

ENERGY SPECTRUM OF SU(2) YANG-MILLS FIELDS WITH SPACE-LIKE SYMMETRIC TWIST

D. DANIEL ^a, A. GONZÁLEZ-ARROYO ^b, C.P. KORTHALS ALTES ^c and B. SÖDERBERG ^d

^a *Theory Group Physics Department, University of Edinburgh, King's Building, Edinburgh EH9 3JZ, UK*

^b *Departamento de Física Teórica C-XI, Universidad Autónoma de Madrid, E-28049 Madrid, Spain*

^c *Centre de Physique, CNRS-Luminy, Case 907, F-13288 Marseille Cedex 9, France*

^d *Institute for Theoretical Physics, University of Lund, Sölvegatan 14A, S-223 62 Lund, Sweden*

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We compute the energy levels of an SU(2) Yang-Mills field in perturbation theory to order g^2 , for a box of finite size and symmetric twist $\mathbf{m} = (1, 1, 1)$. A cubic-invariant spectrum results, with almost degenerate E^{++} and T_2^{++} levels. Various suggestions for further MC measurements are made.

1. Introduction

One of the more fundamental problems of strong interaction physics concerns the existence and properties of glueball particles. Since the experimental situation is by no means clear, this offers a unique opportunity for QCD to yield predictions rather than postdictions. Indeed, even qualitative or rough quantitative results would be a serious evidence in favour of QCD and of our understanding of non-perturbative phenomena. Lattice QCD can show in this domain all its strength and its present-day weaknesses (if any). In principle the problem one has to face is to be able to control the approximations made: finite lattice spacing, finite volume, effect of dynamical quarks, lack of statistics. For this purpose, analytical results which help us to understand the effect of these approximations can be very useful. It is in this context that the present paper has to be inscribed.

An essential limitation of the numerical results obtained from lattice Yang-Mills theory is the finiteness of the volume. This limitation may turn out to be also a physically interesting fact because it enables us to probe the dynamics of Yang-Mills fields in domains not accessible by experiment, but which may turn out to be useful in the context of Kaluza-Klein theories. All in all it would allow us to understand better the non-perturbative dynamics of Yang-Mills

fields. The presence of a finite space with the topology of the torus also allows to introduce a wide class of periodic boundary conditions, called twisted boundary conditions [1]. They can be interpreted as the possibility of confining within space a certain sort of magnetic flux \mathbf{m} which is abelian and conserved. For large enough volumes of space the effect of the boundary conditions should vanish, so that the boundary conditions can be used as an economical method to check volume independence. For small volumes the dynamics can be very much dependent on the value of the magnetic flux: results on the large N limit [2] and perturbative computations on the lattice [3] show this to be the case. Indeed, they even show that the presence of magnetic flux may give rise to a smoother dependence on the volume [4]. The results of this paper also go in this direction and we look forward to Monte Carlo results on the lattice using different twists [5].

In this paper, we will compute the energy spectrum for SU(2) Yang-Mills theory in a box with twist $\mathbf{m} = (1, 1, 1)$. Our computation is done in perturbation theory, which is a good approximation for small volumes, as follows from asymptotic freedom. It was Lüscher [6] who initiated these type of computations with his study of the case $\mathbf{m} = (0, 0, 0)$. The numerical results were presented in ref. [7]. Later on, Koller and van Baal [8] pushed the domain of ana-

lytical results up to intermediate volumes. The main features of the perturbative computations with twisted boundary conditions were given in ref. [9], where the reader is referred to for a more extensive discussion of the formalism. Here, we concentrate on the particular case $\mathbf{m} = (1, 1, 1)$, which is particularly interesting since it does not break cubic symmetry. We put particular emphasis on identifying the quantum numbers of the states. Our goal is to show clearly the different results obtained, compared with those of purely periodic boundary conditions ($\mathbf{m} = (0, 0, 0)$). Ultimately we hope our results will serve to guide and encourage numerical results with these boundary conditions.

