other hand, system people are primarily based on complete and demonstrated implementation that requires unacceptably high levels of abstraction and simplification. So, the first and most important system challenge is to combine useful properties of previous generation of both system and theory papers while eliminating past and some of the current problems.

Other premier system problems include low cost realization and energy efficiency. The last metric is further enhanced to include low power requirement in particular in self-sustainable coverage systems.

Security challenges. Security is one of the premier requirements in many applications and its relative role is rapidly
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Increasing. It already ranges from privacy and trust to resiliency against hardware, software, and physical attacks. Very often sensor networks used to ensure coverage are not attended or may even be deployed in hostile environments. Particularly interesting is the situation when two or more parties are observing each other and simultaneously aim to ensure high coverage while preserving their privacy of action. We expect that game theory techniques will be soon used in this context.

1.2.4 **Focus of This Survey**

In summary, coverage has a great variety of potential formulations and is a premier sensor network and emerging physical security task. In this survey we have three major objectives. The first is to survey of the most popular and most important, in terms of application coverage, tasks and proposed techniques. There are already several thousand coverage techniques. Therefore, it is not even possible to aim to be comprehensive. Instead, we focus on the most effective techniques that target most generic and pervasive coverage formulation.

The second goal is to try to establish the place of coverage in the global picture and its relationship with other sensor network, security, and system design tasks and applications. Our final target is to identify and provide a research impetus for the most important and challenging new coverage research directions.

1.3 **The Coverage Problem**

In this section we discuss the importance of the coverage problem in sensor networks and briefly review the topic of static coverage. In static coverage, the goal is to place the smallest number of sensors in such a way that an area of interest is observable. In comparison, dynamic coverage addresses the situation in which either the sensors or the objects are allowed to move in the area of interest. A special case of dynamic coverage is the exposure problem in which the detection is accomplished if an integral over time of a specific sensing function is large enough to ensure detection and possibly the characterization of the pertinent object.

1.3.1 **Historical Perspective**

As we indicated in the Introduction, coverage is an optimization problem, in particular with a long history in mathematics and crystallography, and more recently in robotics, computational geometry (e.g. art gallery problems), and television and wireless networks. However, the explosion of interest in coverage received a tremendous impetus


Table 1.1: The number of sensor coverage papers according to the Google Scholar database. The first column indicates year. The last two columns indicate the number of papers that address coverage in sensor networks and the total number of papers in the database respectively. The data for 2012 includes only publications indexed in January.

with the emergence of sensor networks somewhere around the turn of the last century.

Table 1 provides the quantification of our claim. It shows the number of sensor coverage papers according to Google Scholar. We see that while the overall number of papers is relatively constant per year, the number of papers with words "coverage in sensor networks" has experienced consistent growth and increased by more than 30 times in the last decade even when normalized against slight growth of the overall number of papers. The overall number of papers is actually increasing every year, but non-trivial latency in paper indexing hides this growth. It also results in reporting somewhat understated growth in the number of coverage papers.

There have been several survey papers completely dedicated to coverage in sensor networks [16] [41] [35] [9]. In addition, several ultra popular comprehensive surveys of sensor networks devoted a substantial space to coverage [5] [64] [33] [7] [99]. Also, a large number of surveys have been published on more specific aspects of coverage [32] [33] in particular using visual sensors [4] [19] [85] and energy efficient coverage [61] [6] [27] [38].

1.3.2 Applications and Architectures

Sensor networks provide a bridge between computational and communication infrastructures (e.g. Internet) and physical, chemical, and biological worlds. The number of potential applications is unlimited. Most often environmental, infrastructure security (e.g. pipelines and building), and military and public security are addressed. More recently, wireless health and medical applications have emerged as one of the most popular research directions.

Initially, Internet research has had a dominating impact on the wireless sensor network research. Energy has been
recognized as one of the most important design metrics. In addition, there has been an emphasize on efficient usage of bandwidth. Ultra low power operation of wireless sensor networks was the focus of many wireless sensor network efforts. Therefore, the ultra low power node with very short communication ranges was accepted as the preferred architecture building block.

However, in the last several years it has been widely recognized that rapid progress in wireless mobile network provides numerous advantages. For example, mobile phones-based participatory sensing that involves human interaction has emerged as the dominant architecture paradigm.

Both applications and architectures have profound ramifications on how coverage problems are formulated and addressed. For example, the use of mobile phone infrastructure eliminated limitations and concerns about communication range that is now much higher than the sensing range of essentially all sensors. Also, the need for localized algorithms is greatly reduced and much more complex definitions of coverage that require much higher processing resources and energy can be now realistically addressed. On the other hand, latency has gained importance over throughput.

Also, each type of application requires new definitions of coverage. For example, medical applications can benefit little from traditional notions of coverage. In order to establish credible medical diagnosis significantly more complex processing is needed that blurs distinctions between coverage and sensor fusion. It also introduces many new aspects such as sizing of sensors and its impact on coverage.

