# Advanced Numerical Techniques and Algorithms to Study Abrupt Climate Change

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Past records show that abrupt climate changes were most common when the climate system was being forced to change most rapidly or when the thermohaline circulation (THC) was weak [1]. So, the question is whether the rapid climate change that we are presently witnessing (Fig. 1), along with model predictions of a weakening of the THC [2], increases the chances of abrupt climate change in the near future.

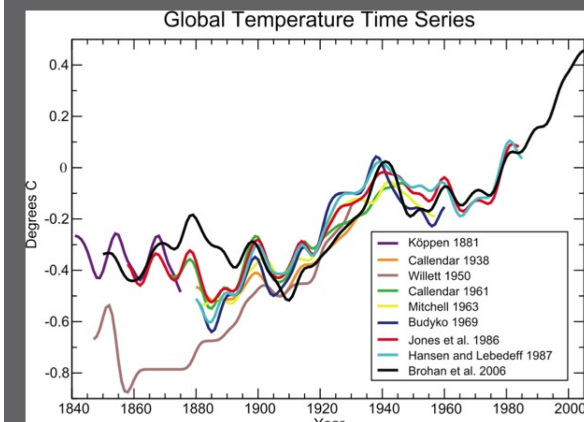


*Fig. 1. The rise in global temperature is one aspect of the unprecedented rapid change in climate that we are presently experiencing. From [1].*

Abrupt climate change is a manifestation of complex nonlinear chaotic behavior in the climate system and occurs when the climate system is forced to cross a threshold, leading to a transition to a new climate state at a rate that is faster than the cause and which is determined internally by the system (e.g., see Fig. 2). Progress was made in understanding the dynamics underlying such complex nonlinear chaotic behavior by using the simplest settings of low-dimensional differential equations and maps, mostly in the 70s and 80s. An extension of techniques that were developed and used for low-dimensional systems (a popular example is the package AUTO) to high-dimensional systems has had to await more recent advances in computational techniques and is only now becoming feasible. To wit, the philosophy and methodology underlying present day oceanic general circulation models (OGCMs), most of which had their origins in the 70s, make them more suited for studying gradual rather than abrupt changes.

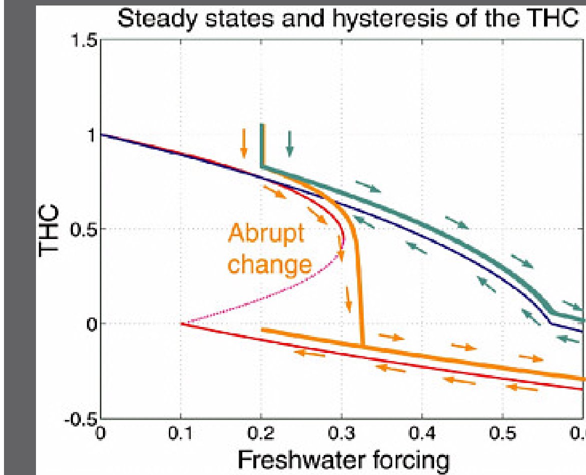


*Fig. 2. Two regimes of Stommel's box model of THC. When diffusion is weak (orange), the model exhibits abrupt change and hysteresis, but not when diffusion is strong (green). From [1].*

In order to help policy makers make informed decisions about safe levels of greenhouse gases in the atmosphere, climate scientists need to provide probabilistic estimates that the climate system will cross some threshold leading to abrupt climate change and probabilistic estimates of the consequences of the resulting climate shift on various aspects of the Earth system. Factors contributing to the uncertainty associated with abrupt climate change involve 1) uncertainty about the location of thresholds as a function of uncertain model parameters and parameterizations, 2) uncertainty about the location of the present climate system in parameter space, and 3) uncertainty about future climate forcing including, for example, future greenhouse gas emissions. Quantifying the risks associated with abrupt climate change therefore requires the systematic exploration of the positions of various nonlinear thresholds as a function of uncertain model parameters and forcing.