To conclude we give a few guidelines for the reader. First, we comment on the fact that, given the nature of this paper, we avoided giving reference to the work on extracting glueball masses from lattice QCD. We refer the readers to the references contained in the review talks given by Berg and Kronfeld and van Baal at the lattice conferences of Seillac and Fermilab [10]. Second, we refer the non-technical reader to sections 4 and 5 of this paper, where the results are summarised and the conclusions given. The essential features of the calculation are reported in sections 2 and 3.

2. General framework

We consider SU(2) Yang–Mills on a spatial hypertorus (box with periodic boundary conditions) of size L^3 . If we formulate the problem in the $A_0 = 0$ gauge, the dynamic variables are the vector potentials $A_i^a(\mathbf{x})$, where $i = 1, 2, 3$ labels the space direction and $a = 1, 2, 3$ is the colour index in the adjoint representation. The canonically conjugate variables are the electric field operators $E_i^a(\mathbf{x})$:

$$[E_i^a(\mathbf{x}), A_j^b(\mathbf{y})] = i^{-1} \delta_{ij} \delta_{ab} \delta^{(3)}(\mathbf{x} - \mathbf{y}) . \tag{2.1}$$

We are interested in the eigenstates of the hamiltonian

$$H = \int d^3x \frac{1}{2} \left(\sum_{a,i} [E_i^a(\mathbf{x})]^2 + \sum_{a,i} [B_i^a(\mathbf{x})]^2 \right) , \tag{2.2}$$

where $B_i^a(\mathbf{x}) = \frac{1}{2} \epsilon_{ijk} (\partial_j A_k^a - \partial_k A_j^a + gf^{abc} A_j^b A_k^c)$ is the magnetic field operator. The whole problem is invariant under gauge transformations depending on the

point of space. The infinitesimal generator of such transformations is $\sum_{b,i} D_i^{ab} E_i^b(\mathbf{x}) \equiv (DE)$. We are interested only in those eigenstates of the hamiltonian which are at the same time eigenstates of zero eigenvalue of (DE) . The latter states are called physical and the restriction to them is a consequence of Gauss' law.

The previous formulae are valid both for infinite space as on a hypertorus. The finiteness and periodicity implies the following boundary conditions for the operator $A_i = \sum_a A_i^a T^a$:

$$\begin{aligned} A_i(\mathbf{x} + L\mathbf{j}_i) &= \Gamma_j A_i(\mathbf{x}) \Gamma_j^\dagger , \\ E_i(\mathbf{x} + L\mathbf{j}_i) &= \Gamma_j E_i(\mathbf{x}) \Gamma_j^\dagger , \end{aligned} \tag{2.3}$$

where T^a are the generators of SU(2) in the spin $\frac{1}{2}$ representation and Γ_j are 2×2 unitary matrices. In addition, the matrices Γ satisfy

$$\Gamma_i \Gamma_j = \exp(\pi i \epsilon^{ijk} m_k) \Gamma_j \Gamma_i , \tag{2.4}$$

where the integer vector \mathbf{m} (defined mod 2) is called topologically conserved magnetic flux [1]. The form of the Γ_j matrices is physically irrelevant, only the value of \mathbf{m} plays a role. When $\mathbf{m} = \mathbf{0}$ one can choose $\Gamma_j = \mathbf{I}$ and one speaks of purely periodic boundary conditions. When $\mathbf{m} \neq \mathbf{0}$ one speaks of twisted boundary conditions [1]. The diagonalization of the hamiltonian depends strongly on the value of the magnetic flux \mathbf{m} . For the purely periodic case and SU(2) see refs. [6,7]. The twisted case has been studied in ref. [9], to which we refer the reader for a more extensive discussion of the formalism. Here we will concentrate on $\mathbf{m} = (1, 1, 1)$ which is a particularly interesting case as we will see.