1.3.3 Real-Time Coverage

Operation in real-time is essential for a majority of coverage applications that use sensor networks. Surprisingly, this topic still does not receive a proportional amount of research and effort. This is unexpected in particular since one of the three tracks of the most prestigious real-time conferences, Real-Time Systems Symposium (RTSS), is dedicated to sensor networks. One of the first and most influential papers in this domain is by Jeong et al. [43] which addresses the problem of observing a net of actual pathways where vehicles move a specified maximal speed. Under a set of assumptions that include the maximal car density the goal is to ensure that all intruding targets are detected before they reach any of the protection points. The objective is to maximize the lifetime of a sensor network that is used for coverage. The algorithm is based on the Floyd-Warshall algorithm to compute the All-Pairs Shortest Paths formulation. In order to maximize the life-time of the network, different sensors are assigned to different duty-cycle schedules. Jeong and his coauthors presented both centralized and localized algorithms for early detection of targets on a graph (i.e. highway or street network). Zahedi et al. [100] further explored the problem of trade-offs between
the quality and duty-cycle (energy) of the sensors.

Trap coverage is a very interesting and natural formulation of coverage that is related to real-time detection and, in particular, latency of detection. It is also a way to address approximate coverage when the number of available sensors is pre-specified. Until now we mainly discussed coverage techniques in which complete coverage of a targeted field is the objective. In trap coverage holes in coverage are allowed but only if their number and their size are below specified measures. One such measure that captures latency of detection is a time that an intruder spends in straight line travel at a specified speed before being detected. Recently, this problem has been addressed both under and not under the assumption that energy-efficiency is one of the requirements [10][57].

1.3.4 Static Coverage

Although our survey is focused on dynamic coverage in sensor networks, it is important to discuss static coverage in which the goal is to cover a specific area using the smallest number of sensors. An alternative formulation is one in which the goal is to cover a maximum subarea of a given area using a specified number of sensors.

Although static coverage is probably conceptually the simplest possible formulation of any coverage problem, almost all of its instantiations are still NP-complete. For example, these instantiations can be often mapped to the dominating set problem. Interestingly, when we consider coverage of a rectangular area using disks, the complexity of the corresponding optimization is not known.

One of the first approaches to address static coverage was presented by Slijepcevic [81]. They proposed two techniques, one uses simulated annealing and the other employs integer linear programming. In addition, D. Tian first as a student and later with his research group proposed a number of techniques for static coverage [89][90].

1.4 Barrier Coverage

In barrier coverage, the objective is to protect the area from unauthorized penetration. We will discuss in detail several types of barrier coverage including perimeter coverage, where the objective is to cover with sensors a narrow strip along the boundary of the region, the maximum breach path problem, where the goal is to find a path that maximizes the minimum distance to any sensor, and the minimum exposure path problem, whose objective is to find a path of minimum exposure, where the exposure of the path defined as the integral of the sensing signal along that path.
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Figure 1.1: Types of belts depending on the boundary type: (a) open belt – when the boundary is connected; (b) closed belt – when the boundary is disconnected

1.4.1 Perimeter Coverage

Problem Formulation

The objective of perimeter coverage is to study ways to detect an intrusion into a protected area by placing sensors near the border of the monitored region. There are two aspects of that problem: the placement problem asks to determine a placement of the sensors that offers optimal or near optimal protection for given resources or costs, and the assessment problem asks, given a placement of sensors, to evaluate how well they protect the area.

Instead of placing sensors on the boundary line, most authors consider instead placement in a belt area, a narrow region between two parallel lines containing the boundary, which we refer to as the outside and the inside of the belt, respectively, where sensors should be placed. If the boundary of the belt region is connected the belt is called open, and otherwise it is called closed (Figure 1.1). We will refer to the short lines in an open belt region connecting the outside to the inside boundary as the left and the right boundary, respectively. A belt with inside and outside boundaries \( l_1 \) and \( l_2 \), respectively, has width \( w \), if for each point \( p_1 \) in \( l_1 \) and each point \( p_2 \) in \( l_2 \) \( \text{dist}(p_1, l_2) = \text{dist}(p_2, l_1) = w \). Here \( \text{dist}(p_i, l_j) \) is defined as the minimum distance between \( p_i \) and any point in \( l_j \).

Since any coverage of the whole area also covers the belt and the belt region is typically much smaller, it is clear that perimeter coverage is often much more cost-effective than the full-area coverage.

Kumar et al. \([55, 56]\) were one of the first to study the perimeter coverage problem in detail. They define two versions of the problem. The weak \( k \)-barrier coverage version considers only breaching paths with lengths equal to the belt width (called orthogonal paths). The rationale behind that restriction to the paths that we want to cover...
Figure 1.2: Placing the sensors on two separating paths results in a strong 2-barrier coverage of the region.

is that an intruder without a prior knowledge of the location of the sensors will likely choose an orthogonal path, since such a path is shortest and hence it minimizes the detection expectation. The strong k-barrier coverage version considers all paths crossing the complete width of the belt (called crossing paths) as possible breach paths. The regions is weakly k-barrier covered (respectively strongly k-barrier covered) if every orthogonal (respectively every crossing) path crosses the sensing region of at least k sensors. We will call the maximum value of k for which the region is k-covered the strength of the coverage.

Strong k-barrier Coverage

Kumar et al. [55, 56] consider two versions of the strong k-barrier coverage placement problem: a deterministic and a probabilistic one. In the deterministic version, sensors are placed on explicitly determined locations, while in the probabilistic one they are placed randomly according to a given probability distribution.

For the deterministic version, they prove that an optimal placement of the sensors in an open belt region is on a set of k shortest paths called separating paths that separate the outside from the inside portion of the belt so that the sensing regions of the sensors touch or overlap inside the belt (Figure 1.2). In the case where the sensing region of each sensor is a disk of radius r, they also prove that the smallest number of sensors necessary and sufficient to cover an open belt region is $k\lceil s/2r \rceil$, where s is the length of a shortest separating path.

