Synchrotron Radiation Spectrum For Galactic-Sized Plasma Filaments

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Abstract—**A detailed analysis of the radiation spectrum for synchrotron-emitting electrons in galactic-sized Birkeland current filaments is presented. It is shown that the number of filaments required to thermalize the emission spectrum to blackbody is not reduced when a non-Maxwellian electron distribution is assumed. If the cosmic background radiation (CBR) spectrum (T = 2.76 K) is due to absorption and reemission of radiation from galactic-sized current filaments, higher order synchrotron modes are not as highly self-absorbed as lower-order modes, resulting in a distortion of the blackbody curve at higher frequencies. This is especially true for a non-Maxwellian distribution of electrons for which the emission coefficient at high frequencies is shown to be significantly less than that for a Maxwellian distribution. The deviation of the CBR spectrum in the high-frequency regime may thus be derivable from actual astrophysical parameters, such as filamentary magnetic fields and electron energies in our model.**

I. Introduction

OBSERVATIONS of ordered and luminous structures in the universe [1] have suggested the possibility that large current-carrying plasma filaments of lengths on the order of tens of pc may exist. The slim "threads" [2] are apparently held together, at least in the center of our own galaxy, by a magnetic field possessing both azimuthal and poloidal components. Electron acceleration (to relativistic energies) in these structures yields an electrodynamic picture of electron currents driven along magnetic-field lines and synchrotron-emitting linear filaments approximately 1-lt. yr.-wide [3].

Large-scale field-aligned filamentary structures (width ~ 35 kpc, length ~ 350 Mpc) have also been suggested to account for some aspects of galactic morphology and rotational curves [4]. In this model, field-aligned currents (often called Birkeland currents) are caused by the formation of double layers in adjacent Birkeland-current filaments, with an associated generation of an axial electric field E_z. Electrons accelerated by the electric field produce an axial current I_z. The Biot–Savart electromagnetic force between two interacting filaments is attractive for two similarly oriented currents (Fig. 1) and is proportional to $1/R$, where R is the distance between filaments [5]. Electrons also spiral around the axial magnetic field. This motion produces an azimuthal component of the particle current. The force between two Birkeland filaments with interacting azimuthal current elements is repulsive, since these currents are antiparallel (Fig. 1). This force is proportional to $1/R^3$ [5].

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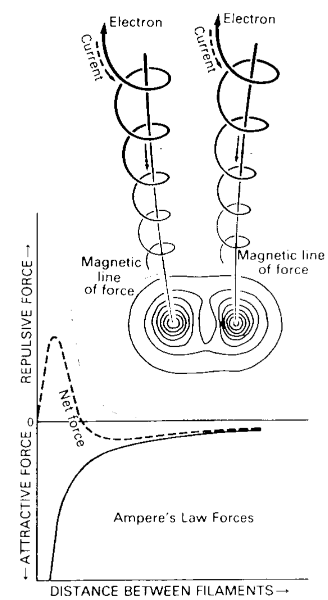

Fig. 1. Schematic drawing of the interaction of two Birkeland filaments, showing the forces between two adjacent filaments. The parallel components of current (dark gray lines) are long-range attractive, while the counter-parallel azimuthal currents (light gray rings) are short-range repulsive. A third force, long-range electrostatic repulsion, is found if the electrons and ions are not present in equal numbers.

The dynamics of the interaction between two field-aligned current-carrying plasma filaments has been simulated by a three-dimensional, fully self-consistent particle-in-cell plasma code [5]. Two interacting filaments will mutually attract until the magnitude of the short range ($\sim R^{-3}$) repulsive force becomes comparable to the long-range ($\sim R^{-1}$) attractive force. It is at this time that the simulations typically show an emissive burst of synchrotron radiation. The emission mechanism is not yet well understood, but may be related to impulsive magnetic forces impressed upon the electrons due to the changing magnetic-field topology during the coalesence phase of two interacting filaments (Fig. 2). The effects may be likened to that of impulsive solar microwave bursts that are believed to be produced by the gyrosynchrotron radiation of electrons accelerated in solar flares [6].