In addition to gauge transformations generated by DE , there is another internal symmetry group isomorphic to $(Z_2)^3$. The representations of this group are labelled by an integer vector (defined mod 2) called electric flux \mathbf{e} . The group can be considered as being that of singular or non-periodic gauge transformations. The reader is referred to ref. [9] for a more extensive discussion. The main point is that we may consider the diagonalization of the hamiltonian separately in every electric flux sector. We will be interested only in the $\mathbf{e} = \mathbf{0}$ sector, but we will dedicate a few words to the other sectors.

The eight values of electric flux are arranged into four groups of two elements labelled by an index $\alpha = 0$,

1, 2, 3. For each group the possible values of electric flux are i_α and $m - i_\alpha$, where i_1, i_2, i_3 are the unit vectors along each direction and $i_0 = m$. Different results are obtained in perturbation theory for each of the four groups, but separation between the two electric fluxes of each group only occurs by non-perturbative effects.

To compute the energy levels of the hamiltonian we make use of perturbation theory (PT). For small box size L we expect PT to be a good approximation as a consequence of asymptotic freedom. The procedure to follow is explained in ref. [9]. First, we write the vector potentials in momentum space. It is possible to diagonalise the boundary conditions (2.3) by choosing an appropriate basis of the Lie algebra of SU(2). The three generators $T(p_c)$ are labelled by a vector called colour momentum

$$p_c \in \{ \pi L^{-1}(0, 1, -1), \pi L^{-1}(1, 0, -1), \pi L^{-1}(1, -1, 0) \}. \quad (2.5)$$

The usefulness of this basis is that now the boundary conditions and the periodicity imply that the possible values of momentum appearing in the Fourier expansion of the vector potentials are

$$p = p_c + 2\pi L^{-1}n, \quad (2.6)$$

where n is an integer vector. Notice that $p = (0, 0, 0)$ is not appearing in (2.6).

If we write the hamiltonian to lowest order in perturbation theory, we get

$$H_{\text{free}} = \sum_{p_c, n, \sigma} |p| a^\dagger(p_c, p, \sigma) a(p_c, p, \sigma), \quad (2.7)$$

where p is given by (2.6), $\sigma = 1, 2$ distinguishes the two transverse polarizations and a^\dagger (a) the creation (annihilation) operators. This means that we have free gluons characterised by the colour index p_c and polarization index σ and with momentum restricted by the boundary conditions. It turns out that a gluon with colour index p_c carries also electric flux. The three values appearing in p_c correspond to $e = \{i_1, i_2, i_3\}$ or to $\{m - i_1, m - i_2, m - i_3\}$ since the electric fluxes e and $m - e$ are degenerate to all orders of perturbation theory.

From (2.7) we can trivially deduce the spectrum of the hamiltonian to this order. The ground state is the Fock vacuum which as all the other states is du-

plicated according to whether the electric flux is $e = 0$ or $e = m$. From now on we will omit any reference to the two-fold degeneracy of states and only refer to $e = (m - i_\alpha)$. The first excited states are the one-gluon states with minimum momentum. There are altogether 24 such states, half of which are indicated in table 1. The other half is obtained by changing the sign of p and is labelled by the same symbol with a bar on top, $\bar{|\rangle}$. The energy of all those states is $E = \pi L^{-1}\sqrt{2}$. Then, there are other one-gluon states with energy $E = \pi L^{-1}\sqrt{6}$, and so on. The energies and quantum numbers of n -gluon states can be easily obtained from those of the one gluon using the additivity of energies, momenta and electric fluxes (mod 2). In the $e = 0$ sector, the sum of all the colour momenta p_c has to be zero (mod $2\pi L^{-1}n$). If we consider the sector with $p = 0$ and $e = 0$, the first excited states above the Fock vacuum is the set of two-gluon states obtained by combining a state of table 1 with one having the opposite momentum:

$$|A; \sigma\rangle \otimes \overline{|A; \sigma'\rangle} \equiv |A; \sigma, \sigma'\rangle, \quad (2.8)$$