For the probabilistic version of the placement problem, Liu et al. [60] show that whether a random placement of sensors in a rectangular belt yields a k-barrier coverage depends on the ratio between the length h and the width $w = w(h)$ of the belt. Specifically, if the sensors are distributed according to a Poisson point process with density $\lambda$, then if $w(h) = \Omega(\log h)$, the region is k-barrier covered with high probability if and only if the density $\lambda$ of the sensors is above certain threshold. If, on the other hand, $w(h) = o(\log h)$, the region does not have a barrier coverage with high probability for any $\lambda$. With high probability (w.h.p.) means that the probability tends to 1 as $h$ tends to
Another interesting question is, given a belt and the positions of a set of sensors placed in it, to determine whether the sensors provide a barrier coverage and to find the strength of such a coverage. Kumar et al. [55, 56] answer that question for open belt regions by reducing the above problem to the problem of finding a set of node-disjoint paths in a graph. They define a coverage graph $G$ whose nodes are the sensors of the network and whose edges connect all pairs of nodes whose corresponding sensors have overlapping sensing regions. They define also two additional nodes $u$ and $v$ and edges between $u$ (respectively $v$) and all nodes whose corresponding sensing regions intersect the left (respectively right) boundary of the belt. Using Menger’s Theorem [96, page 167], they prove that $k$-barrier coverage by the given sensors of the belt is equivalent to the existence of $k$ vertex-disjoint paths between $u$ and $v$ in $G$. Moreover, computing the maximum number of $k$ vertex-disjoint paths between $u$ and $v$ in $G$ can be done in time $O(k^2 n + m)$, where $n$ and $m$ are the number of the nodes and edges of $G$. However, the same proof cannot be used for the closed-belt case since Menger’s Theorem is not applicable to that case. The assessment problem for strong $k$-barrier coverage for closed belt regions is currently still an open problem.

**Weak $k$-barrier Coverage**

Weak barrier coverage allows only crossing paths that are perpendicular to the belt boundary. In [56], Kumar et al. consider sensors that are Poisson distributed with density $np$ and ask the question which values of $np$ produce a weak barrier coverage with high probability. We can think of the parameter $n$ as corresponding to the total number of the sensors and $p$ as the probability of each sensor being awake at any given time. Kumar et al. define function

$$c(s) = 2npr/(s \log(np))$$

and show that, for a belt of width $1/s$ and for any $\varepsilon \in (0, 1)$, if

$$c(s) \geq 1 + \frac{(\log \log(np))^{1-\varepsilon} + (k - 1) \log \log(np)}{\log(np)}$$

(1.1)

for sufficiently large $s$, then all orthogonal lines crossing the belt are $k$-covered w.h.p. as $s \to \infty$. On the other hand, if

$$c(s) \leq 1 - \frac{(\log \log(np))^{1-\varepsilon} + \log \log(np)}{\log(np)}$$

(1.2)

for sufficiently large $s$, then there exists a non-1-covered orthogonal crossing line in the belt w.h.p. as $s \to \infty$. Condition (1.1) is a sufficient condition for achieving $k$-barrier weak coverage and condition (1.2) provides a necessary condition (if the inequality is reversed) for 1-barrier weak coverage. Evidently, there is a gap between the two bounds and finding an optimal $k$-barrier weak coverage condition is an interesting open question.
As noted in [56], the right hand sides of (1.1) and (1.2) tend to 1 as \( s \to \infty \). Hence, asymptotically the critical value for \( c(s) = \frac{2npr}{s \log(np)} \) is 1, meaning that there should be at least \( \log(np) \) sensors deployed in the \( r \)-neighborhood of each orthogonal crossing line in order to produce a weak barrier coverage of the region.

In a different approach to the problem, Li et al. [58] find a lower bound on the probability for a weak \( k \)-barrier coverage, given the size of the region and the number and the distribution of the sensors. Specifically, they show that if the belt region is a rectangle with dimensions \( s \times 1/s \), \( r \) is the sensing radius, the sensors are distributed according to a Poisson point process with density \( np \), and \( B_k \) denotes \( k \)-barrier coverage, then

\[
\Pr(B_k) \geq \left(1 - \sum_{j=0}^{k-1} \frac{(2nr/s)^j}{j!} e^{-2nr/s}\right)^n \cdot \left(1 - \sum_{j=0}^{k-1} \frac{(nr/s)^j}{j!} e^{-nr/s}\right)^2.
\]

Given the placement of the sensors, a natural question to ask is whether those sensors provide a weak \( k \)-barrier coverage. Answering that question is easier in the weak barrier coverage case than the similar question for strong barrier coverage. The reason is that, for weak coverage, the vertical positions of the sensors do not matter as only vertical paths are considered. Hence, the problem can be reduced to a one-dimensional case: just consider the projections of the sensor positions onto the line segment \( S \) defining the internal (or external) belt boundary and determine whether those projections \( k \)-cover that segment. Li et al. [58] present a simple algorithm that considers the set \( Q \) of the endpoints of all sensing intervals on \( S \), i.e., for each point \( x \) on \( S \) corresponding to a sensor projection, we add points \( x - r \) and \( x + r \) to \( Q \). Then \( S \) is swept from left to right keeping track on how many sensors cover each point. The resulting algorithm has time complexity of \( O(N \log N) \), where \( N \) is the number of the sensors.