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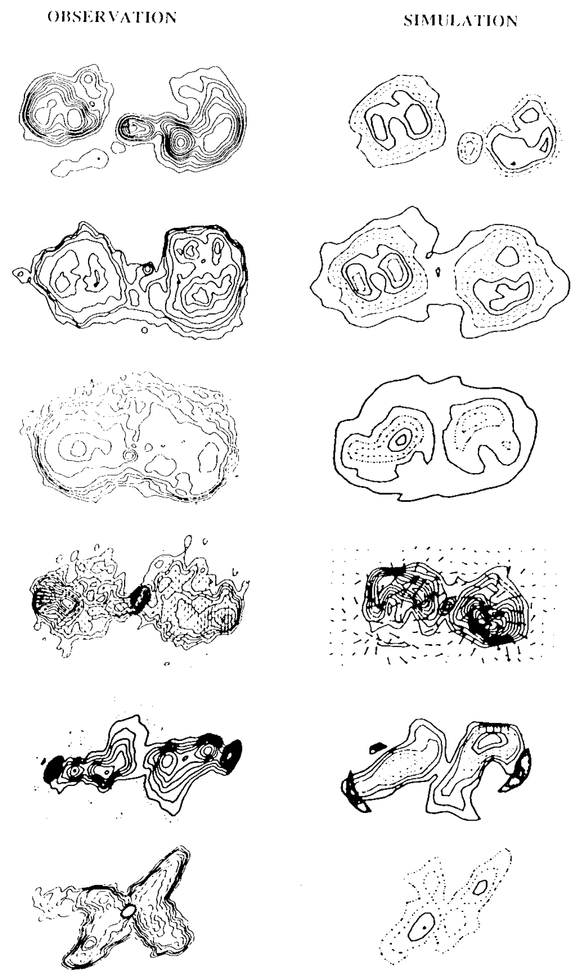

Fig. 2. Coalescence phase of two interacting galactic-sized filaments as described by the synchrotron isophotes at various frequencies of the double radio sources 0844 + 319, Fornax A, 3C192 and 3C315 (left column), and the particle-in-cell simulation analogs at times 10.4 to 58.7 Myr (right column).

From the simulations, the energy density of the intense synchrotron radiation burst is approximately 1.2×10^{-13} erg/cm^3. If the radiation spectrum associated with the synchrotron bursts were blackbody, this energy density would correspond to a temperature of approximately 2 K. The possibility that synchrotron bursts from interacting galactic-sized Birkeland current filaments may be the source of the 2.7 K cosmic microwave background in the universe was first proposed by Peter and Peratt [7]. In this model, the emission spectrum of a burst is just the single-particle spectrum modified by self-absorption effects within the filaments. Such a mechanism can account for the observation of radiation with a blackbody spectrum in the Rayleigh–Jeans regime only if a large number ($>10^{31}$) of filaments is assumed.

In this paper, we present detailed calculations of synchrotron emission and absorption in galactic-sized plasma filaments. The basic model under consideration is pictured in Fig. 1. We assume a large number M of galactic-sized, interacting plasma filaments (axial currents of $I \sim 10^{20}$ A) along the line of sight. Magnetic fields of $B_\theta = 2.5 \times 10^{-4}$ G, $B_z = 2 \times 10^{-4}$ G, and an electric field of $E_z = 60$ mV/m are assumed. For purposes of synchrotron radiation calculations, the azimuthal magnetic field B_θ is neglected. The filamentary plasma is taken to be space-charge neutral with a density $n_i = n_e = 1.8 \times 10^{-3}$ cm^{-3}. From the plasma simulations we have conducted, the longitudinal electron energy is 30 keV, while the mean transverse energy is zero. The temperatures in the transverse and longitudinal direction did not change from their initial value of 3 keV in the simulations.

In the next section we calculate the radiation spectrum along a line of sight due to M filaments. The distribution function of the radiating electrons is assumed to be Maxwellian. This work is a more exact analysis of an earlier study [7]. In Section III we analyze the radiation spectrum of various anisotropic distribution of electrons. Because these distributions are not isotropic Maxwellians, the electrons are not in thermodynamic equilibrium and Kirchhoff's law is not applicable. In the last section the results are discussed in connection to the observed 2.7 K cosmic background radiation spectrum.

II. Vacuum Self-Absorption

For high frequencies such that $\omega^2 >> \omega_p^2$, the real part of the refractive index can be taken to be unity, and the cyclotron emissivity of a plasma is the sum of contributions from each individual electron [8]. A single electron then emits as it does in a vacuum. The typical radiation spectrum of discrete lines at $\omega = m\omega$ (m is an integer) is broadened by summing over a distribution of electrons.