where $A \in \{x, x', y, y', z, z'\}$. There are altogether 24 such states, all degenerate with energy

$$E_0 = 2\sqrt{2}\pi L^{-1}. \quad (2.9)$$

Now we come to the last point which we want to discuss, namely the spacetime symmetries. The infinite volume theory has rotational invariance, but putting the system in a box breaks the rotation group to the cubic group \mathbf{K} . One may question whether the presence of magnetic flux m restricts the invariance to a smaller group. Indeed this is the case for many possible twists such as $m = (1, 0, 0)$ for example. However, our choice $m = (1, 1, 1)$ is interesting because it does not break the cubic group and therefore the comparison with the untwisted $m = 0$ case is most relevant. The reason why the presence of $m = (1, 1, 1)$ does not break the cubic group is the fact that it is defined mod 2. In fact, rotation of $\pi/2$ around the x axis transforms $(1, 1, 1)$ into $(1, -1, 1)$ which is, however, completely equivalent to it. The reader may be worried by the fact that the matrices Γ_j of eq. (2.3) are indeed not invariant under the cubic group. However, this is due to a particular gauge choice (see ref. [9]). As mentioned previously the only physics is in the magnetic flux itself and this can be seen explicitly when writing the hamiltonian in momentum space.

Table 1

One particle states with energy $E_0 = \pi L^{-1} \sqrt{2}$ and colour momentum, electric flux, total momentum and polarization vectors. Only half of the possible momenta is given. The other half is obtained by changing \mathbf{p} into $-\mathbf{p}$. The corresponding polarizations are obtained by $\epsilon_1 \rightarrow \epsilon_1$ and $\epsilon_2 \rightarrow -\epsilon_2$.

State	$\mathbf{p}_c L/\pi$	Electric flux	\mathbf{p}	Polarization ϵ
$ x; 1\rangle$	(0, 1, -1)	(1, 0, 0)	$\pi L^{-1}(0, 1, -1)$	(1, 0, 0)
$ x; 2\rangle$	(0, 1, -1)	(1, 0, 0)	$\pi L^{-1}(0, 1, -1)$	$(1/\sqrt{2})(0, 1, 1)$
$ x'; 1\rangle$	(0, 1, -1)	(1, 0, 0)	$\pi L^{-1}(0, 1, 1)$	(1, 0, 0)
$ x'; 2\rangle$	(0, 1, -1)	(1, 0, 0)	$\pi L^{-1}(0, 1, 1)$	$(1/\sqrt{2})(0, 1, -1)$
$ y; 1\rangle$	(1, 0, -1)	(0, 1, 0)	$\pi L^{-1}(-1, 0, 1)$	(0, 1, 0)
$ y; 2\rangle$	(1, 0, -1)	(0, 1, 0)	$\pi L^{-1}(-1, 0, 1)$	$(1/\sqrt{2})(1, 0, 1)$
$ y'; 1\rangle$	(1, 0, -1)	(0, 1, 0)	$\pi L^{-1}(1, 0, 1)$	(0, 1, 0)
$ y'; 2\rangle$	(1, 0, -1)	(0, 1, 0)	$\pi L^{-1}(1, 0, 1)$	$(1/\sqrt{2})(-1, 0, 1)$
$ z; 1\rangle$	(1, -1, 0)	(0, 0, 1)	$\pi L^{-1}(1, -1, 0)$	(0, 0, 1)
$ z; 2\rangle$	(1, -1, 0)	(0, 0, 1)	$\pi L^{-1}(1, -1, 0)$	$(1/\sqrt{2})(1, 1, 0)$
$ z'; 1\rangle$	(1, -1, 0)	(0, 0, 1)	$\pi L^{-1}(1, 1, 0)$	(0, 0, 1)
$ z'; 2\rangle$	(1, -1, 0)	(0, 0, 1)	$\pi L^{-1}(1, 1, 0)$	$(1/\sqrt{2})(1, -1, 0)$