### Other Perimeter Coverage Results

Kumar et al. establish in [56] that it is not possible to determine locally whether a region is strongly \( k \)-barrier covered or not. This is in contrast to the full area coverage case, where an "yes" answer is not possible, but a "no" answer is, i.e., it is possible in the full coverage case to determine that a region is not \( k \)-covered. In order to deal with the problem of local barrier coverage, Chen et al. [20] introduce the notion of \textit{L-local barrier coverage}. Informally, having \( L \)-local barrier coverage requires that any path contained in a box of length at most \( L \) to be covered (or \( k \)-covered). Hence, \( L \)-local barrier coverage is a generalization of weak coverage for \( L \) equal to zero and to strong barrier coverage for \( L \) equal to the belt length. If \( L \) is sufficiently small, it is possible to locally determine if the region is not \( L \)-locally \( k \)-barrier covered, as proved in [20].

Chen et al. [21] use the idea of \( L \)-local barrier coverage in order to quantify the quality of \( k \)-barrier coverage. Previously, the quality measure has been binary, 1 if there is \( k \)-barrier coverage, and 0 if there isn’t. Chen et al.
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define the quality of $k$-barrier coverage as the maximum value of $L$ for which the belt is $L$-local $k$-barrier covered. If there is no such $L$ then they define the quality as $-1$. They design an algorithm that computes the quality given the sensor positions and a value for $k$. Their algorithm also identifies weak regions that need extra sensors. The property of being able to quantify the quality of barrier coverage is analyzed from another perspective and in much more detail in the next subsections.

1.4.2 Maximum Breach Path

The maximum breach path tries to determine the least covered (the most vulnerable) path between a pair of points. In this context, a measure of how well a path $p$ is covered is the minimum distance between any point of $p$ to any of the sensors. The key conceptual difficulty is that there are continuously many possible paths for the intruder. Nevertheless, this is one of the first problems of coverage in sensor networks that has not only been addressed, but actually solved optimally.

The key idea behind the solution is remarkably simple. The crucial step is to translate this computational geometry and continuous problem into an instance of graph theoretical problem. It is easily accomplished using the notion of a Voronoi diagram. A Voronoi diagram is a tessellation of the space using piecewise linear connected components. If we have two sensors, A and B, the line of separation between them is orthogonal to the line that connects them and passes through the middle of the distance between these two sensors. It is easy to see that during calculation of dynamic coverage, it is sufficient to consider only Voronoi diagram edges and more specifically their weight, which is equal to the distance of the closest point on the Voronoi diagram edge to either one of two sensors that define it. The justification for this observation is that if the intruder does not use for his traversal only Voronoi diagram edges, it will become closer to at least one of the sensors that are used to define the pertinent Voronoi diagram edge.

Now, in order to find if there is a breach in the system of deployed sensors of length $l$, all that is required is to check if there is a path in the graph that is defined on top of the Voronoi diagram, where at least one edge is not larger than a specified value. There are many ways to accomplish this task. Conceptually probably the simplest is one where we iteratively add larger and larger edges until there is a path from the starting point to the ending point. There are several important observations about this approach. One is that one can easily consider the case where different sensors have different sensitivity ranges, or even one can superimpose a grid over the area and define for each field in the grid the level of sensitivity over a single or multiple sensors. All these problems can be easily solved using dynamic programming. The much more in-depth technical presentation of these algorithms can be found in [65][62].
1.4.3 Minimum Exposure Path

As we already said several times, one of the key degrees of freedom in defining the coverage problem is related to the way in which we define the sensitivity with respect to a single or multiple sensors. The exposure is a generalization of dynamic coverage in the sense that it is asked whether it is possible to find a path through a particular field covered with sensors in such a way that the total integral of exposure over time to sensing by all relevant sensors is below the user specified value.

There are two conceptually similar but highly different ways, in terms of implementation, to address this problem. The first one uses rasterization of the pertinent field into a particular grid or some other structure where in each field all points are sufficiently close to each other. This is easy to accomplish by decreasing the size of individual fields. For each small area, we can calculate the amount of exposure for any given period of time. Now, under the natural assumption of constant speed, we can easily use dynamic programming to find the path of minimal exposure from a starting point \( s \) to a destination point \( d \). This task can be easily accomplished in polynomial time that depends on additional constraints that may be imposed on the definition of exposure. This solution was presented by Meguerdichian, who subsequently changed his last name to Megerian, in [68][63].

Another very interesting approach uses variational calculus to solve the exposure problem in a way that guarantees the correct solution (by Veltri et al. [91]). The key idea is to solve a small number of simplified problems such as one where very few sensors are used and to concatenate these locally optimal solutions into one that is globally optimal.

An approximation algorithm for the exposure problem with provable accuracy and polynomial running time was designed by Djidjev [29]. In this algorithm, the points are not placed on a grid covering the region (rasterization), as in the previous algorithms, but only on the edges of a Voronoi diagram for the set of the sensors. This, in effect, replaces a 2-D mesh by an 1-D mesh, significantly reducing the computational complexity of the algorithm. For any given \( \varepsilon > 0 \), the algorithm from [29] can find a path with exposure no more than \( 1 + \varepsilon \) times larger than the optimal. Hence, by reducing the value of \( \varepsilon \), one can get paths with exposures arbitrarily close to the optimal. The running time of the algorithm is proportional to \( n\varepsilon^{-2} \log n \), assuming that the Voronoi diagram does not have angles very close to zero.