If a photon of frequency ω is now emitted by a gyrating particle, it will be absorbed after traversing a distance $1/\alpha(\omega)$, where $\alpha(\omega)$ is called the absorption coefficient of the plasma. If the system has a scale length L, photons of frequency ω less than some critical frequency ω^* will be trapped within the plasma. If the system obeys Kirchhoff's law, then the plasma will radiate as a blackbody for frequencies $\omega < \omega^*$, and radiate freely without self-absorption for $\omega > \omega^*$.

This analysis can be made more quantitative as follows: In the high-frequency regime, the real part of the refractive index may be considered equal to unity [8], [9]. The radiation of a single electron traveling along a spiral in a magnetic field B is given by [10]

$$\eta_\omega = (e^2\omega^2/2\pi c\omega_b)(1 + p^2)^{-1/2} \cdot \sum_{n=1}^{\infty} \delta[n - \nu\sqrt{1 + p^2}](p_\parallel^2 J_n^2 + p_\perp^2 J_n'^2) \quad (1)$$

where $\nu = \omega/\omega_b$ is the harmonic number, $\omega_b = eB/mc$ is the electron gyrofrequency, and J_n and J_n' are Bessel functions and their derivatives. The momenta $p_\parallel$ and $p_\perp$ are expressed in units of mc. The angle between the direction of radiation and the magnetic field (usually denoted by θ) is assumed to be $\pi/2$. This simplifies the calculations and avoids the relativistic complications of defining the observer's frame of reference for $\theta \neq \pi/2$, which has been a subject of frequent controversy in the literature [11]. Finally, the first term within the parentheses of the sum in (1) corresponds to radiation of linearly polarized waves with $\boldsymbol{E} \parallel \boldsymbol{B}$ (the "ordinary" component); the second term gives the radiation of waves for which $\boldsymbol{E} \perp \boldsymbol{B}$ (the "extraordinary" component).

The emission coefficient is given by the average of η_ω over the distribution and is equal to

$$j_\omega = \int \eta_\omega(\boldsymbol{p}) f(\boldsymbol{p})\, d^3p. \tag{2}$$

The absorption coefficient for a high-frequency plasma (index of refraction ~ unity) is defined as [12]

$$\alpha_\omega = \frac{8\pi^3 c^2}{\omega^2} \int \eta_\omega(\boldsymbol{p}')\,[f(\boldsymbol{p}) - f(\boldsymbol{p}')]\, d^3p'. \tag{3}$$

In the classical regime $h\omega << kT$; this simplifies to

$$\alpha_\omega = -\frac{8\pi^3 c^2}{\omega^2} \int \eta_\omega(\boldsymbol{p}) \frac{\partial f(p)}{\partial \epsilon}\, d^3p \tag{4}$$

where ϵ is the energy. The radiation temperature in the classical regime is then defined formally for an isotropic distribution by

$$kT_r = -\frac{\int \eta_\omega(\boldsymbol{p}) f(p)\, d^3p}{\int \eta_\omega(\boldsymbol{p})\,[\partial f(p)/\partial\epsilon]\, d^3p}. \tag{5}$$

In equilibrium, when the particle distribution is Maxwellian, it is easily shown that the radiation temperature T_r is equal to the particle temperature T. For example, a Maxwellian distribution can be written:

$$f(\boldsymbol{p}) = Nf_0\, e^{-\epsilon/kT} \tag{6}$$

where $\epsilon = (\gamma - 1)mc^2$ and γ is the relativistic factor. The normalization is determined from

$$\int 4\pi p^2 f(p)\, dp = N. \tag{7}$$

For $\mu >> 1$ and for radiation emitted at right angles to the field ($\theta = \pi/2$), the substitution of the distribution function in (7) into (1) and (4) yields a series representation for the absorption coefficient [13],

$$\alpha(\omega, \pi/2) = \frac{\omega_p^2}{c\omega_b} \frac{(\mu/\nu)^{5/2}}{2^{3/2}} \sum_{n \geq \nu}^{\infty} \exp\left[-\mu\left(\frac{n}{\nu} - 1\right)\right] \cdot \left[\left(\frac{n}{\nu}\right)^2 - 1\right]^{1/2} \left(\frac{n/\nu - 1}{n/\nu + 1}\right)^n \times \left\{1 + \frac{(n/\nu)^2 - 1}{2\nu}\right\} \tag{8}$$