The colour momenta themselves are not rotationally invariant but the set of all possible momenta is

According to the previous considerations we conclude that it is possible to classify the 24 two-particle states with energy $E_0 = 2\sqrt{2} \pi L^{-1}$ into irreducible representations of the cubic group [11]. These are well known: There are two one-dimensional ones (A_1 and A_2), one two-dimensional (E) and two three-dimensional (T_1 and T_2). The irreducible representations of the rotation group become reducible with respect to the cubic group. Thus, spin zero= A_1 , spin one= T_1 , spin two= $T_2 \oplus E$ and so on. In addition to rotations one can consider parity. Again the presence of twist $\mathbf{m} = (1, 1, 1)$ does not destroy this symmetry. If we split up the 24 states according to these symmetries we find $2(A_1^+ \oplus E^+ \oplus T_2^+) \oplus (T_1^+ + T_2^+) \oplus (A_1^- \oplus E^- \oplus T_2^-)$. The decomposition into irreducible representations is very helpful not only to identify quantum numbers but also to diagonalise the hamiltonian to order g^2 which is done in the next section. Since mixing can only occur within the same representations we only have to diagonalise the 3×3 matrix of T_2^+ states and the 2×2 matrices of A_1^+ and E^+ .

3. Second order perturbation theory

In this section we indicate how the calculation of the splitting ΔE of the energy level (2.9) proceeds to

order g^2 . The computation is done in Coulomb gauge. In this gauge, the hamiltonian reads

$$\begin{aligned}
 H = & H_{\text{free}} + g \int d^3x \text{Tr} \partial_k \mathbf{A}_l [\mathbf{A}_k, \mathbf{A}_l] \\
 & - g^2 \int d^3x \text{Tr} [\mathbf{A}_k, \mathbf{A}_l] [\mathbf{A}_k, \mathbf{A}_l] \\
 & + g^2 \int d^3x \text{Tr} [\mathbf{E}_k, \mathbf{A}_k] \frac{1}{\Delta} [\mathbf{E}_l, \mathbf{A}_l] + O(g^4),
 \end{aligned} \quad (3.1)$$

where H_{free} is given in eq. (2.7). Besides the usual vertices, (3.1) contains the Coulomb term.

Let the polarization vectors be defined as in table 1. Then cubic symmetry dictates the form of the 24×24 matrix ΔE in the space of two particle states with energy E_0 as in eq. (2.9) to be of the following form:

$$\Delta E = \begin{pmatrix} \Omega & \Xi^T & \Xi & \Omega' & \Xi & \Xi^T \\ \Xi & \Omega & \Xi^T & \Xi^T & \Omega' & \Xi \\ \Xi^T & \Xi & \Omega & \Xi & \Xi^T & \Omega' \\ \Omega^T & \Xi & \Xi^T & \Omega & \Xi^T & \Xi \\ \Xi^T & \Omega^T & \Xi & \Xi & \Omega & \Xi^T \\ \Xi & \Xi^T & \Omega'^T & \Xi^T & \Xi & \Omega \end{pmatrix}. \quad (3.2)$$

The first three rows and columns correspond to transitions between two particle states $|A; \sigma, \sigma'\rangle$ with $A=x, y, z$ and the last three rows and columns correspond to transitions with $A=x', y', z'$ (see eq. (2.8)). The entries in this matrix are 4×4 matrices them-

selves given by the four spin combinations. Due to time-reversal parity and cubic invariance they have the following properties:

$$\Xi_{\tau\bar{\tau},\sigma\bar{\sigma}} = \Xi_{\tau\tau,\sigma\sigma} = \Xi_{\tau\bar{\tau},\bar{\sigma}\sigma}$$

and in the basis in which parity is diagonal ($\sigma\bar{\sigma} = 11, 22, 12-21, 12+21$)

$$\Xi = \begin{pmatrix} a_1 & a_3 & a_4 & 0 \\ a_3 & a_2 & a_5 & 0 \\ -a_4 & -a_5 & a_6 & 0 \\ 0 & 0 & 0 & a_7 \end{pmatrix},$$