1.5 Coverage by Mobile Sensors

In the mobile version of the coverage problem, the goal is to cover a region of interest with mobile sensors so that the trajectories of the sensors go through points or areas of interest at predetermined time intervals, form barriers,
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or relocate themselves to better static locations.

1.5.1 Sweep Coverage

Li et al. [59] consider the following problem they call the sweep coverage problem: There are $n$ mobile sensors located in a region that contains $m$ points of interest (POIs) that need to be monitored. The sensors move at the same constant speed $v$ and a POI is considered covered at a given time if a mobile sensor is at that location at that time. Given a coverage scheme (schedule), a POI is considered $t$-sweep covered if it is covered at least once in every time interval of length $t$. The goal is to design a coverage scheme so that each of the $m$ POIs is $t$-sweep covered. A more general version of the problem specifies individual sweep periods $t_i$ for sensor $i$.

It is proved in [59] that the $t$-sweep coverage problem is NP-hard by reducing the traveling salesman problem to it. An even stronger result is proved in the same paper [59], that the $t$-sweep coverage problem cannot be approximated within a factor of less than 2 unless P=NP. It is also shown that for any $\varepsilon > 0$ there exists a polynomial time algorithm for solving the $t$-sweep coverage problem within a factor of $2+\varepsilon$. That algorithm uses the $1+\varepsilon$-approximation algorithm for the traveling salesman problem [8] to construct a short route $r$ visiting all POIs exactly once. Then $r$ is divided into $n$ equal parts, one for each of the $n$ sensors. Finally, each sensor is assigned to monitor one of the parts $p_i$ of $p$ by moving forward and backwards along $p_i$. This algorithm is generalized in [59] for the case of different sweep periods for the POIs, resulting in an algorithm with an approximation ratio of 3.

1.5.2 Optimal Repositioning of Mobile Sensors

The problem of repositioning the sensors so that they provide a better barrier coverage while minimizing the distance they have to travel or the energy they need to consume, is studied in [15, 11, 88]. Bhattacharya et al. [15] assume that $n$ sensors are initially located in the interior of a planar region and study the problem of how to move the sensors to the boundary of the region so that the distance along the boundary between two consecutive sensor positions is the same. Hence, after repositioning the sensor positions will form a regular $n$-gon that is called destination polygon.

We will call the new position of each sensor the destination of that sensor. There are two versions of the problem:

- the min-max problem, aiming to minimize the maximum distance traveled by any sensor.
- the min-sum problem, where the objective is to minimize the sum of the distances traveled by all sensor.

For both problems they consider two type of regions: a unit disc and simple polygon. We will discuss first the algorithms for the min-max problem and then for the min-sum problem.
CHAPTER 1. DYNAMIC COVERAGE PROBLEMS IN SENSOR NETWORKS

The Min-Max Problem

For the min-max problem on a disk region, Bhattacharya et al. call a positive real number \( \lambda \) feasible, if all the sensors can move to the new positions on the boundary of the disk that form a regular \( n \)-gon \( P \) and the maximum distance between an old and a new position of any sensor does not exceed \( \lambda \). Such polygon \( P \) is called \( \lambda \)-feasible. Hence, the min-max problem is equivalent to the problem of finding the minimum feasible number \( \lambda_{\text{min}} \) and a \( \lambda_{\text{min}} \)-feasible polygon. If we can construct an algorithm to check feasibility of any number in time \( T(n) \) and we know an interval containing \( \lambda_{\text{min}} \), then we can do a binary search on that interval, at each step reducing twice the size of the interval containing \( \lambda_{\text{min}} \). Clearly, the interval \([0, 2]\) contains \( \lambda_{\text{min}} \) since the distance between any two points in the disk cannot exceed its diameter. Hence the running time of the resulting algorithm will be \( T(n) \log(1/\varepsilon) \), where \( \varepsilon > 0 \) is the required accuracy. Using a more complex binary search algorithm that uses a finite set of candidate new-position points, Bhattacharya et al. show that the exact value of \( \lambda_{\text{min}} \) can be found in time \( O(T(n) \log n) \).

For testing feasibility of a number \( \lambda > 0 \) for \( n \) sensors on positions \( A_1, \ldots, A_n \) inside a circle \( C \), Bhattacharya et al. construct for each \( i \) a circle of radius \( \lambda \) and center \( A_i \) and consider the two intersection points of that circle with \( C \). The resulting set \( Q \) contains \( 2n \) points. It is shown that, if \( \lambda \) is feasible, then there is a \( \lambda \)-feasible \( n \)-gon one of whose vertices is in \( Q \). Hence, assuming \( \lambda \) is feasible, one can find a \( \lambda \)-feasible \( n \)-gon by checking each of the regular \( n \)-gons that contain a node in \( Q \), whose number is at most \( |Q| = 2n \). Then the problem is reduced to checking whether the vertices \( B_1, \ldots, B_n \) of each of those \( 2n \) polygons can be mapped to distinct points among \( A_1, \ldots, A_n \) so that for each \( i \) the distance between \( B_i \) and \( A_i \) is at most \( \lambda \). The latter mapping problem can be solved using an algorithm due to [40] for finding a perfect matching in a bipartite graph with time complexity of \( O(n^{2.5}) \). The total complexity of the resulting feasibility-checking algorithm is \( O(n^{3.5}) \) and the resulting complexity of the min-max algorithm is \( O(n^{3.5} \log n) \).