where the contribution of both ($\perp$ and $\parallel$) polarizations have been included. For a temperature of 3 keV, the ratio $\mu = mc^2/kT = 167$. The parameter $\omega_p^2/\omega_b c$ is a measure of the optical depth at the cyclotron fundamental. The expression for $\alpha(\omega, \pi/2)$ above involves an infinite series and is often inconvenient for calculations. A more useful form of (8) can be obtained by approximating the sum by an integral for the high harmonic number ν. For $\mu = mc^2/kT >> 1$,

$$\alpha(\omega, \pi/2) = \frac{\omega_p^2}{c\omega_b} \frac{\sqrt{\pi\mu}}{4} (z^2 - 1)^{3/2} \cdot \frac{\mu^2}{x^2} e^{\mu - 2x/(z^2-1)} \left\{1 + \frac{z^2 - 1}{2x}\right\} \tag{9}$$

where $x = \nu/\mu$ and z is determined from the relation

$$\frac{2z}{z^2 - 1} - \ln\frac{z + 1}{z - 1} = \frac{\mu}{x}. \tag{10}$$

The absorption coefficient $\alpha(\omega, \pi/2)$ in (4)–(6) can be explicitly determined in the special case for which $\nu << \mu$:

$$\alpha(\omega, \pi/2) = \frac{\omega_p^2}{c\omega_b} \left(\frac{\pi}{4}\mu e\right)^{1/2} \left(\frac{e\nu}{2\mu}\right)^{\nu - 1/2} [1 + 1/\mu] \tag{11}$$

and for which $y = 9\nu/2\mu >> 1$:

$$\alpha(\omega, \pi/2) = \frac{\omega_p^2}{c\omega_b} \frac{3}{2} \frac{(\pi\mu)^{1/2}}{y} \exp\left[-\mu\left(y^{1/3} - 1 + \frac{9}{20y^{1/3}}\right)\right] \cdot \left[1 + \frac{1}{\mu y^{1/3}}\right]. \tag{12}$$

Chanmugam [14] has compared the results of (11) and (12) with the series derived by Trubnikov (equation (8)) and has found that (11) only gives a crude approximation to the absorption coefficient at low frequencies. Equation (12), however, gives a good approximation to $\alpha(\omega, \pi/2)$ at high frequencies. A typical plot of $\alpha(\omega, \pi/2)$ from (8) is shown in Fig. 3.

Consider a filament of plasma consisting of a homogeneous slab of thickness L with a static uniform axial magnetic field B_0 parallel to the faces of the slab. All other magnetic-field components are neglected. The simplification in treating the filament as a slab instead of a cylinder does not significantly alter the result [15]. In the absence of reflections at the filament boundary, the radiation seen by an observer through the filament is given by

$$\frac{dI}{ds} = -\alpha_\omega(I - S_\omega) \tag{13}$$

where s is the coordinate along the direction of propagation to the observer, and S_ω is the source function $S_\omega = j_\omega/\alpha_\omega$ [10].

If there is now a number M of Birkeland current filaments with identical source functions S_ω at a distance δ apart (Fig. 4), the radiation intensity in the line of sight

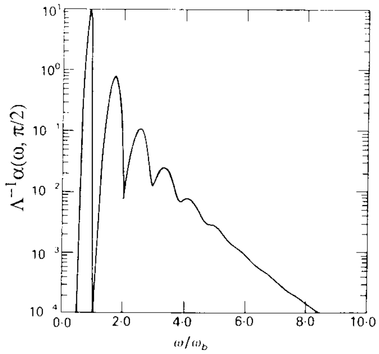

Fig. 3. Plot of $\alpha(\omega, \pi/2)/\Lambda$ versus frequency for a plasma of temperature 30 keV. The quantity Λ was normalized to the width L of the Birkeland filament in calculating the plot.

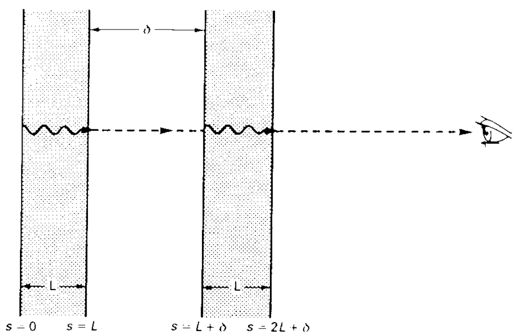

Fig. 4. Schematic drawing of the radiation seen by an observer through two adjacent Birkeland filaments of width L and at a distance δ apart.

to an observer is given by I_M, where

$$I_M = S_\omega[1 - \exp(-M\alpha_\omega L)]. \qquad (14)$$

The argument of the exponential is defined to be the optical depth $\tau = M\alpha(\omega)L$. When $\tau >> 1$ the observer sees a blackbody spectrum, and when $\tau << 1$ the optically thin spectrum of the particle distribution is observed.