$$\Omega = \begin{pmatrix} \omega_1 & \omega_3 & 0 & 0 \\ \omega_3 & \omega_2 & 0 & 0 \\ 0 & 0 & \omega_4 & 0 \\ 0 & 0 & 0 & \omega_5 \end{pmatrix},$$

$$\Omega' = \begin{pmatrix} \omega'_1 & \omega'_3 & 0 & 0 \\ \omega'_3 & \omega'_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \omega'_4 \end{pmatrix}. \tag{3.3}$$

Altogether we have 13 real parameters in the $P = +1$ sector and 3 in the $P = -1$ sector. They determine respectively the 8 levels and 5 mixing angles ($P = +1$) and 3 levels ($P = -1$), where the levels are labelled by the irreducible representations enumerated at the end of section 2.

To order g^2 the graphs are given in fig. 1. The matrix Ω' is zero in this order, and the matrix Ω gets only contributions from the self-energy graphs in fig. 1b; the matrix Ξ gets only contributions from the graphs 1a.

We now proceed to the diagonalization of eq. (3.2) by noting:

(a) $P = \pm 1$ states do not mix; thus ΔE is split into an 18×18 matrix with $P = +1$ and a 6×6 matrix with $P = -1$.

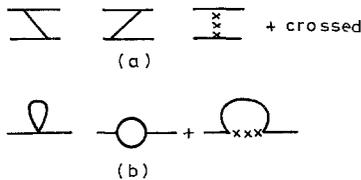


Fig. 1. Graphs to order g^2 . (a) shows the exchange graphs: continuous lines represent gluons, crosses represent the Coulomb force. (b) shows the self-energy graphs.

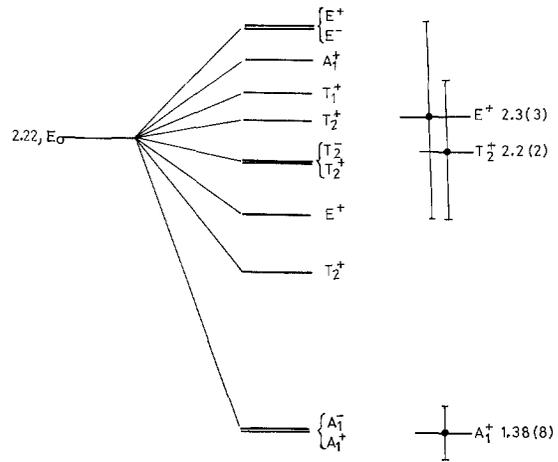


Fig. 2. Splitting of the level E_0 to order g^2 (see eq. (4.1)). The levels are accompanied by the relevant irreducible representation. The results of ref. [5] are also shown. The scale is fixed by fitting to the measured A_1^+ . All numbers refer to masses in lattice spacing units.

(b) The threefold rotations around the $(1, 1, 1)$ axis with eigenvalues w^k ($w = \exp(i2\pi/3)$, $k=0, 1, 2$) serve to block diagonalize the $P = +1$ matrix into three 6×6 matrices.

Then it is an easy matter to diagonalize the remaining 6×6 matrices (see fig. 2 for the result).

Let us note that the self-energy Ω is diagonal in the polarization vectors of table 1. The gluon is polarised along these directions. The self-energy is finite. We also computed the self-energy from the S -matrix using Feynman gauge. The analytic result is the same. Finiteness results here, since we evaluated it on the mass shell ($p^2 = E^2 - p^2 = 0$) and contracted with the polarizations, thereby full projecting out the pole part, which is transverse in the four-dimensional sense. Also the graphs in fig. 1a can be computed from the S -matrix using Feynman gauge, giving the same result. The S -matrix calculation is considerably simpler and becomes compulsive in higher orders.