Tan and Wu [88] improve the complexity of the min-max algorithm for a disk from [15] by using a better characterization of \( \lambda_{\text{min}} \)-feasible polygons. Specifically, they show that, if \( B_1, \ldots, B_n \) are the vertices of an \( \lambda_{\text{min}} \)-feasible \( n \)-gon such that \( |A_iB_i| \leq \lambda \) for all \( i \), then either

(i) for some \( i \) such that \( |A_iB_i| = \lambda \) the line joining \( A_i \) and \( B_i \) contains the center of \( C \), or

(ii) for some \( i \neq j \) \( |A_iB_i| = |A_jB_j| = \lambda \).

Using this fact, one can construct a set of all \( n \) distances of type (i) and all, say \( m \), distances of type (ii). Doing a binary search on that set will yield in \( O(\log(n + m)) \) feasibility tests the value of \( \lambda_{\text{min}} \) and the corresponding \( n \)-gon. Unfortunately, in the worst case \( m \) can be of order \( n^3 \), which implies that the worst-case complexity of the resulting min-max algorithm will be \( O(n^3) \). By employing a more elaborate search procedure, Tan and Wu [88] show that the
complexity of their algorithm can be reduced to $O(n^{2.5} \log n)$.

For the min-max problem in a simple-polygonal region $P$, Bhattacharya et al. [15] show that their algorithm for disk regions can be adapted, resulting in an algorithm of time complexity $O(ln^{3/5} \log n)$, where $l$ is the number of the vertices of $P$. The additional factor of $l$ comes from the fact that the intersection of a circle centered at a sensor and the boundary of $P$ can consist of up to $l$ points, unlike the disk-region problem when it consists of at most two points.

The Min-Sum Problem

Unlike the min-max problem, for the min-sum version no exact polynomial algorithm is known yet, and neither is known whether the problem is NP-hard or not. The reason is that, for the min-sum problem, no characterization of the $\lambda_{opt}$-polygon is known that would allow for reducing the search space from continuous to discrete, as it is in the min-max version. Instead, it is shown in [15] that the destination of at least one sensor in any optimal $n$-gon belongs to a specified short segment along the circle $C$. Based on that fact, the corresponding segment for each of the sensors $A_i$ is discretized by adding $O(1/\varepsilon)$ equally-spaced points, each of which is then considered as a candidate of a destination for $A_i$. Then, for each sensor and candidate, a minimum cost weighted matching problem is solved for a weighted graph whose nodes are the sensors $A_i$ and the vertices of the currently considered $n$-gon candidate, whose edges join each sensor and each polygon vertex, and whose edge weights equal to the Euclidean distances. The matching problem can be solved in $O(n^3)$ time using the algorithm from [54]. The complexity of the resulting min-sum approximation algorithm is $O(n^4/\varepsilon)$ and the approximation ratio is $1 + \varepsilon$. A similar approximation algorithm can be constructed for the min-sum problem for a simple-polygon region with time complexity $O(ln^{5/\varepsilon})$ and approximation ratio $1 + \varepsilon$, where $l$ is the number of the vertices of the polygon.

Tan and Wu [88] consider a special version of the min-sum problem, where the sensors are initially positioned on $C$. For that version, they show that an exact polynomial-time algorithm for the min-sum problem does exist, and its complexity is $O(n^4)$. Their algorithm is based on a characterization of the optimal solution that limits the search space for a destination polygons to a discrete set. Specifically, they show that in any optimal solution, there exists at least one sensor $A_i$ whose destination is $A_i$, i.e., that does not change its position.
There exists a large literature on simultaneous maintenance of coverage and connectivity. As we already stated, originally the sensor research community was targeting wireless sensor nodes with ultra low power radios and multi-hop communication. This type of wireless links has been widely studied experimentally and using statistical generalization in terms of their transmission properties as well as quality of link vs. energy consumption properties. Unfortunately, many of these studies are to a serious extent unrealistic because it was not recognized that the radio consumption model is such that listening is often as expensive as receiving or transmission.

It has been recognized that there exist high positive and negative correlation in link qualities, both spatially and in the temporal domain. Some of the key references in these domains are [17][18][102][73]. With the change of architecture of wireless sensor networks from ultra-low power multi-hop communication to communication using wireless phone infrastructure, many fundamental assumptions about the role of communication in coverage tasks are drastically altered. For example, in this new architecture, it is very rarely the case that communication is the bottleneck and much higher emphasis is on use of sensors in the best possible way.

Multi-objective coverage is one where at least two objectives or two constraints have to be addressed during node deployment or operation. The initial literature focused on maintaining sensing coverage and connectivity in large sensor networks [102][92][98]. In this situation, the key assumption is related to the ratio of communication range and sensing domain. In particular, a very interesting situation is when these two entities are of relatively similar cardinality. These problems may not be an issue in mobile phone-based sensor networks, but multi-objective is bound to emerge as one of the most important definitions of coverage.