For high harmonics of the radiation, the optical depth of the system of M Birkeland current filaments can be determined from (9):

$$\tau = \Lambda M \frac{3}{2} \frac{(\pi\mu)^{1/2}}{y} \exp[-\mu y^{1/3}]$$

where the size parameter $\Lambda = \omega_p^2 L/\omega_b c$.

The optical depth is a function of frequency. Consider, for example, the requirement that the optical depth be unity at $f = 100$ GHz. This is approximately the frequency at the peak of the measured cosmic background radiation spectrum. For $B_z = 2 \times 10^{-4}$ G and a temperature corresponding to $\mu = 167$, the number M of filaments required to thermalize the synchrotron radiation up to 100 GHz is $\ln M \simeq 10^4$. This value is obtained by setting the optical depth τ equal to unity and solving for the number of filaments M. The large magnitude of M is related to the low magnitude of the optical depth τ/M for each individual Birkeland current filament. It is consistent with astronomical observations of low radiation opacity in measurements of galactic spectra. Thus in the case of vacuum self-absorption, a large number of Birkeland current filaments are required to guarantee that the emission spectrum of the interacting galactic-sized filaments will be blackbody up to a frequency of 100 GHz.

III. Non-Maxwellian Distribution Function

In the previous section we investigated the self-absorption of radiation emitted by a collection of electrons obeying a Maxwellian distribution. In general, such a distribution may not be appropriate for the highly directed, nonthermal electrons that are accelerated in Birkeland-current filament regions. A more appropriate distribution function for a mildly relativistic beam ($\gamma^2\beta^2 << 1$) is the bi-Maxwellian distribution discussed in [16]:

$$f(p_\parallel, p_\perp) = n(\pi\alpha_\perp)^{-1}(\pi\alpha_\parallel)^{-1/2} \cdot \exp(-p_\perp^2/\alpha_\perp - p_\parallel^2/\alpha_\parallel) \qquad (15)$$

where $p_\parallel$ ($p_\perp$) is the parallel (perpendicular) particle momentum, and $\alpha_\parallel$ and $\alpha_\perp$ are related to the parallel and perpendicular electron temperatures $T_\parallel$ and $T_\perp$.

For the distribution defined by (15) the system is not in equilibrium, since $T_\parallel \neq T_\perp$. More importantly, the particles are not in equilibrium with the radiation. It is worthwhile to examine the self-absorption characteristics of a particle distribution similar to (15) in order to determine the number of Birkeland filaments required to thermalize the single-particle synchrotron radiation spectrum, and to compare this value with that obtained from a Maxwellian distribution.

Consider now the relativistic distribution function,

$$f(p_\parallel, p_\perp) = f_0 \exp(\gamma mc^2/kT)\,\delta(p_\parallel - p_0) \qquad (16)$$

which is similar to the nonrelativistic distribution function of (15) in the limit that $kT_\parallel/mc^2 \rightarrow 0$. It corresponds to a directed electron beam with $T \equiv T_\perp$ and with a momentum p_0 in the longitudinal direction (along the magnetic field). The quantity $\gamma = \sqrt{1 + p_\parallel^2 + p_\perp^2}$ is the relativistic factor, and the momenta $p_\parallel$ and $p_\perp$ are expressed in units of mc. The constant f_0 is obtained by requiring the normalization,

$$\int f(\boldsymbol{p})\, d^3p = N$$

where N is the total number of electrons. It can be shown then that f_0 is given by

$$f_0 = \frac{N}{(mc)^2}\left(\frac{1}{1 + p_0^2}\right)\frac{\mu'}{1 + 1/\mu'} e^{\mu'} \qquad (17)$$

where we have defined $\mu' = \mu\sqrt{1 + p_0^2}$, and $\mu = mc^2/kT$. Note that in the limit that $p_0 \rightarrow 0$, the distribution function given in (17) reduces to that discussed by Trubnikov and Yakubov [10]. In general, $\mu >> 1$ will be assumed in this work.