4. Results and conclusions

Our "mass" formula is of the form

$$E_{[X]} = E_0 + (g^2/12\pi^2 L)X + O(g^4). \tag{4.1}$$

Here $[X]$ labels the irreducible representations of the

cubic group mentioned below eq. (2.9); E_0 is given in eq. (2.9) and the coefficients X are given in table 2. In fig. 2 one can find the splittings compared with unpublished data by Stephenson and Teper [5] for a volume $4^3 \times 16$ at $\beta=2.3$.

Most of the splitting X is accounted for by the splitting X_e due to the exchange graphs (fig. 1a). They leave an accidental degeneracy in the $A_1^+ - A_1^-$, $T_2^+ - T_2^-$ and $E^+ - E^-$ systems (see fig. 1). This is lifted by the difference δ of the two eigenenergies of the gluon, fig. 1b. This splitting is "accidentally" small because δ is small with respect to the amplitude for a two particle state with both spins up going into a two particle state with both spins down. These splittings are less than a percent; the A_1^+ becomes lighter than the A_1^- and so does the T_2^+ versus the T_2^- (see table 2, column 3). The only appreciable effect of the self-energy is an overall shift of -4.94 downwards (see table 2).

To fit the central value of the measured A_1^+ glueball we need a coupling

$$g^2/4\pi = 0.46. \tag{4.2}$$

For this value of $g^2/4\pi$ one sees that the splittings in fig. 2 stay well within the limits set by the unperturbed neighbour levels, i.e. $E_0 = 2\sqrt{6}\pi/L$ and $E_0 = 0$. So perturbation theory may still be applicable.

The data for the T_2^+ and E^+ glueballs are higher than our $O(g^2)$ prediction using eq. (4.2) and table 2. The error bars are large, though. For one thing, our

Table 2

Values of the variable X in eq. (4.1). X is split into two contributions: one, X_e , from exchange graphs and one, X_s , from self-energy graphs (fig. 1) ($X = X_e + X_s$). The overall shift is subtracted from X_s .

[X]	X_e	$X_s + 4.94$
E^+	32	0.06
E^-	32	0
A_1^+	24	0.12
T_1^+	16	0
T_2^+	$-8 + 8\sqrt{5}$	0.19
T_2^-	0	0
T_2^+	0	-0.62
E^+	-12	-0.06
T_2^+	$-8 - 8\sqrt{5}$	-0.10
A_1^-	-64	0
A_1^+	-64	-0.12

calculation shows the resolution needed in the data. One should keep in mind that the $O(g^4)$ corrections to the mass formula (4.1) (which are under way [12]) might be as large as 25%.

Let us point out the salient differences with the perturbative results of the periodic case [7]:

(i) The (E^+, T_2^+) states are nearly degenerate at a value of $m_{A_1} L \equiv z_{A_1} = 5.56$, whereas in the periodic case this happens only at larger z -values.

(ii) The parity minus states are nearly degenerate with the parity plus states. This may change to order g^4 . In the periodic case they are very different because torons do not allow parity minus states [13].

The twisted z_{A_1} variable starts out at $E_0 L = 2\sqrt{2}\pi$ and decreases like $1/|\log AL|$. Eventually it will increase linearly in L . The periodic z_{A_1} variable starts out at zero and grows like $1/|\log AL|^{2/3}$.

Let us conclude with the following recommendations:

(i) More than ever, we need a z variable that does not suffer from intrinsic volume dependence. For a meaningful comparison between various boundary conditions and a trustworthy extrapolation for large L we need a z variable that does not depend on boundary conditions.

(ii) The resolution of the measurements should be smaller by at least a factor four to disentangle the various levels.

(iii) A measurement of the electric flux energies $E(e=(1, 1, 1))$ and $E(e=(1, 1, 0))$ would be quite interesting. We expect them to saturate quite quickly their asymptotic values, $\sqrt{3}$ and $\sqrt{2}$ times the string tension respectively.

(iv) Finally a measurement of the difficulty accessible parity minus states would be most welcome.

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