For example, in many security applications, it is essential that we observe the enemy while the enemy is not able to observe us. Also, it is easy to imagine that in many types of coverage one has to ensure that fundamentally different types of sensors are able to collect information (e.g. audio and visual sensors). These sensors may have not just different sensitivity ranges, but also they may or may not be directed with various angles of coverage. The key goal here is to make adequate and simple to use sensing models as well as to find which type of sensor fusion is most relevant in a particular application.
1.6. **OTHER COVERAGE ISSUES**

1.6.3 **Localized Algorithms Coverage**

Localized algorithms are those that are executed on a small number of sensor nodes that are close to each other in terms of quality of their communication links and/or in terms of sensed events. Localized algorithms are important for several reasons. They are intrinsically low energy and fault tolerant. Localized protocols usually induce much lower latency and preserve bandwidth. Finally, in very large networks they are the only practical alternative.

A comprehensive but certainly somewhat outdated survey on localized algorithms has been published in 2004 [32]. Several authors have been able to develop localized coverage algorithms that are optimal or competitive with corresponding centralized algorithms [43]. Interestingly, even algorithmic paradigms have been developed for creation of localized algorithms [70] [86]. The key idea is to use as a starting point any regular centralized algorithm. The results of the pertinent centralized algorithm provides statistical knowledge about which information should be used in which way in the corresponding algorithm. The final step is to use statistical validation techniques for the evaluation of the localized algorithm. It is important to emphasize that different instances of the coverage problem should be used for the learning and testing phases. Of course, for best performance the whole procedure is reiterated in a loop until the specified level of discrepancy between the centralized and the localized algorithms is found.

1.6.4 **Lifetime and Energy Efficient Coverage**

It has been realized early that energy is one of the most severe constraints in wireless sensor networks. For example, Srivastava et al. [84] recognized that in the Smart Kindergarten project, batteries have to be changed at least once per day and that in order to instrument a sufficient number of subjects (kids) for the duration of the project one would spend millions of dollars only on batteries. Therefore, a number of approaches have been developed to maintain one or more formulations of coverage while minimizing energy consumption.

The main idea is to schedule different subsets of sensors to be active in any given point of time in such a way that each group of sensors in each subset is sufficient to guarantee the coverage objective while the number of subsets is maximized. It is related to the well known k-coverage problem in graph theoretic literature, which is NP-complete. Interestingly, in many applications with a relatively small number of nodes (up to several hundred), one can obtain the optimal solution using integer linear programming (ILP) [69][47]. It is interesting to note that also there is a very large number of survey papers that are completely dedicated to energy efficient strategies in wireless ad hoc and sensor networks [61][6][38][27]. In particular, a large number of heuristics have been developed to maintain network coverage using low duty cycle sensors [30][26][44][71].
1.6.5 Fault Tolerance and Errors

There are two major sources of sensing data errors that have been widely considered. The first is that sensor measurement may provide incorrect values. The second source of error is less dangerous for the accuracy and the correctness of the evaluation of coverage and is related to missing data.

There are three main types of errors that have high impact on coverage algorithms and applications. The first is related to readings of detection sensors. The second is associated with location errors [82]; these are particularly important for mobile sensors. These two types of errors may be both in terms of missing data or incorrect measurement. The final type is related to communication using lossy links and is of the missing data nature. Note that once real-time issues are considered a new type of error related to late-arriving data emerges. It is important to note that in more complex scenarios new types of errors may play important roles. For example, if nodes use a sleep mode for energy conversation, errors in time synchronization may be of essential importance [43] [37].

There is a tremendous amount of literature in sensor measurement data. By far the most popular approach is to assume independent errors that follow a Gaussian distribution. A number of interesting and theoretically important results are established under these assumptions. Unfortunately, the actual properties of real errors in data essentially always have highly nonparametric distributions and rather high spatial temporal correlations. It has been demonstrated that assuming a Gaussian error distribution may result in location errors that are several orders of magnitude higher than if non-parametric models that consider correlations are used for location discovery [31]. Conceptually, the most difficult problem with error modeling is that in many applications corresponding signals are non-stationary.

There have been several efforts to accurately and realistically model errors of individual sensors [51] [34] and errors and communication links of a system of sensor and wireless nodes [49] [48].

There is a complex interplay between error properties and optimization techniques used for calculating or optimizing coverage. In some situations there are readily available provably optimal solutions. For example, if the coverage problem can be optimally solved using error free data and if an error model is Gaussian, convex programming addresses the same problem in presence of error optimally. Unfortunately, this situation rarely has practical benefits [52]. The impact of realistic error models is discussed in detail using several sensor networks applications [82].

In many scenarios, sensor networks for coverage are deployed in hostile environments where repair is either difficult or essentially impossible. In some scenarios the environment is harsh and may have highly negative impact on the reliability of the sensors. In essentially all scenarios in which sensor networks are used to establish coverage are not attended by humans. Therefore, it has been recognized that there is a need for fault-tolerant coverage.
1.6. OTHER COVERAGE ISSUES

The most natural and the most popular way to ensure fault tolerance is through the use of redundancy [24]. In particular k-cover algorithms simultaneously provide both energy efficiency and fault tolerance [47] [1] [53]. Interestingly, a much more efficient approach can be derived when tolerance is treated within the framework of sensor fusion [50] [75].