An analysis of the spectrum due to synchrotron emission for particles obeying the distribution function (equation (16)) will now be summarized. A detailed analysis is given elsewhere [17]. The emission coefficient for radiation emitted at right angles to the field ($\theta = \pi/2$) is easily calculated to be:

$$j_\omega = \frac{A^*}{m} \sum_{n \geq m\sqrt{1+p_0^2}} \cdot e^{-\mu n/m} \left[p_0^2 J_n^2 + \frac{n^2}{m^2} J_n'^2 \left(1 - \frac{1+p_0^2}{n^2/m^2}\right)\right] \quad (18)$$

where

$$A^* = \left(\frac{Ne^2\omega^2}{2\pi c\omega_b}\right) \frac{1}{1+p_0^2} \frac{\mu'}{1+1/\mu'} e^{\mu'}.$$

The sum over n extends to $n \leq m\sqrt{1+p_0^2}$. The arguments of the Bessel functions in (18) are given by $n\beta_n$, where

$$\beta_n = \left(\frac{m}{n}\right)\sqrt{\left(\frac{n}{m}\right)^2 - 1 - p_0^2} \quad (19)$$

and the prime in (18) denotes differentiation with respect to the argument. Converting the sum in (18) to an integral and using [10] the asymptotic Carlini approximation in the high-frequency regime $n \gg 1$, the expression for j_ω is easily shown to be

$$j_\omega = \frac{A^*(1+2p_0^2)}{2\pi m} \int_1^\infty d\xi \, e^{-\mu' z\xi} \quad (20)$$

where

$$\xi = \frac{n}{m}(1+p_0^2)^{-1/2} \quad (21a)$$

$$z\xi = \xi - 2x + \xi x \ln \frac{\xi+1}{\xi-1} \quad (21b)$$

and we have defined $x \equiv m/\mu$ and $\mu' = \mu\sqrt{1+p_0^2}$. The quantity μ' is typically large for our case ($kT = 3$ keV corresponds to $\mu' \geq 167$) so that the method of steepest descents can be used. The calculation is straightforward, and the result is [10]

$$j_\omega = \frac{Ne^2\omega^2}{2\pi c\omega_b} \frac{1+2p_0^2}{\sqrt{1+p_0^2}} \frac{1}{x} \frac{1}{\sqrt{2\pi\mu'}} \frac{e^{-\mu'(z_0-1)}}{\sqrt{z_0}\sqrt{\cosh t - 1}} \quad (22)$$

where the saddle points are determined from the equation,

$$\sinh t = t + \frac{1}{x} \quad (23)$$

and the following definition was used:

$$z_0 = x(\cosh t - 1). \quad (24)$$

The absorption coefficient α_ω^i for each component $i = [o, e]$ of the ordinary and extraordinary waves is given in general by

$$\alpha_\omega^i = \frac{8\pi^3 c^2}{\omega^2} \int \eta_\omega^i \frac{f(\epsilon', p_\parallel') - f(\epsilon, p_\parallel)}{\hbar\omega} d^3p \quad (25)$$

where $f = f(\epsilon, p_\parallel)$ is a distribution function which is an explicit function of the energy $\epsilon = \gamma mc^2$ and longitudinal momentum $p_\parallel$ only. For a plasma with a Maxwellian distribution, (25) reduces to [10]:

$$\alpha_\omega^i = \frac{8\pi^3 c^2}{\omega^2} j_\omega^i \left(\frac{\exp(\hbar\omega/kT) - 1}{\hbar\omega}\right) \simeq \frac{8\pi^3 c^2}{\omega^2 kT} j_\omega^i. \quad (26)$$

At low frequencies ($\hbar\omega \ll kT$), (26) corresponds to the Rayleigh–Jeans flux, $I_{RJ} = \omega^2 kT/8\pi^3 c^2$. In the classical limit ($\hbar \to 0$), (25) becomes

$$\alpha_\omega^i = \frac{8\pi^3 c^2}{\omega^2} \int \eta_\omega^i \left(-\frac{\partial f}{\partial \epsilon} - \frac{\cos\theta}{c} \frac{\partial f}{\partial p_\parallel}\right) dp. \quad (27)$$