Techniques required to study abrupt changes are complementary to those used in present day ocean models. For example, present day OGCMs cannot compute or track unstable equilibria—the simplest dynamical objects that underlie abrupt changes. So also, thermodynamic spinup of present day OGCMs is extremely slow: The time step which is determined by the fastest gravity wave speeds is between a few seconds to a few hours depending on spatial resolution, whereas the times associated with setting up deep circulation is of the order of thousands of years.

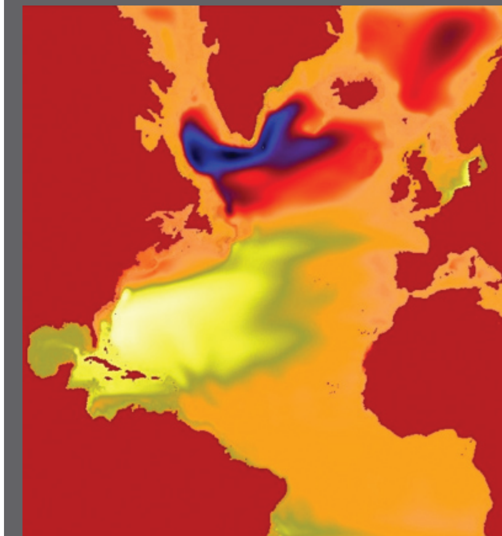

*Fig. 3. Modeled sea surface height in the North Atlantic. With the new techniques, a time step of a day was used. The time step using the traditional methodology is limited to about an hour.*

We have now developed new and complementary numerics based on Jacobian Free Newton Krylov techniques in a popular OGCM that enables studying thresholds and abrupt change scenarios. That is, with this approach, we can now track changes in ocean circulation as key parameters are changed. This approach has been implemented in various other problems of interest to the Laboratory as in [4].

Further, when used in place of traditional time stepping algorithms, this method allows for a time step that is of relevance to the physical phenomenon that is being studied. The time step is not limited by the fastest modes of the system [3]. For example, Fig. 3 shows the sea surface height as modeled in a simplified setup of the North Atlantic. In this case, the maximum timestep in the traditional methodology is an hour, whereas the present computation used a timestep of a day. Other advantages include a consistent and uniform treatment of terms in the governing equations. Work is underway to further improve the efficiency of these schemes. As an example, a high degree of improvement in efficiency is achieved (Fig. 4) by "preconditioning" the gravity waves, which happen to be the fastest waves in the system. Thus with this and other related techniques that we are presently working on, we expect to significantly ameliorate the spinup problem as well.

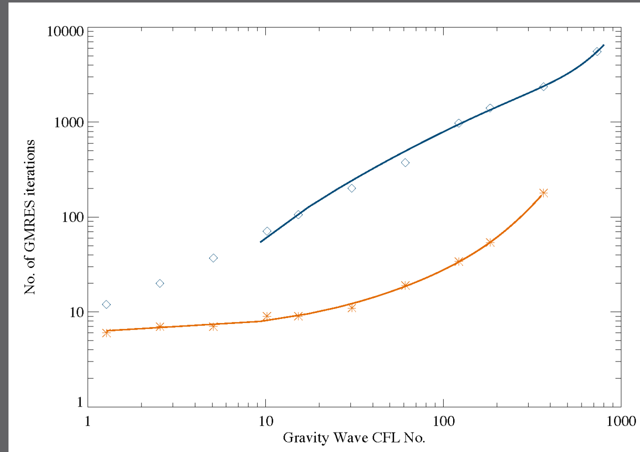


*Fig. 4. Impact of a preconditioner on the efficiency of the Newton-Krylov methodology. The upper line shows the number of iterations of generalized minimal residual method (GMRES) required without the preconditioner and the lower line with the preconditioner. GMRES is a costly component of the scheme.*

[1] Abrupt Climate Change: Inevitable Surprises, National Research Council, (2002).
[2] Intergovernmental Panel on Climate Change, Fourth Assessement Report, (2007); http://www.ipcc.ch
[3] B.T. Nadiga et al., Computer and Mathematical Modeling, 44, 870-886, (2006).
[4] D.A. Knoll et al., Journal of Scientific Computing, 25, 213-30, (2005).

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