1.6.6 Dealing with Uncertainty

Coverage under uncertainty in terms of locations of nodes has been widely studied [36] [25] [72] [87] [13]. Many of these effort use mathematically sophisticated concepts (e.g. homology) or verification techniques. We expect that soon other uncertainty degrees of freedom will be addressed. For example, probabilistic or, even better, statistical guarantees in the presence of uncertainty, about actual actions of other side (attacker, intruder) will be practically essential in many applications. One such potential framework to address these issues is the use of game theory.

1.6.7 Visual Coverage

One of the key predecessors of coverage is tasks in computational geometry such as art gallery observation by a limited number of agents. It is assumed that an agent can detect object at an arbitrary distance unless the object is hidden by a wall. The problem asks to deploy the smallest number of art gallery employees in such a way that there does not exist any area of the gallery that is not observed by at least one employee. In many security applications, as well as in entertainment applications, visual information is of the ultimate importance. Therefore, in the last five years, visual coverage emerged as one of the most popular topics. There are several surveys that treat this important problem in great detail [19][85].

In addition, there is a survey by Georgia Institute of Technology researchers that covers multimedia wireless sensor networks that is concerned with both data acquisition and data transmission [4]. The main conceptual difference between the standard definition of coverage and visual coverage is that cameras are subject to directional field of view and that they have rather large but nevertheless limited sensing range. A very important assumption is about the ability to rotate camera as required by tracking or coverage needs. As a consequence of these intricate sensor models, very intriguing and challenging optimization problems arise. It is surprising that a significant number of them can be solved in provably optimal ways using polynomial time complexity algorithms [83][101][2][39][3][14][78][97][42].
1.6.8 Security

Security is one of the most important parameters in many mobile and unattended systems. In addition to papers published at many wireless, sensor, and security conferences even dedicated conferences for wireless security attract large numbers of submissions. Essentially all security issues related to system security directly apply to coverage in sensor networks. It is not surprising that security of coverage results is of high importance. After all, coverage problems are very often directly related to security applications. There is a large number of surveys on security in sensor networks [76] [28] [22].

In addition there are at least two security challenges that are specific for sensor networks and coverage. The first is the issue of physical attacks. Usually security attacks require sophisticated mathematical, software, or system techniques. Therefore, it can be undertaken only by experts in these fields and significant efforts. However, reading of sensors can be easily altered using corresponding source of excitation. For example, one can easily increase the temperature of a sensor or alter speed of acoustic signal propagation using dust. These type of attacks can easily result in greatly incorrect distance, location, or other measurements [23]. The development of techniques that mitigate or even better eliminate such impacts are of high importance.

The second issue is that in addition to the correct measurements one need to ensure that each of the measurements is collected by a sensor deployed by a trusted party at exact location where the sensor is initially deployed at exactly the time when it claimed that data is collected. Recently several such solutions that utilize notion of public physical unclonable function (PPUF) [12] have been developed [77] [67]. The key idea is to combine challenges and/or GPS as inputs to one or more PPUFs. The characteristics of PPUF are such that any attempt to separate or replace them destroys their characteristics and therefore security properties.

1.6.9 Emerging Directions

Initial efforts on coverage in sensor networks have formulated and solved several canonical problems. There are exponentially many new formulations that consider more and more issues or accept more complex and detailed sensing models as well as object movement. While many of them are interesting and technically challenging, there is still an ongoing search for killer applications of large and profound practical importance. Also, several basic problems such as static coverage with respect to static objects are still not completely answered.

There are too many new applications for any survey or even book to cover. Due to space limitations we just very briefly go through two new applications: mobile wireless health [46] and energy harvesting [66] [95] [93] [94]. In addition, we will briefly discuss the related and intriguing emerging topic of local sensing using global sensors [80].
1.7. CONCLUSION

We illustrate issues in coverage problems using a very small crosscut of wireless health research, specifically, medical shoes. Medical shoes are instrumented with a large number of sensors that record pressure below each small area of a soul and several other types of sensors (e.g. accelerators) [75] [74]. These remarkably simple systems are capable of facilitating remarkable broad sets of diagnoses and of supporting a wide spectrum of medical treatments. However, these systems are rather expensive and have high energy budgets. It has been recently demonstrated that both can be reduced by more than an order of magnitude by using the notion of semantic coverage. Semantic coverage does not detect all events but only ones that are relevant for medical purposes [95] [93] [94]. Therefore, in a sense it provides a natural bridge between coverage and general sensor fusion that is driven by applications.

We use the term, global sensors, for large sensors that simultaneously sense multiple locations. Probably the best illustration is one where a single sensor is used to sense pressure from any of $k$ keyboards. At first this approach to coverage of events (one where any single key of a keyboard senses pressure) may sound counter-intuitive. However, it results in great energy sensing. For example, if we just want to detect if any key activated when we have standard one key - one sensor scheme we need as many sensor reading as there are keys. However, if each sensor covers $k$ keys, this requirement is reduced by a factor of $k$ times. Judicious placement of such global sensors can ensure complete coverage of keys while reducing energy requirements by more than an order of magnitude [80] [79]. Although the first algorithms have been proposed and they are very effective, we still know rather little of advantages and limitations of the use of global sensors for local sensing.

1.7 Conclusion

We have surveyed the history, state-of-the-art, and trends of coverage in sensor networks. Since comprehensive and complete coverage is out of the question due to the tremendous amount of research, we placed emphasize on the most important conceptual and practical issues. Even then, only a small slice of research results are covered. Nevertheless, we hope that this paper will help practitioners and facilitate starting research in obtaining a better global picture of coverage in sensor networks.

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