The radiation intensities $\eta_\omega^{(o)}$ and $\eta_\omega^{(e)}$ for a single electron are not known separately, but the sum $\eta_\omega = \eta_\omega^{(o)} + \eta_\omega^{(e)}$ of the ordinary and extraordinary components is given by (1). The absorption coefficient $\alpha_\omega = \alpha_\omega^{(o)} + \alpha_\omega^{(e)}$ can then be shown to be equal to

$$\alpha_\omega = \frac{8\pi^3 c^2}{\omega^2} \left[-\frac{j\omega}{kT} - \frac{\cos\theta}{c} \int \eta_\omega \frac{\partial f}{\partial p_\parallel} dp \right]. \quad (28)$$

When $\theta = \pi/2$, the ratio j_ω/α_ω is exactly equal to the Rayleigh–Jeans flux, and we recover Kirchhoff's law:

$$\alpha_\omega = \frac{8\pi^3 c^2}{\omega^2 kT} j_\omega. \quad (29)$$

Note that the temperature T appearing in (29) is the perpendicular electron temperature of the distribution function defined in (15), and is not the usual temperature of a system in equilibrium which Kirchhoff's law (equation (29)) usually describes.

A comparison can now be made of the relative magnitudes for the absorption coefficients (or, alternatively, the optical depths) of a Birkeland filament containing radiating electrons which obey either a Maxwellian or a non-Maxwellian electron distribution function. At $\theta = \pi/2$, the emission coefficient j_ω^M for a Maxwellian distribution is given by [10]

$$j_\omega^M = \frac{Ne^2\omega^2}{2\pi c\omega_b} \frac{1}{\sqrt{\cosh t - 1}} \frac{1}{x} \frac{e^{-\mu(z_0-1)}}{z_0\sqrt{2\pi\mu}} \quad (30)$$

where t and z_0 are defined by (23) and (24). The ratio of the two emission coefficients is from (22) and (30):

$$\frac{j_\omega}{j_\omega^M} = \sqrt{z_0} \frac{1+2p_0^2}{\sqrt{1+p_0^2}} \exp\left[-\mu(z_0 - 1)(\sqrt{1+p_0^2} - 1)\right]. \quad (31)$$

For $p_0 \to 0$, (31) becomes

$$\frac{j_\omega}{j_\omega^M} = \sqrt{z_0}. \quad (32)$$

Note from (23) and (24) that $z_0 \geq 1$, so that $j_\omega \geq j_\omega^M$. For high mode numbers such that $x \equiv m/\mu \gg 1$, one obtains $t \sim (6/x)^{1/3}$ and $z_0 \sim x^{1/3}$. A plot of z_0 versus t (valid for all values of x) is shown in Fig. 5. From (31)

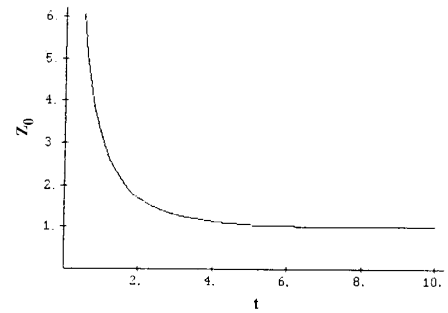

Fig. 5. Plot of the quantity z_0 versus t. The magnitude of z_0 is related to the ratio of emission coefficients of a non-Maxwellian distribution to that of a Maxwellian distribution (cf. equation (32)).

for $p_0 \to 0$, it is seen that the emission coefficient for the distribution of electrons described by (16) is larger than that for a Maxwellian plasma.

For a finite particle velocity ($p_0 \neq 0$) along the field lines the situation is quite different, as is easily seen in (31). For a larger emission coefficient, the absorption coefficient defined by Kirchhoff's law (cf. (29)) is also larger. Care must be taken at extrapolating this result to non-Maxwellian electron distributions, however, since Kirchhoff's law is not always valid.

Consider the case of 30 keV electrons for which $\gamma = 1.06$, so that $p_0 = \gamma\beta \approx 0.352$. In this parameter regime almost all modes ($m \geq 2$) have $j_\omega << j_\omega^M$. For the lower frequencies, $j_\omega \geq j_\omega^M$. In general, as the parameter $\mu = kT/mc^2$ decreases, an increasing number of modes satisfies the relation $j_\omega \geq j_\omega^M$. A non-Maxwellian distribution of radiating electrons is thus more likely to radiate as a blackbody in the low-frequency regime than a Maxwellian distribution. For higher frequencies, the converse is true.

IV. Discussion

Recent measurements of the cosmic microwave background radiation over a wide range of frequencies have confirmed that there exists a significant distortion of the perfect blackbody spectrum in the submillimeter waveband [18], [19]. A number of possible explanations have been put forward for this distortion. One possibility, based on the Sunyaev–Zeldovich effect [20], suggests that microwave photons interact with hot gas to distort the cosmic background radiation (CBR) spectrum [19]–[21]. Another interpretation is that the spectrum can be matched by summing two independent radiation fields, one blackbody, and one graybody [22]. Still another scenario [23] suggests that reradiation from cosmological dust from population III stars in the universe would show up as excess radiation in the far infrared. This is similar to an older model by Hoyle *et al.* [24], [25], who had invoked a model based on a universe of interstellar needle-like grains to account for a steady-state etiology for the CBR.

All of these theories are not without difficulties. For example, the inverse Compton scattering model described by Sunyaev and Zeldovich [20] would distort the observed blackbody curve by lowering the number of photons (i.e., cooler temperatures) in the Rayleigh–Jeans regime and raising the number of photons (i.e., higher temperatures) in the Wien regime. Temperatures for the CBR as low as 2.6 K are needed in the Rayleigh–Jeans (i.e., classical) regime. This is too low to agree with the actual measurements. Furthermore, the mechanism for heating the gaseous medium to the required high temperature is still not understood. Two recent proposals are stellar explosions with a nonstandard stellar mass function [26] and superconducting cosmic strings [27].

In the model described in this paper, large filamentary structures (~ 350 Mpc long) interact to cause bursts of synchrotron radiation from electrons accelerated along magnetic field lines. The synchrotron radiation is thermalized by the radiating electrons themselves and perhaps by ambient thermal plasma in the Birkeland current filaments. For a large number M of these filaments, the optical depth τ of radiation passing along a given line of sight to an observer can be shown to be sufficient to cause the observed spectrum to be blackbody in the classical Rayleigh–Jeans part of the curve.

In laboratory filamentary plasmas a similar phenomenon occurs for high-current discharges, such as low-inductance vacuum sparks and the plasma focus (or exploding wire) [28]. In all these cases the plasma currents are held together (or "pinched") by their self-magnetic field in the azimuthal direction. Another recent application of the study of plasma filaments to astrophysics has been the hypothesis that there is a pinched plasma mechanism to the origin of galactic cosmic rays. This hypothesis, by Vlasov *et al.* [29], appears to be in good agreement with the observations. Finally, it is interesting to note that radiative bursts from laboratory-produced plasma filaments occur over a broad spectral range, from the microwave region to the hard X-ray region.

One feature of the CBR model described here is that the radiation spectrum need not be blackbody for all frequencies, but only blackbody up to a given critical frequency. This critical frequency is a function of the distribution of radiating electrons, and also on parameters such as the plasma density and magnetic field within the Birkeland current filament. Another feature is that the model is based on observable physical properties of plasma that are relatively understood in the laboratory and not on exotic proposals of very massive objects or superconducting cosmic strings. In addition, because each current filament has a rather small optical depth, it is not hard to reconcile the presence of these filaments to known observations.

The calculations here do not address the problem of predicting the CBR spectrum for high frequencies in the Wien regime $h\nu \geq kT$. In this frequency range, the physics is complicated by quantum effects and the calculations are quite involved. As was seen in Section III for $h\nu < kT$, the lower-order modes are self-absorbed more readily into a blackbody spectrum than the higher-order modes, especially for non-Maxwellian distribution functions. Another feature of the high-frequency regime $h\nu \geq kT$ is that stimulated emission is relatively unimportant. If the spectrum would be indicative of single-particle emission (as opposed to that of blackbody), it could be a useful way to determine critical filament parameters such as magnetic fields and electron velocities.

In conclusion, the possibility that the cosmic background radiation spectrum is due to self-absorption of synchrotron radiation within large plasma filaments has been examined. It has been determined that in general a very large number of filaments is required to ensure the radiation spectrum to be blackbody up to a frequency ~ 100 GHz. For the lower frequencies of the Rayleigh–Jeans spectrum, non-Maxwellian distribution functions appear to be more efficient in thermalizing the radiation than Maxwellian distributions. For higher-order modes, the optical depth of a typical non-Maxwellian electron distribution function is much smaller than the corresponding Maxwellian value. Ambient thermal plasma effects are relatively unimportant for the higher-order modes and for reducing the number M of the required filaments. Predictions in the Wien regime $h\nu \geq kT$ require a nonclassical treatment. Calculations in this frequency regime are planned for future studies.

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